Cooperative Problem Solving in Physics
A User's Manual

Why? What? How?

STEP 1 — **Recognize the Problem**
What's going on?

STEP 2 — **Describe the problem in terms of the field**
What does this have to do with ...... ?

STEP 3 — **Plan a solution**
How do I get out of this?

STEP 4 — **Execute the plan**
Let's get an answer

STEP 5 — **Evaluate the solution**
Can this be true?

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Acknowledgments

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Finally we would like to thank Arnold Arons for his inspiration and guidance that has lasted a lifetime.

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Introduction

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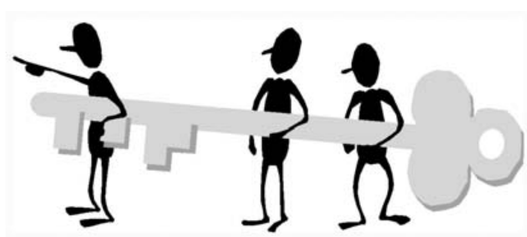
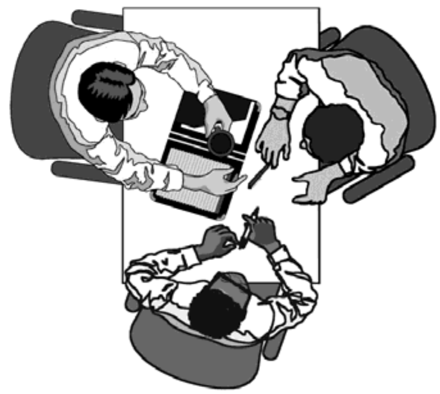
- ✓ Steps needed to adapt CPS.
- ✓ What to expect after implementing CPS.
- ✓ Our "Laws of Instruction" and frequently used icons in this book.

The most effective teaching method depends on the specific goals of a course, the inclination of the instructor and the needs of the students, bound by the constraints imposed by the situation. There is no known "best" way to teach. Cooperative Problem Solving is one teaching tool that may fit your situation. It is not the "magic bullet" that, by itself, will assure that all of your students achieve your goals for the course. It is, however, based on a solid research foundation from cognitive psychology, education, and physics education. We have over two decades of experience testing and refining Cooperative Problem Solving at it is used by many professors teaching thousands of students and different institutions. Cooperative Problem Solving can be used as the major focus of a course, or as a supplement in combination with other teaching tools.



What is Cooperative Problem Solving (CPS)? This book is designed to answer this question. First lets describe what you would see if you observed a 50-minute class engaged in Cooperative Problem Solving. As they walk into the class, students sit in groups of three, facing each other and talking. The instructor begins class by talking about 5 minutes, setting the goal for this session. For example, the instructor might remind the class that they have just started two-dimensional motion, that the problem they will solve today was designed to help them understand the relationship between one-dimensional and two-dimensional motion, and they will have 35 minutes to solve the problem. The instructor also informs the students that at the end of 35 minutes, one member from each group will be randomly selected to put part of their solution on the board. The instructor then gives each group a sheet with the problem and all the fundamental equations they have studied in class to this point in time.



The class is quiet for a few minutes as students read the problem, then there is a buzz of talking for the next 30 minutes. No textbooks or notes are open. Group members are talking and listening to each other, and only one member of each group is writing on a piece of paper. They mostly talk about what the problem is asking, how the objects are moving, what they know and don't know, and what they need to assume, the meaning and application of the equations that they want to use, and the next steps they should take in their solution. There are disagreements about what physics applies to the problem and what that physics means in this situation.

The instructor circulates slowly through the room, observing and listening, diagnosing any difficulties the group is having solving the problem, and occasionally interacting with the groups that the instructor judges need help. This pattern of listening to groups and short interventions continues for about 30 minutes. At the end of that time, the instructor assigns one member from each group to draw a motion diagram and write the equations they used to solve the problem on one of the boards on the walls. That member can ask for help from the remaining group members as necessary. The instructor then tells the class to examine what is on the board for a few minutes to determine the similarities and differences of this part of each group's solution. The instructor then leads a class discussion that highlights the similarities and differences and clarifies which are correct and which are incorrect. As the students are about leave class, the instructor hands out a complete solution to the problem. Almost everyone looks over the solutions. Some groups celebrate and others groan.

The description above gives a image of a CPS class but does not include all the preparations and scaffolding that go along with implementing CPS. In the remainder of this chapter we first outline how to use this book. This is followed by an outline of how CPS influences both content and course structure. The next section in this chapter includes a checklist for preparing to implement CPS, a brief outline of the steps needed to adapt CPS, and what to expect after implementing CPS. Finally, the last section of the chapter introduces some "laws of instruction" and icons that are frequently used in this book.

How To Use This Book

This book has four parts, described briefly below:

Part I. Teaching Physics Through Problem solving. These six chapters provide background about the unsuccessful problem-solving strategies of beginning students, how context-rich problems and a problem-solving framework help students engage in real problem solving, and why Cooperative Problem Solving (CPS) is a useful tool for teaching physics through problem solving.

Part 2. Using Cooperative Problem Solving. These four chapters describe the foundation of cooperative problem solving and provide detailed information about how to implement CPS for maximum effectiveness. This information includes what are appropriate group problems, how to manage groups, and the course structure and grading practices necessary to implements CPS. The last chapter provides the research evidence that CPS improves both students' problem-solving skills and their conceptual understanding of physics.

Part 3. Teaching a CPS Session. These three chapters provide information about how to prepare for a cooperative problem solving session, how to implement the session, and how to monitor and intervene with groups as they are solving problems.

Part 4. Personalizing a Problem solving Framework and Problems. These three chapters provide some advanced techniques building upon knowledge from previous chapters and assuming that you have already tried using cooperative grouping. This part provides details on how to construct a problem-solving framework for your students, how to write context-rich problems, and how to judge or adjust the difficulty of a context-rich problem.

The second page of each part summarizes the purpose and content of each chapter of that part (pages12, 80, 134, and 164). You may want to read these summaries before you to help guide your reading.

Random Access

We have tried to write this book so that you can jump in anywhere you find an issue of interest. Of course you can also read it straight through. For to achieve that flexibility, each chapter contains frequent references to other related chapters. There is also some ***repetition*** of content to allow this type of random access reading.

Endnotes and References

The endnotes at the end of each chapter provide comments as well as references to seminal research papers or research reviews if you are interested in pursuing a subject further. Since the research articles and reviews often apply to more than one chapter, a bibliography is provided at the end of the book. To keep the list of references to a minimum, we have left out many important papers and books that can be found in the references of those cited.

Course Content and Structure for Cooperative Problem Solving

Topics Covered

The use of Cooperative Problem Solving (CPS) has only minor implications for how many topics you can incorporate into your course, and none, as far as we know, for the order of those topics. Those decisions must be based on other factors such as the success of your students in meeting the goals of your course. CPS has been designed for courses with a goal of having the majority of students reaching the appropriate level of a qualitative and quantitative understanding of all of the topics in the course. There is no method of instruction or course structure that can be successful if the course content is presented too rapidly for the average student in the class to have a chance of understanding it.

Class Size and Course Structure

The Cooperative Problem Solving techniques described in this book can be used under conditions that are less than ideal. We teach at a large research-orientated state university with over 2000 students per semester taking physics courses taught in this manner. About one quarter of these students are in algebra-based courses and the rest are in calculus-based courses. Essentially all of these students take physics because their major requires it. The number of physics majors in any of the introductory courses is negligible.

The courses at our University have the very traditional structure of three large fifty-minute lectures per week, accompanied by smaller two-hour laboratories and fifty-minute discussion sections. Many different professors teach the lecture sections with the help graduate and undergraduate teaching assistants primarily responsible for the coordinated laboratories discussion sections. The teaching personnel changes each semester. This is not the best structure for CPS instruction but it is still effective.

At other institutions where Cooperative Problem Solving has been used effectively, teaching situations range from large classes with separated lecture, laboratories, and discussion sections, to smaller classes with all functions interwoven, taking place in a single room. Naturally each professor adapts CPS to fit his or her teaching style and situation. These techniques are being used at large research-oriented universities, comprehensive state universities, private universities colleges, and community colleges.

Implementing Cooperative Problem Solving

If you decide to teach a physics course using Cooperative Problem Solving (CPS), you need to be prepared to:

- ☑ Explicitly show students how to use the fundamental principles and concepts of physics to solve problems (Chapters 4, 5 and 11).
- ☑ Use problems that require students to make decisions based on a working knowledge of physics (Chapters 3 and 8).
- ☑ Have students solve problems while coached in groups, as well as solve problems by themselves (Chapters 7, 12 and 13).
- ☑ Grade student problem solutions for communicating the application of physics while using sound problem solving techniques as well as the correctness of the solution. Grade students in a manner that emphasizes individual achievement but does not discourage cooperation (Chapter 9).
- ☑ Assume that all parts of the instructional process must be repeated every time a new topic is introduced.

Of course, "the devil is in the details." Successful implementation requires attention to the learning process as well as the teaching process. In designing Cooperative Problem Solving we tried many things that did not work, although they seemed to be reasonable extensions of fundamental learning research or common sense experience. Even techniques that work well with one class may not give as large an effect in another type of class. This is not surprising because the learning process is complex. As with any complex system, the parameters that influence learning must be tuned for each particular set of constraints to achieve large effects. This book describes the sensitive parameters that we have found in our research and development work. They must be matched to the goals and constraints of your situation.

Overview of Adapting Cooperative Problem Solving

There is no point in you making exactly the same mistakes we did in getting Cooperative Problem Solving to work. Based on the experience of faculty at other institutions that have adapted these techniques to their situations, we suggest that you begin by following as much of our prescription as possible in your situation. After you have tried it once, you will have a good idea of what to change to make it work better for you.

The following is an outline of what to do. The details and rationale are given in the remainder of this book.

Step ❶ Determine the most important goals for student learning in your class. For example, the goals below are the top goals from a survey of the engineering faculty that require their students to take introductory physics at the University of Minnesota. See *Appendix A* for a copy of the survey.

- Know the basic principles underlying all physics.
- Be able to solve problems using general qualitative logical reasoning within the context of physics.
- Be able to solve problems using general quantitative problem-solving skills within the context of physics.
- Be able to apply the physics topics covered to new situations not explicitly taught by the course.
- Use with confidence the physics topics covered.

Step ❷ Get or write context-rich problems that require students to:

- Directly use the physics concepts you want to teach;
- Directly address the goals of your course; and
- Practice their weakest problem solving skills.

Appendices B and C contain examples of context-rich problems in mechanics and electromagnetism. See Chapter 3 for a description of context-rich problems, Chapter 15 for how to write context-rich problems, and Chapter 16 for how to judge and adjust the difficulty of those problems.

Step ❸ Adopt a research based, problem-solving framework that you want your students to use to solve problems. Make every step of the process explicit to the students.

- Always demonstrate the same logical and complete problem-solving process throughout the course no matter what the topic.
- Explain all the decisions necessary to solve the problem.
- Show every step, no matter how small, to arrive at the solution.
- Hand out or have on the web examples of complete solutions to problems showing how you expect students to communicate their thought process in problem solutions.
- Allow each student to make their own reasonable variations of the framework for their solutions.

See Chapter 6 for additional information about demonstrating a problem-solving framework. Chapter 4 provides a description and example of a research based problem-solving framework, and Chapter 5 provides examples of complete problem solutions using the framework. Chapter 14 describes how to personalize a framework to match your preferences and the needs of your students.

