AAPT UNITED STATES PHYSICS TEAM
AIP 2014

USA Physics Olympiad Exam

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Important Instructions for the Exam Supervisor

- This examination consists of two parts.
- Part A has four questions and is allowed 90 minutes.
- Part B has two questions and is allowed 90 minutes.
- The first page that follows is a cover sheet. Examinees may keep the cover sheet for both parts of the exam.
- The parts are then identified by the center header on each page. Examinees are only allowed to do one part at a time, and may not work on other parts, even if they have time remaining.
- Allow 90 minutes to complete Part A. Do not let students look at Part B. Collect the answers to Part A before allowing the examinee to begin Part B. Examinees are allowed a 10 to 15 minutes break between parts A and B.
- Allow 90 minutes to complete Part B. Do not let students go back to Part A.
- Ideally the test supervisor will divide the question paper into 4 parts: the cover sheet (page 2), Part A (pages 3-5), Part B (pages 7-8), and several answer sheets for two of the questions in part A (pages 10-13). Examinees should be provided parts A and B individually, although they may keep the cover sheet. The answer sheets should be printed single sided!
- The supervisor *must* collect all examination questions, including the cover sheet, at the end of the exam, as well as any scratch paper used by the examinees. Examinees may *not* take the exam questions. The examination questions may be returned to the students after April 15, 2014.
- Examinees are allowed calculators, but they may not use symbolic math, programming, or graphic features of these calculators. Calculators may not be shared and their memory must be cleared of data and programs. Cell phones, PDA's or cameras may not be used during the exam or while the exam papers are present. Examinees may not use any tables, books, or collections of formulas.

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INSTRUCTIONS

DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

- Work Part A first. You have 90 minutes to complete all four problems. Each question is worth 25 points. Do not look at Part B during this time.
- After you have completed Part A you may take a break.
- Then work Part B. You have 90 minutes to complete both problems. Each question is worth 50 points. Do not look at Part A during this time.
- Show all your work. Partial credit will be given. Do not write on the back of any page. Do not write anything that you wish graded on the question sheets.
- Start each question on a new sheet of paper. Put your AAPT ID number, your name, the question number and the page number/total pages for this problem, in the upper right hand corner of each page. For example,

  AAPT ID #
  Doe, Jamie
  A1 - 1/3

- A hand-held calculator may be used. Its memory must be cleared of data and programs. You may use only the basic functions found on a simple scientific calculator. Calculators may not be shared. Cell phones, PDA's or cameras may not be used during the exam or while the exam papers are present. You may not use any tables, books, or collections of formulas.
- Questions with the same point value are not necessarily of the same difficulty.
- **In order to maintain exam security, do not communicate any information about the questions (or their answers/solutions) on this contest until after April 15, 2014.**

Possibly Useful Information. You may use this sheet for both parts of the exam.

$g = 9.8\ \mathrm{N/kg}$	$G = 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$
$k = 1/4\pi\epsilon_0 = 8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2}$	$k_\mathrm{m} = \mu_0/4\pi = 10^{-7}\ \mathrm{T \cdot m/A}$
$c = 3.00 \times 10^8\ \mathrm{m/s}$	$k_\mathrm{B} = 1.38 \times 10^{-23}\ \mathrm{J/K}$
$N_\mathrm{A} = 6.02 \times 10^{23}\ (\mathrm{mol})^{-1}$	$R = N_\mathrm{A} k_\mathrm{B} = 8.31\ \mathrm{J/(mol \cdot K)}$
$\sigma = 5.67 \times 10^{-8}\ \mathrm{J/(s \cdot m^2 \cdot K^4)}$	$e = 1.602 \times 10^{-19}\ \mathrm{C}$
$1\mathrm{eV} = 1.602 \times 10^{-19}\ \mathrm{J}$	$h = 6.63 \times 10^{-34}\ \mathrm{J \cdot s} = 4.14 \times 10^{-15}\ \mathrm{eV \cdot s}$
$m_e = 9.109 \times 10^{-31}\ \mathrm{kg} = 0.511\ \mathrm{MeV/c^2}$	$(1+x)^n \approx 1 + nx$ for $\lvert x \rvert \ll 1$
$\sin\theta \approx \theta - \frac{1}{6}\theta^3$ for $\lvert\theta\rvert \ll 1$	$\cos\theta \approx 1 - \frac{1}{2}\theta^2$ for $\lvert\theta\rvert \ll 1$