Step ❹ Require that your students practice solving problems in small groups while you or another instructor provide timely guidance.

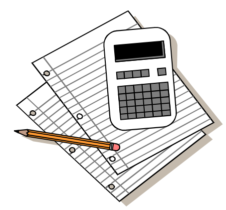
- Use cooperative groups of three, or at most four.
- Structure the groups so that they really work together on the solution.
- Use a problem appropriate for group work.

Make sure no notes or books are available to students while they solve problems in groups.

- Grade the results of these group problems, only occasionally, for a logical and complete solution using correct physics reasonably communicated.

See Chapter 7 for a description of how Cooperative Problem Solving is different from having students work in groups, and Chapters 8 - 9 for appropriate group problems, structuring and managing groups, and grading. See also Chapters 11-13 for how to prepare for and teach a Cooperative Problem Solving session.

Step ❺ Encourage students to solve homework problems by themselves using correct physics communicated in a logical and complete manner.



- If homework is graded, make sure the grade depends on logical and complete solutions, not just a correct answer.
- If you cannot grade homework in this manner (we don't have the resources) make at least one problem on your tests an obvious modification of a homework problem. Point this out to your students. A week or two before each test, give students a sample test to work on at home.

See Chapter 9 for how to grade problem-solving performance.

Step ❻ Give the same type of problems on your examinations that your students solve in their groups. Grade them based on the student behavior you wish to encourage

- Grade on an absolute scale to encourage student cooperation.
- Grade for well-communicated problem solutions presented in an organized and logical manner.
- Give enough time to actually solve problems based on making decisions about physics and writing complete solutions.

See Chapters 9 for how to grade problem-solving performance.

After becoming comfortable with the basics of Cooperative Problem Solving, you can improve student achievement by using all of your course resources to address your goals. For example, all demonstrations can be presented as examples of problem solving. In addition, laboratories can be structured so that they give students practice in problem solving that can be checked by measurement. See Appendix D or visit our website (http://groups.physics.umn.edu/physed/research.html) for more information about problem solving labs.

What to Expect After Implementation of CPS

If the implementation of Cooperative Problem Solving (CPS) is going well, by about half way through your term you should notice some changes in your class. When you visit discussion sections you will hear students talking to each other about physics concepts, and beginning to use the language of physics. Student problem solutions on exams will look neater and more organized and thus easier to grade. By the end of the term you will find that student drop-out rates, it they were high, will have decreased. Student conceptual knowledge as measured by objective exams such as the Force Concept Inventory will have improved. The problem-solving performance of your students will also improve. Your students will be more willing to tackle new and unusual problems. They will tend to begin problems by thinking about what physics to apply. Typically, you cannot expect things to go smoothly until the second time you have taught a course after you implement any significant change.

Of course you will not be 100% successful and not all your students will like CPS. Judge your success by comparing to results of previous classes. Research shows that in traditional classes, about 20% of the students show significant improvement. Our goal is to reach 2/3 of the students 2/3 of the time. In the beginning, try not to worry if one of five groups is dysfunctional. That's an 80% success rate! Spend most of the time on most of the students. Next time around you can try to tweak the course structure to reach the others. Make incremental changes. Try to improve your results just a little bit each year.

Students are usually the most conservative element in the educational process. They may not like how things are now taught, but they resist any changes. If the course is noticeably different than it was the previous year, students will blame their difficulties on those changes. Be patient and supportive of your students at the beginning. You must believe that the changes are for the better. If you don't have confidence in what you are doing, then you can't expect them to. Naturally you will meet the most resistance from students that have been successful previously. Human beings do not like to change, especially if they have been successful. On the other hand, students who were expecting to have trouble will be immediately grateful. As long as you are firm and positive and the students are more successful than they imagined, they will be positive about the changes at the end of the course. Interestingly we have found that the strongest support for Cooperative Problem Solving comes from some of the students who resisted the most at the beginning. In the hands of a very good student this is a very powerful tool and they recognize it.

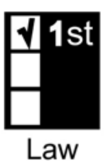
Our Laws of Instruction and Other Frequently Used Icons

To guide the actual implementation of a course design, we have invented, only half seriously, four "Laws of Instruction" in analogy with the "Laws of Thermodynamics." In the same spirit as classical thermodynamics, these "laws" describe robust empirical observations that are based on the current state of knowledge of human behavior and learning.[1] The overriding principle of human behavior addressed by the laws is that most human beings do not like to change their behavior. As with thermodynamics, our "laws" are statistical in nature -- you will certainly know of specific counter examples.

Our Laws of Instruction are described below.

If you don't grade for it, students won't do it.

It would be wonderful if we lived in a world in which students were intrinsically motivated to learn new things, and our physics class was the only class students were taking. But it just isn't so. Humans expend the minimum energy necessary to survive. Students expend the minimum energy necessary to get what they consider a "good" grade.



Doing something once is not enough.

The most effective way for humans to learn any complex skill is apprenticeship. For example, a new apprentice would learn tailoring in a busy tailor shop, where he or she is surrounded both by master tailors and other apprentices, all engaged in the practice of tailoring at varying levels of expertise. Masters teach apprentices through a combination of activities called modeling, scaffolding, coaching and fading.[2] Repetition of these activities is essential.

Modeling. The apprentice repeatedly observes the master demonstrating (or modeling) the target process, which usually involves many different but related sub-skills. This observation allows the apprentice to build a mental model of the processes required to accomplish the task.

Scaffolding. Scaffolding is structure that supports the learning of the apprentice. Scaffolding can include a compelling task or problem, templates and guides, practice tasks, and collections of related resources for the apprentice. Usually scaffolding is removed as soon as possible but may need to be reintroduced later for a new context.

Coaching. The apprentice then attempts to execute each process with guidance and help from the master (i.e., coaching). A key aspect of coaching is the support, in the form of reminders or help that guides the apprentice to approximate the execution of the entire complex sequence of skills, in their own way. The interaction with other learners provides the apprentice with instant feedback through peer coaching and a calibration of progress, helping focus the effort needed for individual improvement.

Fading. Once the apprentice has a grasp of the entire process, the master reduces the scaffolding (i.e., fading), providing only limited hints, refinements, and feedback to the apprentice, who practices by successively approximating smooth execution of the entire process. The interplay between observation, scaffolding, peer interactions, expert coaching, and increasingly independent practice helps the apprentice develop self-monitoring and correction skills and integrate those skills with other knowledge to advance toward expertise.

Problem solving is a complex mental skill. Like learning a complex physical skill, students learn physics problem solving best in an environment in which problem solving is modeled, the process is scaffolded, they are coached in the process, and the structure and coaching fades as the students become better problem solvers. All of this should happen in what is called an environment of expert practice in which the student should be able to answer the following three questions at any time in the course:

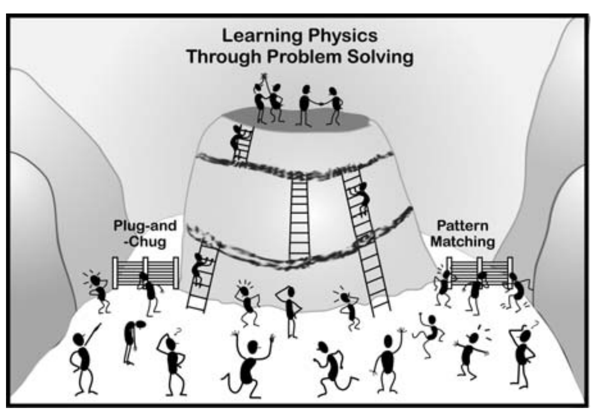
1. Why is whatever we are now learning important?
2. How is it used?
3. How is it related to what I already know?

[See Chapter 6 for a more detailed description of cognitive apprenticeship.]

Don't change course in midstream; structure early then gradually reduce the structure.

Humans are very resistant to change. It is easier on both instructors and students to start with what may seem a rigid structure (e.g., a problem-solving framework, roles for working in groups), then fade gradually as the structure is no longer needed. It is almost impossible to impose a structure in the middle of the course, after you discover that students need it.

Make it easier for students to do what you want them to do and more difficult to do what you don't want.



Humans will persist in a previously successful behavior until it is no longer viable for survival. Learning a new way of thinking is a difficult, time consuming, and frustrating process, like climbing a steep mountain. For most students, expert-like problem solving is a new way of thinking. Students will try to run around this mountain by using their unsuccessful problem-solving strategies (e.g., plug-and-chug and pattern-matching, see Chapter 2), especially if they seem to almost work.

For students to learn physics through problem solving, it is not enough to model problem solving, scaffold and coach (provide ladders up the mountain), and gradually fade the support. You must also supply barriers (fences) to make it obvious to students that their novice problem-solving strategies are unsuccessful.

Icons

In addition to the icons for the four Laws of Education, we have used the following icons to highlight certain information within each chapter.

Remember

This icon is a reminder of related ideas, often described in other chapters of the book, that are helpful in understanding the current topic.

Warning

A warning icon denotes a research-based recommendation does not match a traditional, common sense teaching practice.

Tip

Teaching tips are practices that has been found useful in implementing Cooperative Problem Solving (CPS).

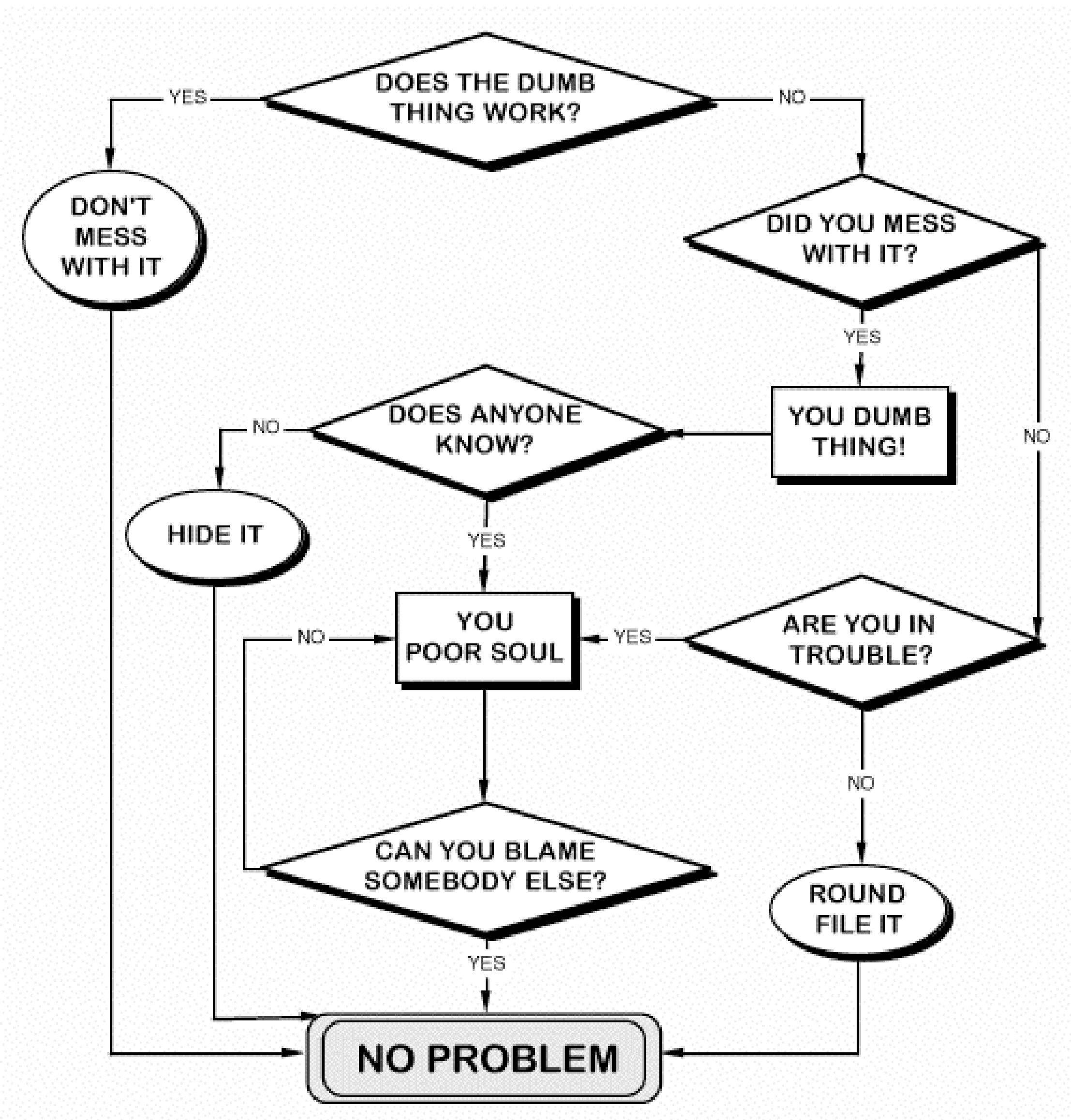
Endnotes

1 For a summary of the research on how people learn, see Bransford, J.D., Brown, A.L., and Cocking, R.R. (Eds) (2000), *How people learn: Brain, mind, experience, and school,* Washington DC, National Academy Press. For a description of seven principles of learning based on this research, see National Research Council. (2002), *Learning and understanding: Improving advanced study of mathematics and science in U.S. high schools,* Washington, D.C., National Academy Press. Research-based principles of learning are also described in Reddish, E.F. (2003), *Teaching physics with the physics suite*, New York NY, John Wiley & Sons Inc.

2 See, for example: Collins, A., Brown, J.S. & Newman, S.E. (1989), Cognitive apprenticeship: Teaching the crafts of reading, writing and mathematics, in *Knowing, Learning, and Instruction: Essays in Honor of Robert Glaser* edited by L.B. Resnick, Hillsdale NJ, Lawrence Erlbaum, pp. 453 – 494; and Brown, J. S., Collins, A., & Duguid, P. (1989), Situated cognition and the culture of learning, *Educational Researcher*, **18**(1), 32-42.

Part 1

Teaching Physics Through Problem Solving



How to Avoid Solving Problems
(unknown internet author)

In this part . . .

You can explore some of the underlying justification for the techniques we describe in this book. If you just want to know "how to do it," you can skip this part.

Chapter 2 describes *why* most instructors teach physics through problem solving, the difference between student and expert problem solving, and what students typically learn.

Chapter 3 gives two applications of our 3rd Law of Instruction. The first is the design of context-rich problems and the second is the control of the equations that students use to solve those problems.

Chapter 4 describes the features of a general problem-solving framework, and gives an example of a problem-solving framework used in introductory physics courses. Included in the example is a running commentary explaining the purpose and rationale for each part of the solution using this framework.

Chapter 5 describes three different ways of presenting a problem solving framework to students – flow charts of problem-solving decisions, answer sheets for problem solutions, and example problem solutions using the answer sheets.

Chapter 6 presents the rationale for Cooperative Problem Solving – what difficulty does it address? We also present a theoretical framework for the structure of these techniques.

Chapter 2
Connecting Students, Physics, and Problem Solving

In this chapter:

- ✓ Why physics faculty teach physics through problem solving
- ✓ Knowledge organization – expert and novice networks of ideas.
- ✓ Students' novice problem-solving strategies, the plug-and-chug and pattern-matching strategies.
- ✓ Who benefits from traditional teaching

Students come to us with well-developed knowledge, including their personal physics ideas[1], and expectations about how and what they will learn in a physics class.[2] Decades of research have shown that students' personal ideas about physics often do not match established physics concepts. For example, about 80% of students entering our calculus-based introductory physics course think that during a collision between a large truck and a small car, the force of the truck on the car is larger than the force of the car on the truck. Student's personal ideas that do not match physics concepts are often called preconceptions, naive conceptions, alternative conceptions or misconceptions.

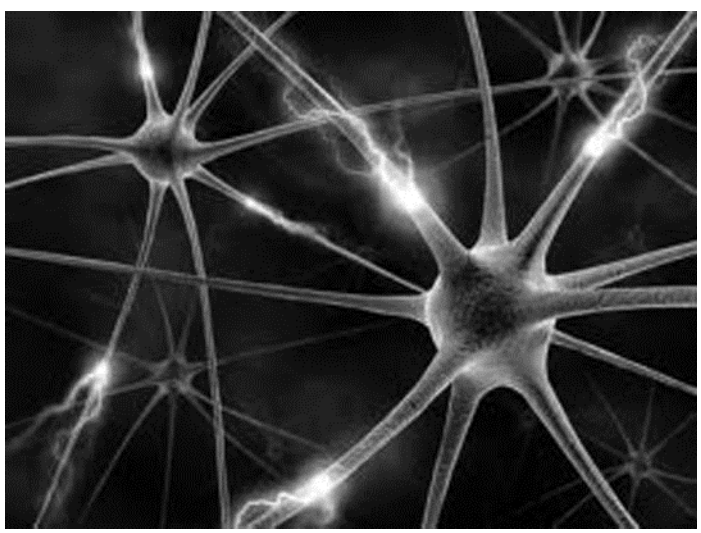
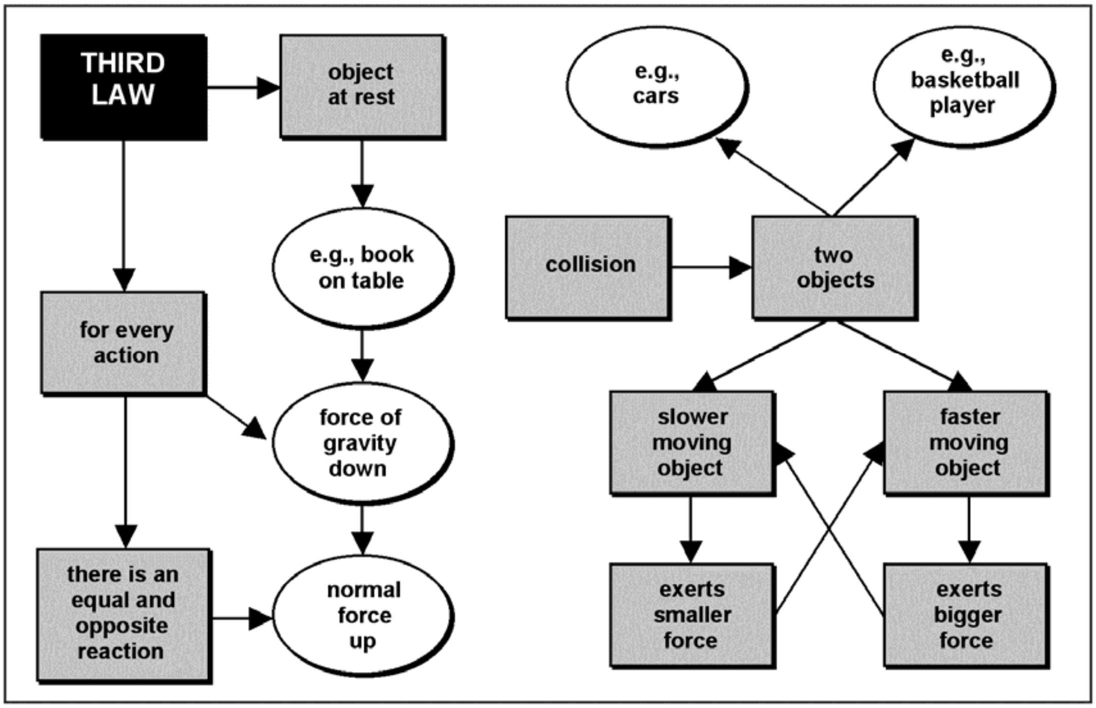


Figure 2.1. Model of neurons firing in the brain

Figure 2.2. Example of a network of a student's personal ideas about the Newton's third law of motion.



Although many details remain to be determined, there is a consistent model of cognition that has emerged from neuroscience and cognitive science. In this model, elements of knowledge are networks of connected neurons. When someone uses the knowledge represented by a particular network, the neurons in that network are activated – increase in their firing rate (see Figure 2.1). Networks arise from building associations among neurons through synapse growth. These neural connections are built when different neurons are activated at approximately the same time. This process is summarized by the slogan "neurons that fire together, wire together" attributed to Hebb. [reference Hebb, D (1949). *The organization of behavior*. New York: Wiley.]
Knowledge stored in long-term memory is the way those neurons are linked, and the way those linked structures are activated.[3]

For example, a student's knowledge of the Newton's third law can be represented as a network of these connections of ideas which are themselves networks of neural connections, as illustrated in Figure 2.2.[4] Some of this knowledge might be relatively isolated and some interwoven with that of other activities, such as driving a car or playing basketball. The biological process of learning is complex, requiring the establishment and deletion of connections that differ in detail for every individual. Because everyone filters their perceptions through their knowledge network, instructional input by itself, is not sufficient for a majority of students to learn physics or any other field. Neither clear explanations, nor dramatic demonstrations, nor laboratory activities are enough.

In this chapter we first describe *why* physics is traditionally taught through problem solving and what students learn. The second section describes what students usually learn when they solve physics problems.

Why Teach Physics Through Problem Solving?

Tradition

When experts solve a physics problem they link fundamental physics principles and concepts, such as Newton's laws of motion or the conservation of energy, to their knowledge of a physical situation.[5] For example when throwing a ball over short distances, experts know that the drag force can be ignored; the acceleration in the vertical direction is a known constant; and there is no acceleration in the horizontal direction. Using this knowledge they can predict the motion of the ball through logic, including mathematics, based on the initial velocity of the ball. An example of part of the network of knowledge of an expert, such as yourself, is shown in Figure 2.3.

Solving problems in this manner requires a deep understanding of fundamental physics concepts, including their utility in particular situations. A correct solution embodies both correct physics concepts and their proper interconnection to other ideas that are related to the physical situation of the problem. This is what we want our students to learn.

In addition to our own traditions, there are external reasons to teach physics through problem solving.

Other Majors in Our Courses

Surveys of university faculties show that gaining experience in problem solving is one of the primary reasons that other majors require their students to take introductory physics. (An example of the survey we used is included in Appendix A). This is as true for the departments that require algebra-based physics as it is for the science and engineering departments that require calculus-based physics.

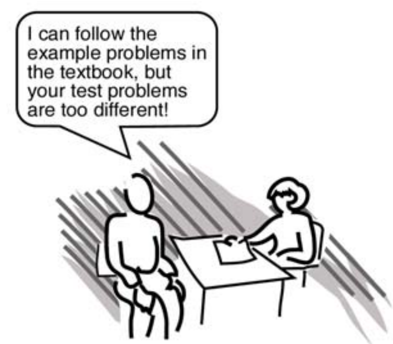
Employment of Physics Majors

Surveys of physics majors, after they have graduated and are functioning in jobs, shows that problem solving is the primary skill from physics that they use, as shown in Figure 2.4. It is interesting to note that interpersonal skills, including teamwork skills, is the second skill used most often by graduates in their jobs.

What do students typically learn through problem solving?