Part A

Question A1

Inspired by: http://www.wired.com/wiredscience/2012/04/a-leaning-motorcycle-on-a-vertical-wall/

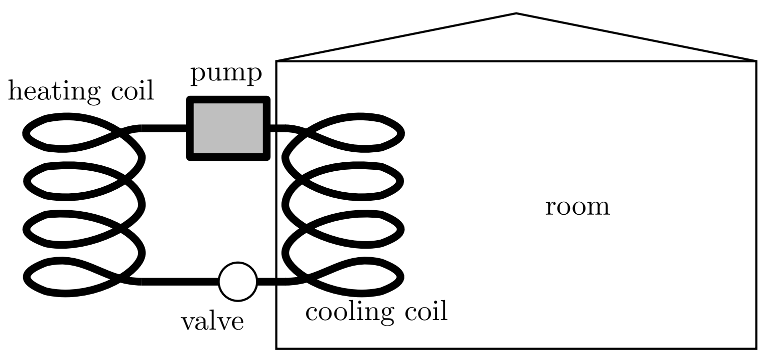
A unicyclist of total height h goes around a circular track of radius R while leaning inward at an angle θ to the vertical. The acceleration due to gravity is g.

a. Suppose $h \ll R$. What angular velocity ω must the unicyclist sustain?

b. Now model the unicyclist as a uniform rod of length h, where h is less than R but not negligible. This refined model introduces a correction to the previous result. What is the new expression for the angular velocity ω? Assume that the rod remains in the plane formed by the vertical and radial directions, and that R is measured from the center of the circle to the point of contact at the ground.

Question A2

A room air conditioner is modeled as a heat engine run in reverse: an amount of heat Q_L is absorbed from the room at a temperature T_L into cooling coils containing a working gas; this gas is compressed adiabatically to a temperature T_H; the gas is compressed isothermally in a coil *outside* the house, giving off an amount of heat Q_H; the gas expands adiabatically back to a temperature T_L; and the cycle repeats. An amount of energy W is input into the system every cycle through an electric pump. This model describes the air conditioner with the best possible efficiency.



Assume that the outside air temperature is T_H and the inside air temperature is T_L. The air-conditioner unit consumes electric power P. Assume that the air is sufficiently dry so that no condensation of water occurs in the cooling coils of the air conditioner. Water boils at 373 K and freezes at 273 K at normal atmospheric pressure.

a. Derive an expression for the maximum rate at which heat is removed from the room in terms of the air temperatures T_H, T_L, and the power consumed by the air conditioner P. Your derivation must refer to the entropy changes that occur in a Carnot cycle in order to receive full marks for this part.

b. The room is insulated, but heat still passes into the room at a rate $R = k\Delta T$, where ΔT is the temperature difference between the inside and the outside of the room and k is a constant. Find the coldest possible temperature of the room in terms of T_H, k, and P.

c. A typical room has a value of $k = 173\ \text{W}/^\circ\text{C}$. If the outside temperature is 40°C, what minimum power should the air conditioner have to get the inside temperature down to 25°C?

Question A3

When studying problems in special relativity it is often the invariant distance Δs between two events that is most important, where Δs is defined by

$$(\Delta s)^2 = (c\Delta t)^2 - \left[(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2\right]$$

where $c = 3 \times 10^8$ m/s is the speed of light.[1]

a. Consider the motion of a projectile launched with initial speed v_0 at angle of θ_0 above the horizontal. Assume that g, the acceleration of free fall, is constant for the motion of the projectile.

   i. Derive an expression for the invariant distance of the projectile as a function of time t as measured from the launch, assuming that it is launched at $t = 0$. Express your answer as a function of any or all of θ_0, v_0, c, g, and t.