We all have students that are reasonably successful at getting the correct answer to some of the end-of-chapter problems in the text, but have difficulty solving the more complex problems in the text and on exams. Many beginning students attempt to answer physics problems in a manner that we do not recognize as problem solving.

Their solution strategy is to connect the known quantities in a problem with a similar quantities in an equation, and then perform mathematical manipulations until the answer appears. A typical solution of such a student is shown in Figure 2.5. This strategy, known as "plug and chug", promotes learning of physics by memorizing equations and practicing the mathematical manipulation of those equations.[6] Most students employing this strategy do not do well in a university physics course. They complain that there is too much to learn. From their point of view they are correct.

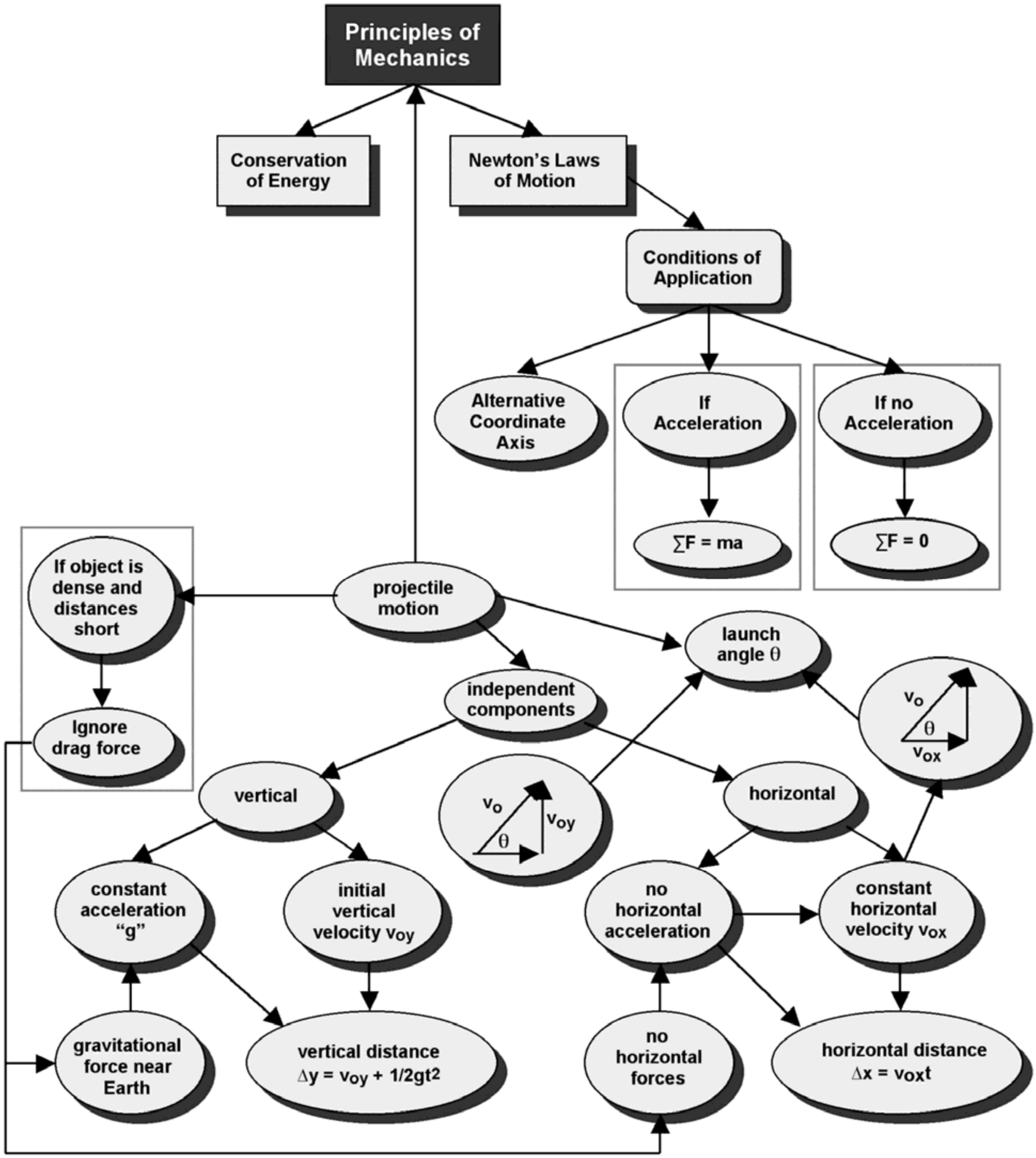


Most students who survive their course have usually generalized their strategy from trying to learn the specific formula to solve each problem, to trying to learn the *pattern of equations to solve different classes of problems*. Each class of problems is characterized by a literal feature of the problem, such as the specific action of the objects involved. These students try to remember solution patterns, usually from example solutions in the textbook or the lecture, and attempt to force their solution to fit the pattern.[7]

For example, a student might try to remember the pattern of mathematical steps for solving problems involving "objects sliding down an incline plane" or "objects moving in a circle." Pattern-matching students often remark that they can't figure out how to *begin* a problem. No matter how many worked out solutions the instructor gives, they are always asking for more worked examples.

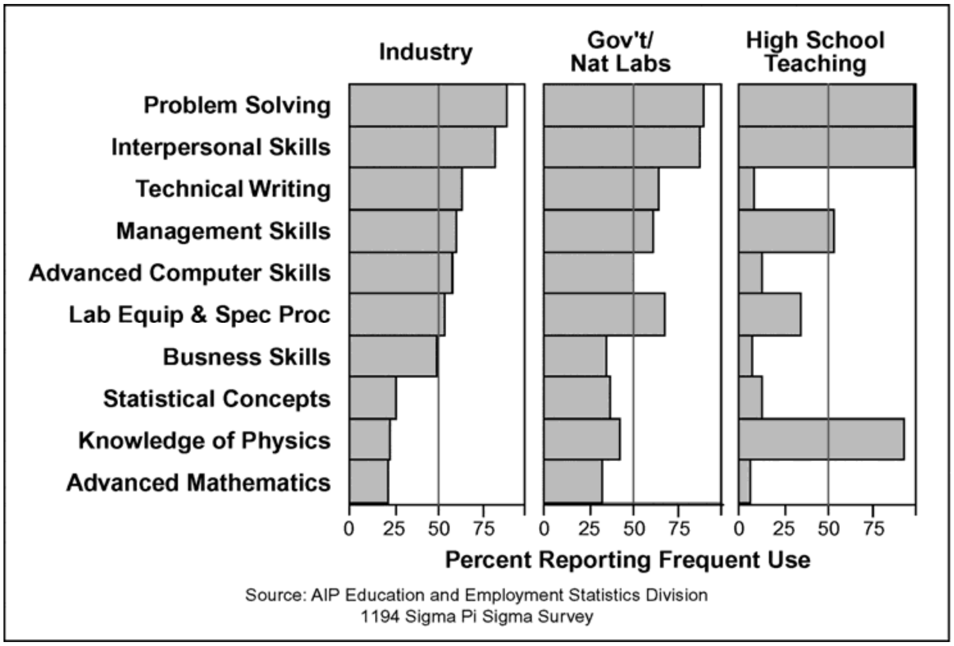
The student solution in Figure 2.6 illustrates this "pattern-matching" strategy. This student tried to match the solution pattern for "objects launched upwards" to this situation. As the solution illustrates, a small variation in the problem can derail such students -- they are not able to generalize a problem type to similar problems with different objects, events, or constraints.[8] Even if the pattern-matching behavior results in the correct answer, the process itself is not very effective for learning either physics concepts or their application through problem solving. Pattern matching leads to an incoherent and fragmented network of knowledge -- one that is not organized around fundamental concepts. For example, compare the network of knowledge of an expert (Figure 2.3) with that of a pattern-matching student shown in Figure 2.7.

Figure 2.3. Example of a small part* of the expert network of knowledge for solving a novel projectile-motion problem.



* Not shown are the links to many experiences with projectile motion and the links to the conservation of energy.

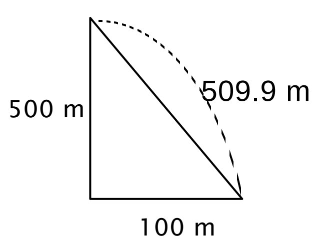
Figure 2.4. Survey of employees with physics bachelor's degrees



The plug-and-chug and pattern-matching behavior of students is documented by research into what are called is called "novice" problem-solving strategies. Even successful students who can answer the end-of-chapter textbook problems typically do not have a grasp of the basic physics concepts involved.[9] In other words, research shows that we, the physics faculty, have been correct all along. Students who do poorly on our examination problems really do not understand the basic concepts of physics.

Figure 2.5. Example of a plug-and-chug novice problem-solving strategy (taken from an actual student test solution in an algebra based physics class)

Problem: A boulder, rolling horizontally at a constant speed, goes off the edge of a 500-ft cliff. How fast would it need to be rolling to hit 100 feet from the base of the cliff?



$$\tan\theta = \frac{opp}{adj}$$

$$\theta = \tan^{-1}\frac{100}{500} = 11.3^\circ$$

$$500^2 + 100^2 = \sqrt{260000} = 509.9m$$

$$x_y = v_o t + \frac{1}{2}at^2$$

$$500 =$$

$$t = \frac{x}{v}$$

$$x_y = at^2$$

$$t^2 = \frac{500\ m}{9.8\ m/s^2}$$

$v = at \qquad v = \frac{x}{t}$

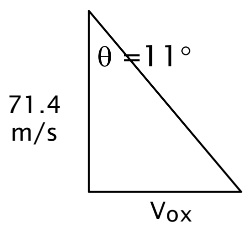
$\frac{x}{t} = at$

$\frac{x}{a} = t^2$

$t^2 = (9.8\ m/s^2)(500\ m)$

$t^2 = 51.0\ s$

$t = 7\ sec.$



$x = x_o + v_o t + \frac{1}{2}at^2 \qquad a = g = 9.8\ m/s$

$x - x_o = v_o t + \frac{1}{2}at^2 \quad \Rightarrow \quad \frac{x - x_o}{t} = v_o + \frac{1}{2}gt$

$\cancel{\frac{x - x_o}{t} - \frac{1}{2}gt = v_o \quad \Rightarrow \quad \frac{0 - 500\ m}{7\ s} - \frac{1}{2}\left(9.8\frac{m}{s^2}\right)(7\ s) = v_o}$

$71.4 - 34.3 = 37.13\ m/s$

$\frac{x}{t} = v$

$\tan 11^\circ = \frac{v_{ox}}{71.4\ m/s}$

$\boxed{v_{ox} = 13.9\ m/s}$

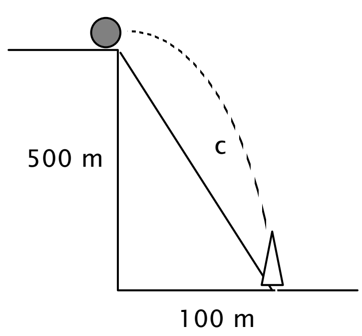
$\frac{500\ m}{7\ sec} = v_y$

$v_y = 71.4\ m/s$

The boulder would have to roll at 13.9 m/s.

Figure 2.6. An example of a novice "pattern-matching" problem-solving strategy (this is an actual student solution on a test in an algebra based physics class)

Problem: A boulder, rolling horizontally at a constant speed, goes off the edge of a 500-ft cliff. How fast would it need to be rolling to hit 100 feet from the base of the cliff?