   ii. The radius of curvature of an object's trajectory can be estimated by assuming that the trajectory is part of a circle, determining the distance between the end points, and measuring the maximum height above the straight line that connects the endpoints. Assuming that we mean "invariant distance" as defined above, find the radius of curvature of the projectile's trajectory as a function of any or all of θ_0, v_0, c, and g. Assume that the projectile lands at the same level from which it was launched, and assume that the motion is *not* relativistic, so $v_0 \ll c$, and you can neglect terms with v/c compared to terms without.

b. A rocket ship far from any gravitational mass is accelerating in the positive x direction at a constant rate g, as measured by someone *inside* the ship. Spaceman Fred at the right end of the rocket aims a laser pointer toward an alien at the left end of the rocket. The two are separated by a distance d such that $dg \ll c^2$; you can safely ignore terms of the form $(dg/c^2)^2$.

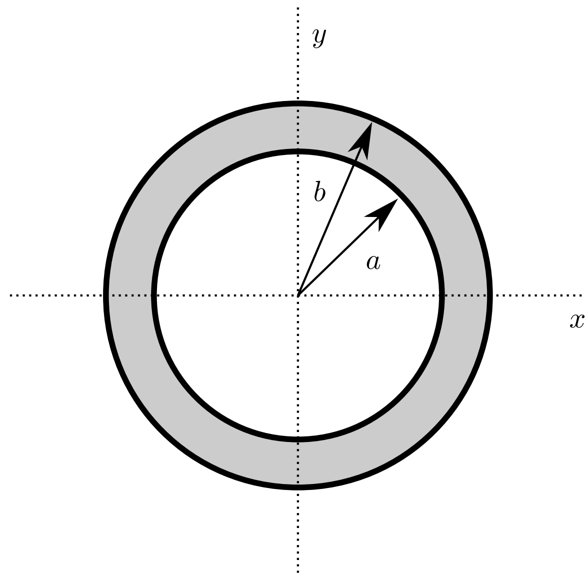
   i. Sketch a graph of the motion of both Fred and the alien on the space-time diagram provided in the answer sheet. The graph is *not* meant to be drawn to scale. Note that t and x are reversed from a traditional graph. Assume that the rocket has velocity $v = 0$ at time $t = 0$ and is located at position $x = 0$. Clearly indicate any asymptotes, and the slopes of these asymptotes.

   ii. If the frequency of the laser pointer as measured by Fred is f_1, determine the frequency of the laser pointer as observed by the alien. It is reasonable to assume that $f_1 \gg c/d$.

[1] We are using the convention used by Einstein

Question A4

A positive point charge q is located inside a neutral hollow spherical conducting shell. The shell has inner radius a and outer radius b; $b - a$ is not negligible. The shell is centered on the origin.



a. Assume that the point charge q is located at the origin in the very center of the shell.

   i. Determine the magnitude of the electric field outside the conducting shell at $x = b$.

   ii. Sketch a graph for the magnitude of the electric field along the x axis on the answer sheet provided.

   iii. Determine the electric potential at $x = a$.

   iv. Sketch a graph for the electric potential along the x axis on the answer sheet provided.

b. Assume that the point charge q is now located on the x axis at a point $x = 2a/3$.

   i. Determine the magnitude of the electric field outside the conducting shell at $x = b$.

   ii. Sketch a graph for the magnitude of the electric field along the x axis on the answer sheet provided.

   iii. Determine the electric potential at $x = a$.

   iv. Sketch a graph for the electric potential along the x axis on the answer sheet provided.

   v. Sketch a figure showing the electric field lines (if any) inside, within, and outside the conducting shell on the answer sheet provided. You should show at least eight field lines in any distinct region that has a non-zero field.

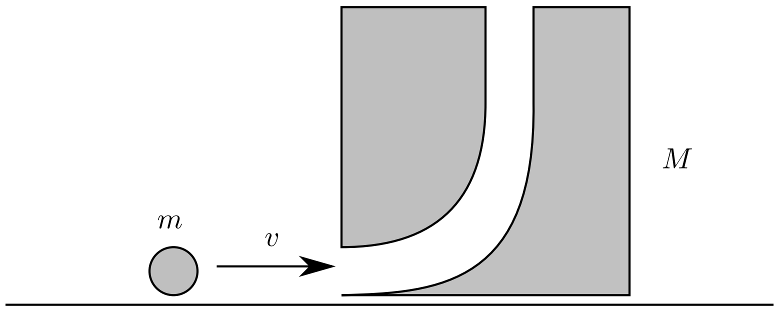
STOP: Do Not Continue to Part B

If there is still time remaining for Part A, you should review your work for Part A, but do not continue to Part B until instructed by your exam supervisor.

Part B

Question B1

A block of mass M has a hole drilled through it so that a ball of mass m can enter horizontally and then pass through the block and exit vertically upward. The ball and block are located on a frictionless surface; the block is originally at rest.



a. Consider the scenario where the ball is traveling horizontally with a speed v_0. The ball enters the block and is ejected out the top of the block. Assume there are no frictional losses as the ball passes through the block, and the ball rises to a height much higher than the dimensions of the block. The ball then returns to the level of the block, where it enters the top hole and then is ejected from the side hole. Determine the time t for the ball to return to the position where the original collision occurs in terms of the mass ratio $\beta = M/m$, speed v_0, and acceleration of free fall g.

b. Now consider friction. The ball has moment of inertia $I = \frac{2}{5}mr^2$ and is originally not rotating. When it enters the hole in the block it rubs against one surface so that when it is ejected upwards the ball is rolling without slipping. To what height does the ball rise above the block?

Question B2

In parts a and b of this problem assume that velocities v are much less than the speed of light c, and therefore ignore relativistic contraction of lengths or time dilation.

a. An infinite uniform sheet has a surface charge density σ and has an infinitesimal thickness. The sheet lies in the xy plane.

   i. Assuming the sheet is at rest, determine the electric field $\vec{\mathbf{E}}$ (magnitude and direction) above and below the sheet.

   ii. Assuming the sheet is moving with velocity $\vec{\mathbf{v}} = v\hat{\mathbf{x}}$ (parallel to the sheet), determine the electric field $\vec{\mathbf{E}}$ (magnitude and direction) above and below the sheet.

   iii. Assuming the sheet is moving with velocity $\vec{\mathbf{v}} = v\hat{\mathbf{x}}$, determine the magnetic field $\vec{\mathbf{B}}$ (magnitude and direction) above and below the sheet.

   iv. Assuming the sheet is moving with velocity $\vec{\mathbf{v}} = v\hat{\mathbf{z}}$ (perpendicular to the sheet), determine the electric field $\vec{\mathbf{E}}$ (magnitude and direction) above and below the sheet.

   v. Assuming the sheet is moving with velocity $\vec{\mathbf{v}} = v\hat{\mathbf{z}}$, determine the magnetic field $\vec{\mathbf{B}}$ (magnitude and direction) above and below the sheet.

b. In a certain region there exists only an electric field $\vec{\mathbf{E}} = E_x\hat{\mathbf{x}} + E_y\hat{\mathbf{y}} + E_z\hat{\mathbf{z}}$ (and no magnetic field) as measured by an observer at rest. The electric and magnetic fields $\vec{\mathbf{E}}'$ and $\vec{\mathbf{B}}'$ as measured by observers in motion can be determined entirely from the local value of $\vec{\mathbf{E}}$, regardless of the charge configuration that may have produced it.

   i. What would be the observed electric field $\vec{\mathbf{E}}'$ as measured by an observer moving with velocity $\vec{\mathbf{v}} = v\hat{\mathbf{z}}$?

   ii. What would be the observed magnetic field $\vec{\mathbf{B}}'$ as measured by an observer moving with velocity $\vec{\mathbf{v}} = v\hat{\mathbf{z}}$?

c. An infinitely long wire wire on the z axis is composed of positive charges with linear charge density λ which are at rest, and negative charges with linear charge density $-\lambda$ moving with speed v in the z direction.

   i. Determine the electric field $\vec{\mathbf{E}}$ (magnitude and direction) at points outside the wire.

   ii. Determine the magnetic field $\vec{\mathbf{B}}$ (magnitude and direction) at points outside the wire.