$v_i = ?$

$x = 500\ m$

$y = 100\ m$

$\Delta t = ?$

$\sqrt{500^2 + 100^2} = 510\ m$

$c = 510\ m$

$\Delta x = v_i \Delta t + \frac{1}{2}g\Delta t^2$

$\Delta x = v\cos\theta + \frac{1}{2}g\Delta t^2$

$\Delta y = v\sin\theta + \frac{1}{2}g\Delta t^2$

$\Delta x - \Delta x = v\cos\theta\Delta t + \frac{1}{2}g\Delta t^2 - \Delta x = 0$

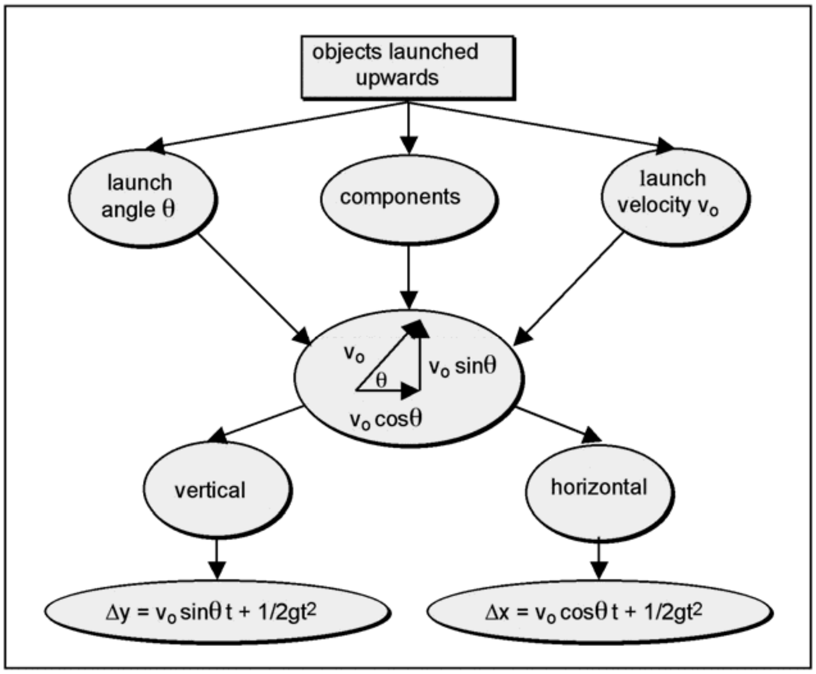
$\Delta y - \Delta y = v\sin\theta\Delta t + \frac{1}{2}g\Delta t^2 - \Delta y = 0$

$v\cos\theta\Delta t + \frac{1}{2}g\Delta t^2 - \Delta x = v\sin\theta\Delta t + \frac{1}{2}g\Delta t^2 - \Delta y$

$v\cos\theta\Delta t - v\sin\theta\Delta = \Delta x - \Delta y$

$v\Delta t(\cos\theta - \sin\theta) = 100 - 500 = -400\ m$

Figure 2.7. Example of a novice student's pattern-matching network of knowledge for solving projectile motion problems



Students often complain that such problems are not clear or tricky. Over time many instructors tend to modify their test problems so that students can answer the problems using their novice problem-solving strategies. Then they can get the right answer and still not understand either the physics concepts or the process of problem solving. The very human response to minimizing student frustration often drives instructors in the direction of building questions which minimize learning.

Who benefits?

Real problem solving is an additional teaching tool that requires each student to examine his or her own mental connections. This process can be difficult, time consuming, and frustrating. Most human beings work very hard to avoid engaging in this process, as illustrated in the problem-solving flow chart that circulated around the internet some years ago (shown on cover page to Part 1, page 11). The vast majority of students entering our introductory physics courses do not engage in real problem solving. Without instruction in problem solving, those who survive tend to use a pattern-matching strategy while unsuccessful students engage in the plug-and-chug strategy.

In the classroom, it is useful to remember the old practice of miners using canaries to detect poisonous gasses underground. Unsuccessful students are more sensitive to weak teaching practices, so like the canaries in mines, it is easy to see their failure. If a significant number of students are dropping out or failing a course, then it is probable that the successful students who, like the miners survive, are also being harmed.

Good teaching practices most obviously benefit the typically unsuccessful students, but they equally benefit the best students. In the next three chapters in Part 1, we describe some teaching practices that benefit *both* the typically successful and unsuccessful students.

Endnotes

1 There is a vast literature on college students' personal physics ideas -- also called alternative conceptions, preconceptions, naive conceptions, and misconceptions. A sampling of articles include: Trowbridge, D.E. & McDermott, L. C. (1981), Investigation of student understanding of the concept of acceleration in one dimension, *American Journal of Physics*, **49**(3), 242-253; Clement, J. (1982), Students' preconceptions in introductory mechanics, *American Journal of Physics*, **50**(1), 66-71; McCloskey, M. (1983), Naive theories about motion, in *Mental Models*, D.S. Gentner and A. Stevens (Eds), Hillsdale, New York: Lawrence Elbraum; McDermott, L.C. (1984), Research on conceptual understanding in mechanics, *Physics Today*, **37**(7), 24-32; Goldberg, F.M. & McDermott, L.C. (1987), An investigation of student understanding of the real image formed by a converging lens or concave mirror, *American Journal of Physics*, **55**(2), 108 –11; Cohen, R., Eylon, B. & Ganiel, U. (1983), Potential difference and current in simple electric circuits, *American Journal of Physics*, **51**, 407 - 412; Guth, J. (1995), An in-depth study of two individual students' understanding of electric and magnetic fields, *Research in Science Education*, **25**(4), 479-490; Furio, C. & Guisasola, J. (1998), Difficulties in learning the concept of electric field, *Science Education*, **82**(4), 511-26; Loverude, M.E., Kautz, C.H., & Heron, P.R.L. (2002), Student understanding of the first law of thermodynamics: Relating work to the adiabatic compression of an ideal gas, *American Journal of Physics,* **70**(2), 137; and Kautz, C.H., Heron, P.R.L., Shaffer, P.S. and McDermott, L.C. (2005), Student understanding of the ideal gas law, Part II: A microscopic perspective, *American Journal of Physics,* **73**(11), 1064-1071. For an overview of some of these studies, see McDermott, L.C. & Redish, E.F. (1999), Resource letter PER-1: Physics education research, *American Journal of Physics*, **67**, 755-767.

2 There is a growing research base about students' expectations or beliefs about the nature of physics knowledge and how to learn physics that affect student learning in a class. See, for example: Redish, E.F., Saul, J.M., & Steinberg, R.N. (1998), Student expectations in introductory physics, *American Journal of Phys*ics, **66**, 212-224; Hammer, D. (2000), Student resources for learning introductory physics, *American Journal of Physics*, Physics Education Research Supplement, **68**(S1), S52-S59; Elby, A., and Hammer, D. (2001). On the substance of a sophisticated

epistemology. *Science Education*, **85**, 554-567; Hammer, D. (1994), Epistemological beliefs in introductory physics. *Cognition and Instruction*, **12** (2), 151-183; and Perkins, K. Adams, W. K., Finkelstein, N. D., Pollock, J. S., & Wieman, C. E. (2005), Correlating student attitudes with student learning using the Colorado Learning Attitudes about Science Survey, *Proceedings of the 2004 Physics Education Research Conference*, Rochester, NY: PERC Publishing

[3] Directly from an article by Jonathan Tuminaro and Edward Redish (2007), Elements of a cognitive model of physics problem solving: Epistemic games, *Physical Review ST Physics Education Research*, **3**, 020101. There are many books about the emerging models of cognition. See, for example, Anderson, J.R. (2004), *Cognitive science and its implications*, 6th Ed, Worth Publishing; Fuster, J. (1999), *Memory in the cerebral cortex: An empirical approach to neural networks in the human and nonhuman primate*, MIT Press; and Fuster, J. (2003), *Cortex and mind: Unifying cognition*, Oxford University Press.

[4] In this brief presentation, we are using the more familiar term, ideas, rather than the smaller elements of knowledge with which students think. For a *brief* discussion of these elements of knowledge, see the introduction to an article by Jonathan Tuminaro and Edward Redish (2007), Elements of a cognitive model of physics problem solving: Epistemic games, *Physical Review ST Physics Education Research*, **3**, 020101.

[5] For some seminal papers about how experts and novice solve problems, see Chi, M.T.H., Feltovich, P. J., & Glaser, R. (1981), Categorization and representation of physics problems by experts and novices, *Cognitive Science*, **5**, 121-152; Eylon, B. & Reif, R (1984), Effects of knowledge organization on task performance, *Cognition and Instruction*, **1**(1), 5-44; and Larkin, J., McDermott, J., Simon, D. and Simon, H. (1980), Expert and novice performance in solving physics problems, *Science*, **208**, 1335-1342.

[6] Schoenfeld, A. H. (1983). Beyond the purely cognitive: Belief systems, social cognitions, and meta-cognitions as driving forces in intellectual performance, *Cognitive Science*, **8**, 173-190

[7] Of course, there are more than two novice strategies for solving problems. Different researchers use different research methodologies (e.g., videotaping individual students thinking aloud while solving a problem with structured or semi-structured questions by the interviewer; videotaping groups of students solving homework problems), students from classes with different pedagogies, and describe the novice strategies in different ways. See, for example: Tuminaro, J & Redish E.F. (2007), Elements of a cognitive model of physics problem solving: Epistemic games, *Physical Review ST Physics Education Research*, 3, 020101; Walsh, L.N., Howard, R.G., and Bowe, B. (2007), Phenomenographic study of students' problem solving approaches in physics, *Physical Review ST Physics Education Research*, **3**, 020108.

[8] See, for example, Chi, M., Feltovich, P.J., & Glaser, R. (1981), Categorization

and representation of physics problems by experts and novices, *Cognitive Science*, **5**, 121-152; Reif, F., & Heller, J.I (1982), Knowledge structure and problem solving in physics, *Educational Psychologist*, **17**, 102-127; Ferguson-Hessler, M.G.M., & de Jong, T. (1987), On the quality of knowledge in the field of electricity and magnetism, *American Journal of Physics*, **55**(6), 492-497; Elio, R. & Scharf, P.B. (1990), Modeling novice-to-expert shifts in problem solving strategy and knowledge organization, *Cognitive Science*, **14**(4), 579-639; and Zajchowski, R., & Martin, J. (1993), Differences in the problem solving of stronger and weaker novices in physics: Knowledge, strategies, or knowledge structure? *Journal of Research in Science Teaching,* **30**, 459-470.

[9] For reviews of expert-novice research, see Maloney, D.P. (1994), Research on problem solving in physics. In D.L. Gabel (Ed.*), Handbook of Research in Science Teaching and Learning*, (pp. 327-354), NY, Macmillan. See also, Hsu, L., Brewe, E., Foster, T. M., & Harper, K. A. (2004), Resource letter RPS-1: Research in problem solving. *American Journal of Physics*, **72**(9), 1147-1156.

Chapter 3
Combating Problem Solving That Avoids Content

In this chapter:

- ✓ How context-rich problems help students engage in real problem solving
- ✓ The relationship between students' problem-solving difficulties and the design of context-rich problems
- ✓ How to discourage students from indiscriminate formula memorization.

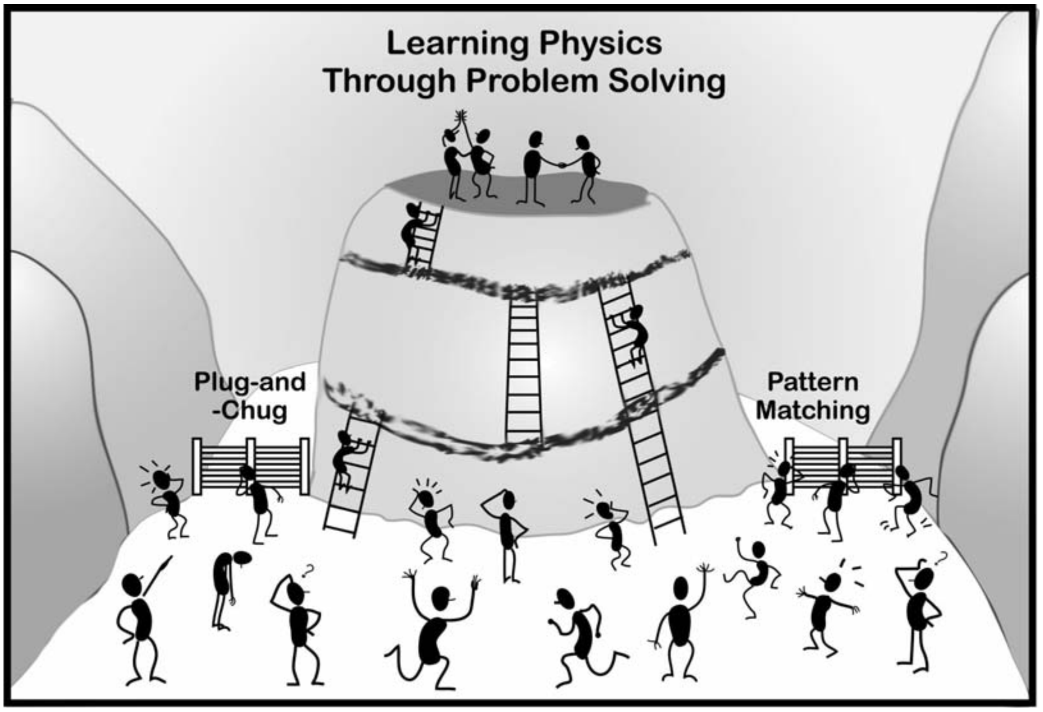
Real problem solving has been described as the process of arriving at a solution when you don't initially know what to do.[1] This means that problem solving requires making decisions. Making decisions involves the connection and application of different types of knowledge (e.g., concepts, facts, and procedures) to construct a satisfactory solution. Because problem solving uses general-purpose tools, such as fundamental physics concepts, it *should* be a good vehicle for assisting in the learning of those concepts.

Of course problem solving is often a difficult, time consuming, and frustrating process -- like climbing a steep mountain. Most students either give up or avoid this mountain by using their novice strategies such as plug-and-chug or pattern-matching. Consequently, our Third Law of Education needs to be applied, as illustrated in Figure 3.1. [See the Introduction, pages 9-10, for a complete description of our Laws of Education.] The Third Law is:

Make it easier for students to do what you want them to do, and more difficult to do what you don't want.

In this chapter and chapters 4 and 5, we describe applications of this Law of Education that helps *all* students become more competent problem solvers. Two applications of the Third Law are described in this chapter. The first is the design of context-rich problems, and the second is the restriction of the "formulas" allowed on exams.

Figure 3.1. Illustration of the Third Law of Education: *Make it easier for students to do what you want them to do (ladders) and more difficult to do what you don't want (fences).*



Design of Context-rich Problems: How They Help Students Engage in Real Problem Solving

We have developed a type of question, context-rich problems, that are designed to encourage students to engage in real problem solving while discouraging students' natural tendency use novice problem-solving strategies (see Chapter 2 for novice problem-solving strategies). The goal of this type of problem is to give students practice incorporating physics into their existing knowledge network. If students understand the basic physics involved, the problem should be easy to comprehend and straightforward to solve. If students do not have that understanding, they should not be able to make progress toward a solution at the point that the physics concept arises.

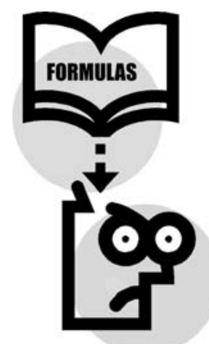
In other words, it should be obvious to the student, as well as the instructor, where the difficulty lies so that they can examine their own connections to that physics and get help if necessary. The following is an example of such a problem that might be given early in an introductory physics course that begins with kinematics:

Figure 3.2. The Traffic Accident context-rich problem

You have a summer job with an insurance company and are helping to investigate a tragic accident. At the scene, you see a road running straight down a hill that is at 10° to the horizontal. At the bottom of the hill, the road widens into a small, level parking lot overlooking a cliff. The cliff has a vertical drop of 400 feet to the horizontal ground below where a car is wrecked 30 feet from the base of the cliff. A witness claims that the car was parked on the hill and began coasting down the road taking about 3 seconds to get down the hill. Your boss drops a stone from the edge of the cliff and, from the sound of it hitting the ground below, determines that it takes 5.0 seconds to fall to the bottom. You are told to calculate the car's average acceleration coming down the hill based on the statement of the witness and the other facts in the case. Obviously, your boss suspects foul play.

Some of the features of this context-rich problem are explained below. These features are common to all context-rich problems. Some features are designed to encourage students to incorporate physics principles and problem solving practices into their knowledge network by making decisions based on their existing ideas.(ladders). Other features are designed to discouraging students' natural tendency use novice problem-solving strategies (fences). Above all, this type of problem requires the student to make and link decisions in a logical and organized manner, the hallmark of real problem solving.

Feature 1. It is difficult to use a few equations and plug in numbers to get an answer.



The student "knows" that acceleration is velocity divided by time, but no velocity is given and there are two different times in the Traffic Accident problem. The student also "knows" that velocity is distance divided by time, so one time is to get the velocity and the other is to get the acceleration. Unfortunately for the student, there are two distances in the problem. Which one should be used? There is also an angle given. Does the student need to multiply something by a sine or cosine?

Feature 2. It is difficult to find a matching solution pattern to get an answer.

Going down the hill at a known angle looks like an "inclined plane problem." Is the acceleration just g sinθ? But what about the other numbers in the problem? When the car goes off the cliff, this is a "projectile problem." You can calculate an acceleration from the distance (which one?) and the time the car falls. Why would that be the acceleration down the hill?

Feature 3. It is difficult to solve the problem without first analyzing the problem situation.

It is difficult to understand what is going on in this problem without drawing a picture and designating the important quantities on that picture.

- Making the situation as real as possible, including a plausible motivation, helps students in the visualization process. (ladder)
- Making the student the primary actor in the problem also helps the visualization process. This also avoids gender and ethnic biases that can inhibit learning. The other actors in the problem are as generic as possible, so the student's visualization is not hampered by unfamiliar names or relationships. (ladder)
- Students are forced to practice visualization because no picture is given to them. What are the velocity and acceleration of the car at interesting positions in its motion? What are those positions? (fence)
- The visualization of a realistic situation gives the student practice connecting "physics knowledge" to other parts of the student's knowledge structure. This makes the physics more accessible, and so more easily applied to other situations. What does a car going off a cliff have to do with dropping a stone? Does physics really apply to reconstructing accidents? (ladder)
- In real situations, assumptions must always be made. What are reasonable assumptions? What is the physics that justifies making those assumptions? Can friction be ignored? Where? Is the acceleration down the hill constant? Do you care? This gives students practice in idealization to get at the essential physics behind complex situations. (ladder)

Feature 4. Physics cues, such as "inclined plane", "starting from rest", or "projectile motion", are avoided.

Avoiding physics cues not only makes it difficult for students do pattern matching, it encourages students to build the connections between physics and their existing knowledge structure.

- Using common words helps students practice connecting physics knowledge to other things they know. This makes the physics more capable of being applied to other situations. Here the car goes down a hill and begins by being parked. (ladder)
- Physics cues tend to set students thinking along a predetermined path.

They do not require students to examine their physics concepts to determine which are applicable. Here the student must apply physics knowledge to decide on where the acceleration of the car is constant and where the velocity, or a component of the velocity of the car is constant. (fence)

Feature 5. Logical analysis using fundamental concepts is reinforced.

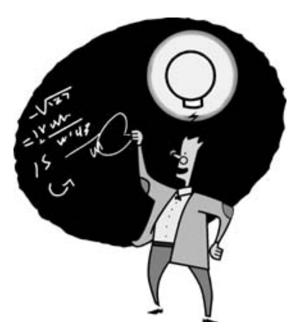
Logical analysis is reinforced because there is no obvious path from the information given to the desired answer. Each student must construct that path incrementally.

- Using a logical analysis helps to determine which information is relevant and which is not. The extra information is not put in to confuse the student but to force informed decision making. It is information that they would likely have in that situation and could be relevant. It might show that there is a choice between two equally valid solution paths or actually be irrelevant so it would only be used if the student has incorrect or fragile physics knowledge. (ladder and fence)
- The answer to the problem can be arrived at in a straightforward manner after a logical analysis using the most fundamental physics concepts, in this case the definition of average acceleration and average velocity as well as the connection between average and instantaneous velocity for constant velocity. (ladder)
- A logical analysis is necessary because this question cannot be answered in one step. (fence)

If you think such context-rich problems are too difficult for most of your students at the beginning of your class, you are right. This problem was designed for either: (1) an instructor demonstration of a logical and coherent framework for solving this problem, or (2) for a group of students to use the framework to co-construct a solution. In the next chapters we describe problems-solving frameworks and the reason for cooperative problem solving.

Relationship Between Students' Problem-solving Difficulties and the Design of Context-rich Problems

For contrast, here is the same traffic-accident problem (Figure 3.2) with only the "essentials."



> A block starts from rest and accelerates for 3.0 seconds. It then goes 30 ft. in 5.0 seconds at a constant velocity.
>
> a. What was the final velocity of the block?
>
> b. What was the average acceleration of the block?

Figure 3.3. Table of student difficulties and context-rich problem design features

Student Difficulty	Symptom	Design Feature
Visualizing a physical situation	Physically impossible results. No pictures or diagrams drawn.	No pictures given. Situation realistic.
Connecting physics to reality	Physically impossible results. Difficulty in applying knowledge to slightly different situations. Difficulty applying knowledge consistently, even within a single situation.	Situation realistic. Reasonable motivation. Avoid "physics" words such as inclined plane, inelastic collision, frictionless, . . .
Recognizing the underlying fundamental physics of a situation. Idealization.	Difficulty in applying knowledge to slightly different situations.	Realistic situation that reduces in a straightforward way to a simple situation.
Application of fundamental concepts.	Difficulty in applying knowledge to slightly different situations.	Decisions necessary to determine which concepts to apply and which quantities are relevant.
Integrating knowledge into a coherent conceptual framework.	Misconceptions remain.	Realistic situation described. Misconception will prevent a correct solution.
Executing a logical analysis.	Random equations	Problem requires more than one mathematical and logical step.
Over reliance on pattern matching	Solving the "wrong" problem either through over-simplification or misreading of the problem.	The solution that does not obviously repeat the pattern of textbook examples.
Lack of mathematical rigor.	Frequent algebraic mistakes. Mathematical "magic" in solutions.	Problem can be solved in a straightforward way using fundamental physics.
Gender or ethnic bias.	Lack of interest or intellectual involvement. Difficulty visualizing physical situations.	Actors are "you" and unnamed acquaintances. Situations are perceived as realistic. There is a motivation.