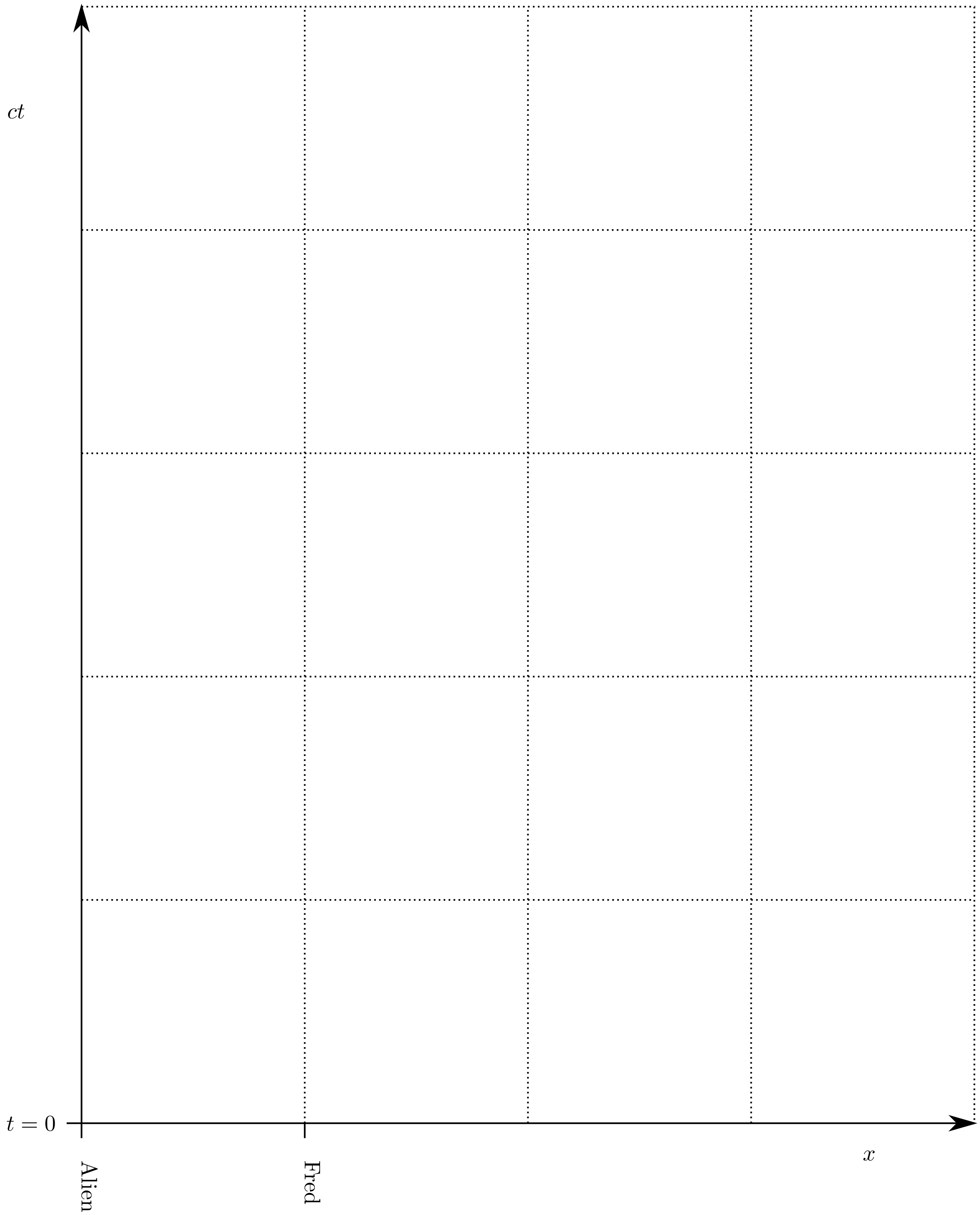
   iii. Now consider an observer moving with speed v parallel to the z axis so that the negative charges appear to be at rest. There is a symmetry between the electric and magnetic fields such that a variation to your answer to part b can be applied to the magnetic field in this part. You will need to change the multiplicative constant to something dimensionally correct and reverse the sign. Use this fact to find and describe the electric field measured by the moving observer, and comment on your result. (Some familiarity with special relativity can help you verify the direction of your result, but is not necessary to obtain the correct answer.)

Answer Sheets

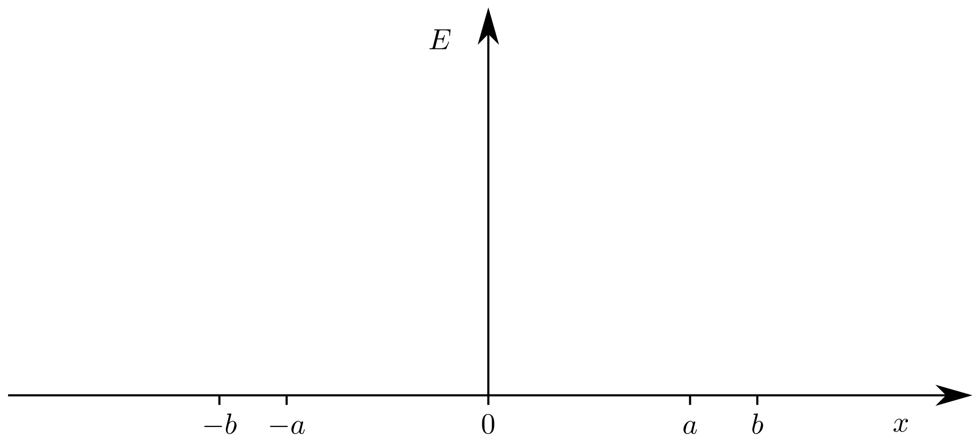
Following are answer sheets for some of the graphical portions of the test.

Answer for Part A, Question 3

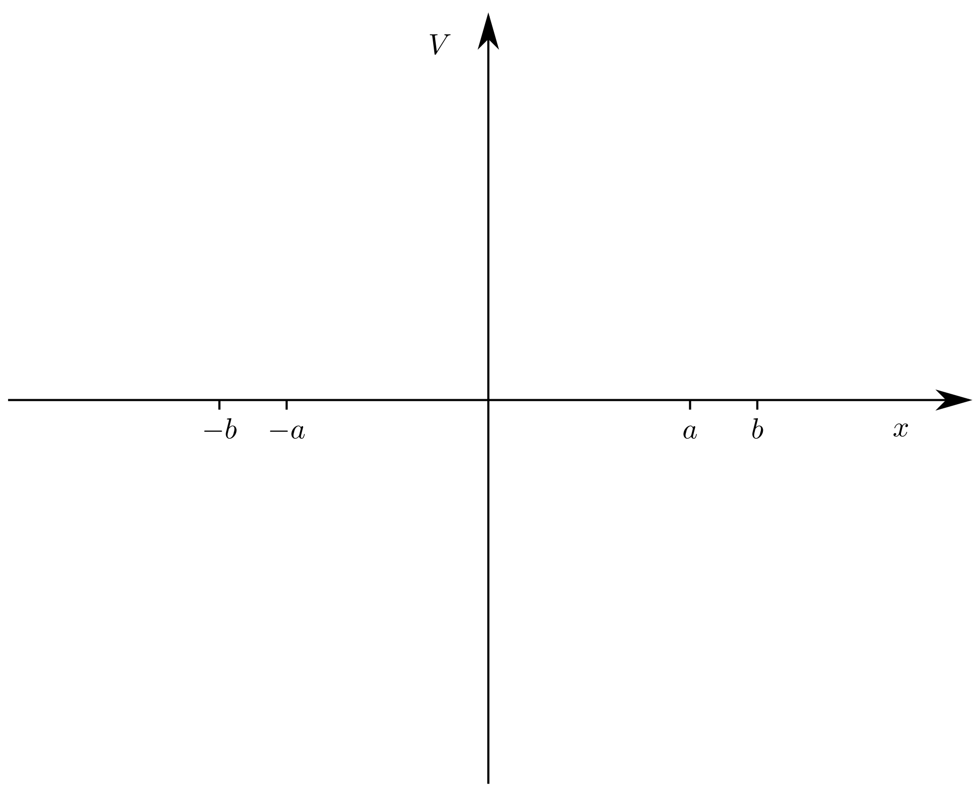
Space-time graph for accelerated rocket. The positions of Fred and the Alien at $t = 0$ are shown.