For an expert, such as you, the two problems are identical and are solved identically. For the introductory student, however, they are very different. The second form of the problem actually encourages students to use their novice, plug-and-chug or pattern-matching tactics. The student is not allowed to practice decision-making because the problem is already broken down into single-concept steps.

This problem is difficult for students to visualize because no realistic context is given. Most importantly, a problem stripped down to its essentials does not help students connect physics with other parts of their knowledge.[2]

The first two columns of the table in Figure 3.3 give a summary of what research indicates are some major difficulties students have solving physics problems.[3] The last column summarizes specific features of context-rich problems designed to help students overcome these difficulties.

How to Discourage Students From Indiscriminate Formula Memorization.

Many students believe that solving a physics problem requires them to know the right equation(s) for that particular problem. So part of their plug-and-chug and pattern-matching techniques is the memorization of *all* the equations in each chapter of their textbook. These equations are of equal importance to students. They do not distinguish the mathematical formulations of fundamental principles from equations that are the consequence of those fundamental principles to specific circumstances or for specific types of interactions. Worse yet, the equations memorized for one chapter are promptly forgotten while memorizing the formulas for the next chapter.

So how should the Third Law of Education be applied here?

3rd
Law

Make it easier for students to do what you want them to do and more difficult to do what you don't want.

What can you do to promote the development of a coherent, connected network of knowledge organized around fundamental concepts, while discouraging the memorization of *disconnected* formulas?

Warning

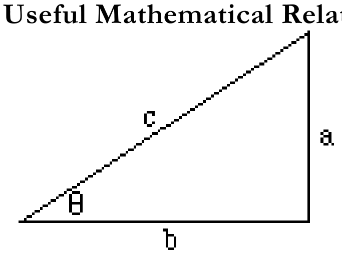
At first glance the answer may seem simple: allow students to write equations on a single file card or sheet of paper and bring their equations to your exams and examinations. While this solution discourages memorization, it does not address the difficulty students have in developing a coherent knowledge base. Those who have tried this technique know too well that students will write as small as possible to cram everything onto a single piece of paper. They then complain

Figure 3.4. Example information sheet for exams: algebra-based course

This is a closed book, closed notes quiz. Calculators are permitted. The only formulas that may be used are those given below. Credit is given *only* for logical and complete solutions that are clearly communicated. In the context of a complete solution, partial credit will be given for a well-communicated solution based on correct physics.

- 5 points: A useful picture, defining the question, and giving your approach.
- 7 points: A complete physics diagram defining the relevant quantities, identifying the target quantity, and specifying the relevant equations.
- 6 points: Planning the solution by constructing specific equations and checking for sufficiency
- 5 points: Executing the plan to get algebraic and numerical answer
- 2 points: Evaluating the validity of the answer.

Useful Mathematical Relationships:



For a right triangle: $\sin\theta = \frac{a}{c}$, $\cos\theta = \frac{b}{c}$, $\tan\theta = \frac{a}{b}$,

$a^2 + b^2 = c^2$, $\sin^2\theta + \cos^2\theta = 1$

For a circle: $C = 2\pi R$, $A = \pi R^2$

For a sphere: $A = 4\pi R^2$, $V = \frac{4}{3}\pi R^3$

If $Ax^2 + Bx + C = 0$, then $x = \dfrac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

Fundamental Concepts:

$$\bar{v}_r = \frac{\Delta r}{\Delta t}$$

$$\bar{a}_r = \frac{\Delta v_r}{\Delta t}$$

$$v_r = \lim(\Delta t \to 0)\frac{\Delta r}{\Delta t}$$

$$a_r = \lim(\Delta t \to 0)\frac{\Delta v_r}{\Delta t}$$

$$\sum F_r = ma_r$$

$$\Delta E_{system} = E_{transfer}$$

$$E_f - E_i = E_{in} - E_{out}$$

$$KE = \frac{1}{2}mv^2$$

Under Certain Conditions:

$$\bar{v}_r = \frac{v_{ri} + v_{rf}}{2}$$

$$a = \frac{v^2}{r}$$

$$F = \mu_k F_N$$

$$F_s \leq \mu_s F_N$$

$$F = k\Delta r$$

$$F = \frac{Gm_1m_2}{r^2}$$

$$F = \frac{k_e q_1 q_2}{r^2}$$

$$E_{transfer} = \sum F_r \Delta r$$

$$PE = mgy$$

$$PE = \frac{1}{2}kx^2$$

$$PE = \frac{Gm_1m_2}{r}$$

$$PE = \frac{k_e q_1 q_2}{r}$$

Useful constants: 1 mile = 5280 ft, 1 ft = 0.305 m, $g = 9.8\ m/s^2 = 32\ ft/s^2$, 1 lb = 4.45 N, $G = 6.7 \times 10^{-11}\ N\ m^2/kg^2$, $k_e = 9.0 \times 10^9\ N\ m^2 / C^2$

that they would have done better on the exam if they had just had the foresight to write down one more equation.

We have found it useful to supply students with only the information they need to solve exam problems from the basic ideas emphasized by the course. An example of such an information sheet used in an algebra-based course is shown in Figure 3.4. There are three features of the information sheet that are designed to promote student development of a coherent, integrated network of knowledge and reinforce logical analysis of a problem using fundamental concepts.

Supply a limited number of equations that students may use.

Supply *only* the equations that state the fundamental physics principles and concepts that are stressed in the course. Students are not allowed to use any other equations to solve a problem. A distinction is made between physics that underlies everything and the important physics that depends on a specific but reasonably general situation. The symbols are not defined to encourage students to know the meaning of the equations.

The choice of equations depends on the course and the emphasis of the instructor. For example, the kinematics equations in Figure 3.4 would be given to the students in the algebra based physics course. In the calculus-based course, we replace the equation for the special case of constant acceleration

$$\bar{v}_x = \frac{v_{xi} + v_{xf}}{2}$$

that builds on students intuitive understanding of an average with

$$x_f = \frac{1}{2} a_x \Delta t^2 + v_{ox} \Delta t + x_o ,$$

an equation directly connected by calculus to the solution of the equation defining acceleration. [See Chapter 11, page 141 for an example of an information sheet for a calculus-based course.

The information sheet grows with time.

Nothing is ever taken off the sheet, but new equations and needed constants are added. The “old” equations are often used in problems for new topics to emphasize that the underlying physics does not depend on the context. For example, the information that is not shaded was supplied for the second exam in the course, the information in light-gray was added by the end of 8 weeks, and the information in darker gray by the end of about 12 weeks in the course.

Allow students to use calculators.

Students love their calculators and feel persecuted if they are not allowed to use them on exams. Unfortunately modern calculators can hold an enormous amount of information. Since all students have an information sheet with all the equations that they are allowed to use, there is no advantage in using the calculator memory to store equations for exams. The same holds true for any temptation to bring written crib sheets to exams. If students are also required to communicate their solution in a logical and complete manner, the ability and temptation to cheat is almost completely eliminated. For these reasons, we see no benefit of restricting the use of calculators in CPS.

Endnotes

[1] The definition of a problem and problem solving has not changed substantially in the last forty years. For example, see Newell, A., & Simon, H.A. (1972), *Human problem solving*. Englewood Cliffs, NJ, Prentice-Hall, Inc; Hayes, J.R. (1989), *The complete problem solver (2nd ed.)*, Hillsdale NJ; Lawrence; and Martinez, M. (1998), What is problem solving? *Phi Delta Kappan*, **79**, 605-609.

[2] A number of researchers have concluded that the typical problems assigned in physics courses are actually counterproductive to learning physics. Other types of "nonspecific goal problems" and "ill-structured problems" have been found to move students towards more expert-like problem solving. See, for example, Sweller, J., Mawer, R. & Ward, M. (1982), Consequences of history: Cued and means-end strategies in problem solving, *American Journal of Psychology*, **95**(3), 455-483; and Shekoyan, V., and Etkina, E. (2008), Introducing ill-structured problems in introductory physics recitations, *Proceedings of the 2007 Physics Education Research Conference*, PERC Publishing, Rochester, NY, 951, 192-195. In our own research, we also found that standard problems seemed to promote the use of the novice, plug-and-chug or pattern-matching strategies rather than the use of a more logical and organized strategy. See Heller, P. & Hollabaugh, M. (1992), Teaching problem solving through cooperative grouping. Part 2: Designing problems and structuring groups, *American Journal of Physics*, **60**(7), 637-644.

[3] For reviews of student difficulties solving problems, see Maloney, D.P. (1994), Research on problem solving in physics, in D.L. Gabel (Ed.*), Handbook of Research in Science Teaching and Learning*, (pp. 327-354), NY, Macmillan; and Woods, D. (1989), Problem solving in practice*,* in D.L. Gabel (ed.*), What Research Says to the Science Teacher Vol. 5*, National Science Teachers Association. See also, Hsu, L., Brewe, E., Foster, T. M., & Harper, K. A. (2004), Resource letter RPS-1: Research in problem solving. *American Journal of Physics*, **72**(9), 1147-1156.

Chapter 4
Building Content into Problem Solving

In this chapter:

- ✓ An answer to the question: What is a problem-solving framework?
- ✓ An example of a problem-solving framework for introductory physics
- ✓ An example of an "ideal" student's solution using this framework
- ✓ Problem solving as a series of translations
- ✓ Initial objections to teaching a problem-solving framework

When is a question a "problem?"

If you know how to arrive at a solution, even if you don't know the answer, then the question is NOT a problem for you.

Whether or not a question is a problem depends on the *viewpoint* of the person seeking a solution. If that person knows how to arrive at a solution, even if they don't know the answer, then the question is not a problem for them.[1] If the question is not a problem, it is not necessary to use a logical problem-solving framework that emphasizes the understanding of fundamental concepts and the practice of important problem solving skills. This is the case whether or not a person's solution is correct. It takes a great deal of confidence for a person to approach a question as a problem and embark on a solution based *only* on physics, logic, and mathematics without knowing the path that will be travelled. Most people require the additional guidance of a *framework* to embark on this journey.

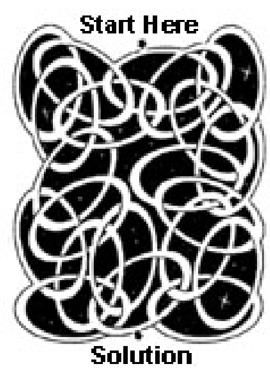
Unfortunately most students do not come into our introductory physic classes with a robust and logical general problem-solving framework. A sure indication is when students tell you they don't even know how to get started on a problem. Without a reasonable framework, students little choice other than to continue to apply the unproductive novice strategies with which they entered the course. [See Chapter 2 for a description of the plug-and-chug and pattern-matching novice strategies for solving problems.]

Consequently, if your goal is to teach physics through problem solving, then your students will need an explicit example of a problem-solving framework that directs their efforts toward making connections both among physics concepts and between those concepts and the rest of their knowledge. The framework should be a logical and organized guide to build a problem solution. It gets students started, guides them to what to consider next, organizes their mathematics, and helps them determine if their answer is correct.

In this chapter, we first describe a general problem-solving framework used by experts in all fields, then give an example of that framework tailored to our introductory, algebra-based physics course. If you want to explore what it is like to use a problem-solving framework, the next section provides an example of an "ideal:" student's problem solution arrived at by using our framework. Included in the example is a commentary explaining the purpose and rationale for each part of the framework. In Chapter 14 we describe how you could personalize a problem-solving framework for the introductory physics course you teach.

What Is a Problem-solving Framework?