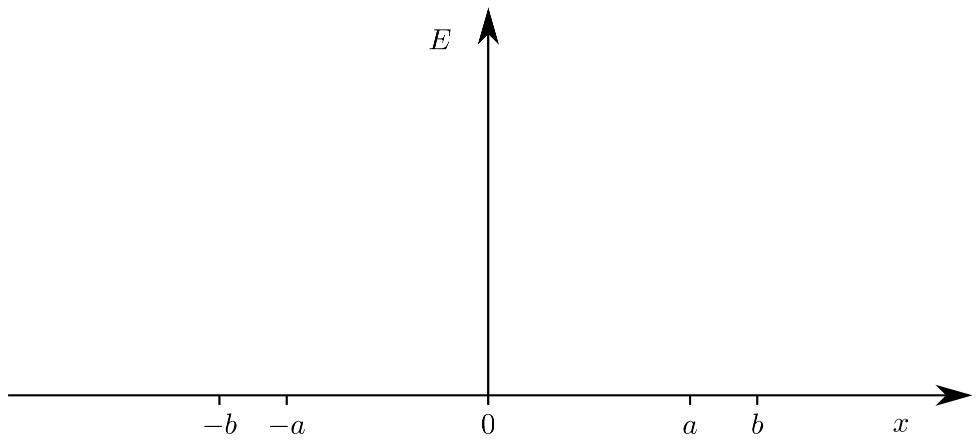
Answer for Part A, Question 4



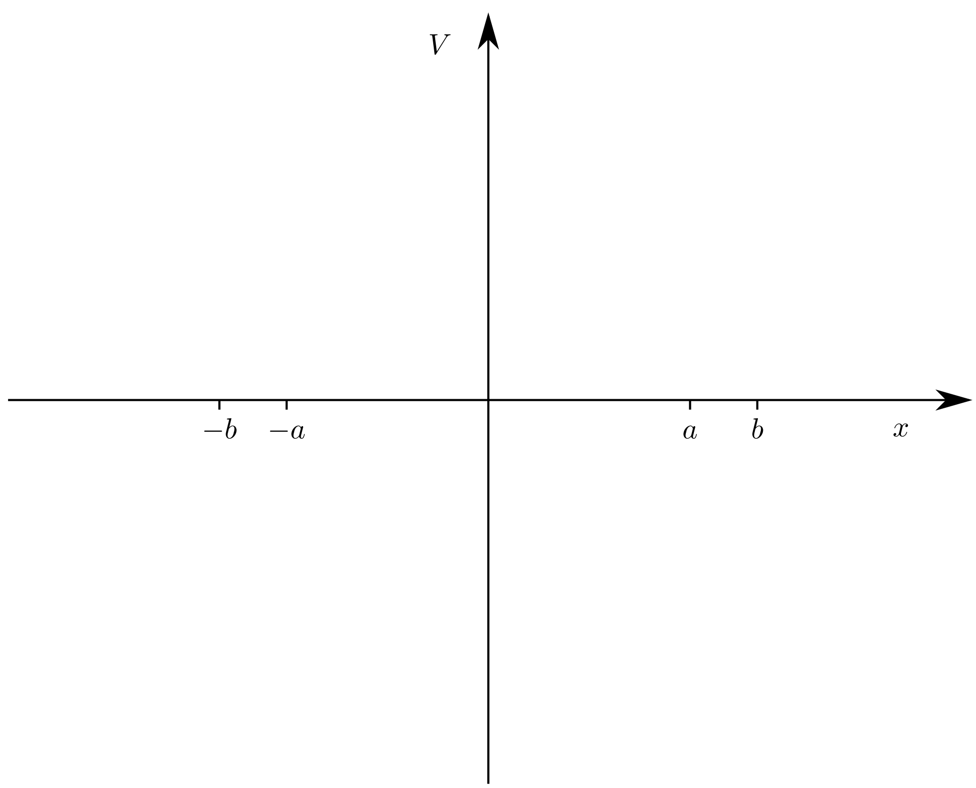
Answer for Part A, Question 4



Answer for Part A, Question 4



Answer for Part A, Question 4



Answer for Part A, Question 4