There is no formula for true problem solving. Problem solving is a process similar to working your way through an unfamiliar forest. You navigate your way *step by step*, making some false moves but gradually moving closer toward the goal. Each step is more likely to succeed the choice is guided by some fundamental principle. But what are these "steps" and what guides your decisions?



We have known for a long time that humans generally follow the same steps to solve any problem. Many psychologists and educators have described these steps in slightly different ways.[2] One of the most influential descriptions is by the mathematician George Polya (1945):[3]

1. Understand the Problem (i.e., define the problem)

2. Devise a Plan

3. Carry Out the Plan

3. Look Back (i.e., check your results)

Of course, these steps are a simplification of a complex process. A person solving a problem *overlaps* steps. For example, you may begin to devise a plan while you are defining the problem. You also *backtrack* to earlier steps. For example, in devising a plan for a solution, you may decide that you have not

Figure 4.1. The general problem-solving framework used by experts in all disciplines.

Step 1. Understand the Problem

Bring the problem into focus by describing the situation and goal(s) as precisely as possible.

Describe the problem in the terms developed by your field of expertise.

- ♦ Translate the situation and goals into the fundamental concepts of your field using the notation developed by your field.
- ♦ Decide the reasonable idealizations and approximations you need to make.

Step 2. Devise a Plan

Apply the specialized techniques (heuristics) of your field to develop a plan, using the concepts of your field to connect the situation with the goal.

Re-examine the description of the problem if a solution does not appear possible.

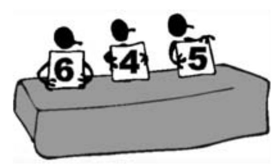
Step 3. Carry Out the Plan

Follow your plan to the desired result.

Re-examine your plan if you cannot obtain the desired result.

Step 4. Look Back.

Determine how well your result agrees with your knowledge of similar behavior, use limiting behavior that you understand.

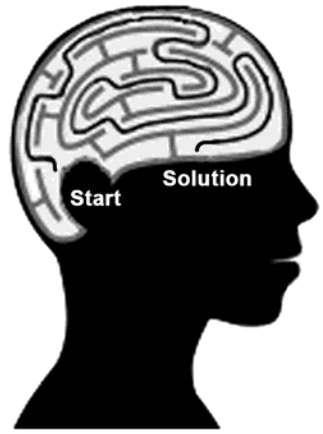


fully understood the problem and go back to consider the situation. When you carry out your plan, you may find that the plan is not complete or that it really does not lead to a solution, so you modify the plan. Finally, you may *skip* some steps altogether, depending on your background and experience.

You probably recognize Polya's steps as self-evident. But what guides our decisions through the several possible paths in our problem-solving? Polya introduced the word *heuristics* for the thinking tools by which problems are solved. A heuristic is a rule of thumb – a strategy that is both powerful and general, but not absolutely guaranteed to work.

For example, one general heuristic used in Step 1 (understanding the problem) is to determine the goal, the unknowns, the data (givens), and the conditions that relate the data. Another heuristic used to devise a plan (Step 2) is called working backwards.[1] Start with the ultimate goal and then decide what would constitute a reasonable step just prior to reaching that goal. Then ask yourself what the step would be just prior to that, and so on until you reach the initial conditions of the problem. Another general heuristic is to break a problem into sub-problems that you can solve.

If you are thinking that you do not use general heuristics to solve textbook problems, then you are right. For you, textbook problems, are not real problems. You know roughly what path to take to get a solution.[4] You can work forwards from fundamental physics concepts and the known information towards the solution.[5]



When faced with an atypical or novel situation, however, it is a problem for you. You then probably use techniques similar to the Polya's general heuristics. It is often difficult for expert problem solvers to articulate these techniques because they are often automated and deeply integrated into a large and specialized knowledge structure. For example, expert physicists faced with an unfamiliar or novel problem will often:[6]

- Use analogies with systems they understand better.
- Search for potential limitations to the analogy.
- Refer to mental models based on visual and kinesthetic "intuition" to try to understand how the target system would behave.
- Investigate the target system with extreme-case arguments, probing how the system would work if different parameters were pushed to zero or infinity.

And, of course, when faced with unfamiliar or novel problems, at some point experts work backwards from the target unknown, dividing the problem into sub-problems that can be solved. It turns out that experts in all fields solve unfamiliar or novel problems in a similar way. This general problem-solving framework is shown in Figure 4.1.

A Physics Example: The Competent Problem-solving Framework

Because you are a expert problem solver, you probably do not pay much attention to your own problem-solving framework, outlined in general terms in Figure 4.1. You don't even have to use a problem-solving framework to solve introductory physics problems because they are not really problems for you. However, if you want your students to solve problems as a tool for learning physics, they will need to use a problem-solving framework that emphasizes the application of fundamental concepts and the connection of those concepts to their existing knowledge using generally useful problem solving skills.

Figure 4.2. The Competent Problem-solving Framework: Algebra Version

1. **Focus the Problem.** Establish a clear mental image of the problem.

   A. Visualize the situation and events by sketching a useful picture.
   - Draw a picture showing how the objects are related spatially, they are moving, and they are interacting. The drawing should show the time sequence of events, especially for those times when an object experiences an abrupt change.
   - Write down the relevant known and unknown information, giving each quantity a symbolic name and adding that information to the picture.

   B. Precisely state the question to be answered in terms you can calculate.

   C. Identify physics approach(es) that might be useful to reach a solution.
   - Which fundamental principle(s) of physics (e.g., kinematics, Newton's Laws, conservation of energy) might be useful in this situation.
   - List any approximations or problem constraints that are apply to this situation.

2. **Describe the Physics**

   A. Draw any necessary diagrams with coordinate systems that are consistent with the approach(es) you have chosen.
   - Define consistent and unique symbols for any quantities that are relevant to the situation.

   \

   B. Identify the target quantity(s) that will provide the answer to the question.

   C. Assemble the appropriate equations to quantify the physics principles and constraints identified in your approach.

3. **Plan a Solution**

   A. Construct a logical chain of equations from those identified in the previous step, leading from the target quantity to quantities that are known.
   - Begin with the quantitative relationship that contains the target variable. Identify other unknowns in the equation.
   - Choose a new equation for one of these unknowns. Keep track of any additional unknowns.
   - Continue this process for each unknown.

   B. Determine if this chain of equations is sufficient to solve for the target quantity by comparing the number of unknown quantities to the number of equations.

   C. Write down a verbal description of the solution steps you will take to solve this chain of equations so that no algebraic loops are created. Work from the last equation to the first equation that contains the target quantity.

4. **Execute the Plan**

   A. Follow the outline from in the previous step.
   - Arrive at an *algebraic* equation for your target quantity by following your verbal description of the solution steps.
   - Check the units of your final algebraic equation before putting in numbers.
   - If quantities have numerical values, substitute them in your final equation to calculate a value for the target quantity.

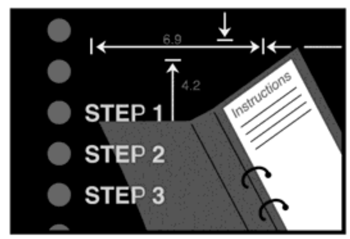
5. **Evaluate the Answer**

   A. Does the mathematical result answer the question with appropriate units?

   B. Is the result unreasonable?

   B. Is the answer complete?

This framework should be designed so that students practice an expert problem solving behavior of examining the conceptual aspects of the problem before launching into mathematical calculation. This causes each student to examine their physics knowledge to which helps remediate weakly held misconceptions and prevents new ones from forming. Such a framework should also emphasize that mathematical calculation is only one small part of a problem solution. Introductory students do not tend to generalize new techniques, so the framework that you want them to use must be *explicitly* demonstrated each time you introduce a new topic. Constructing such a problem-solving framework may be difficult for you because you do not need a framework to solve introductory physics questions because they are not problems for you. Fortunately, there are several available implementations of the Polya's general problem-solving framework that have been used to help teach introductory physics. You can modify one of these to fit your taste and the needs of your students.



Several classroom studies show that the explicit teaching of these frameworks does result in better problem solving.[7,8] A concrete example of a specific implementation of a problem solving framework for students in our algebra-based course is shown in Figure 4.2. We based this framework, called the *Competent Problem-solving Framework*, on our own research and the research of others on student difficulties in solving physics problems. [See Chapter 14, page 173 for the problem-solving framework we use in our calculus-based course for scientists and engineers.] In particular this framework is constructed to address specific weaknesses in the problem solving of those students based on the analysis of written problem solutions, informal interviews of students during office hours, and observations of students working in cooperative groups. Specific parts of the framework were tested by removing them to see if the student difficulties reoccurred.

The Competent Problem-solving Framework has five steps. Each step consists of *specific actions* that lead the student to decisions that confront their difficulties and guide them to the next decision point in the solution. You will, no doubt, recognize most of the actions as things you expect your students to do. The placement of the decisions in one step rather than another is an artificial consequence of having to delineate steps. For a true expert, this process is a continuous whole.

Our problem-solving framework, like all other research-based frameworks, is more proscriptive than the framework you use to solve unfamiliar problems because it is designed both as a tool for learning physics and to move students from novice problem solving toward a more expert-like problem solving.[9] This is why we call it the *Competent* Problem-solving Framework.[10] We do not expect students to become expert problem solvers by the end of an introductory physics course.

The details of the framework that you decide to teach should be tailored to the needs and backgrounds of *your* students and to your own approach to your introductory course. Chapter 14 provides some suggestions about how you could personalize a problem-solving framework.

Example of an "Ideal" Student's Problem Solution for an Algebra-based Course

To be more concrete, Figure 4.3 (pages 42 – 49) shows how to use the Competent" Problem-solving Framework to solve a problem suitable for introductory students at the beginning of an algebra-based physics class. The solution is on the left (even) side of the page. The right (odd) side of the page contains commentary about how the details would support particular goals in an introductory course. [See Chapter 14, pages 175 for a problem-solving framework for the calculus-based course.]

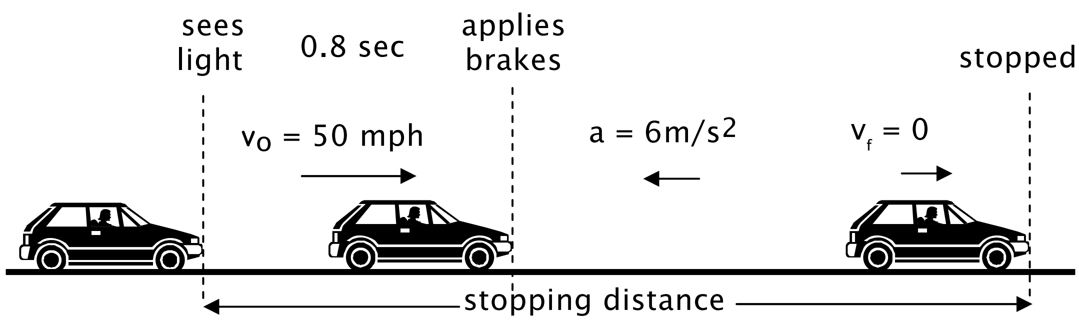
As you read this example, you may want to take some notes to help you personalize a problem-solving framework for your own students and situation, as described in Chapter 14. What parts of this Competent Problem-solving Framework matched the goals for your introductory course? What parts did not match your goals? What parts are not needed by your students? What is missing that your students need? What wording needs changing to better match your students? If so, how would you change it?

Figure 4.3. Example of an "ideal" student problem solution, with commentary

The Problem: You are driving at 50 mph on a freeway when you wonder what your stopping distance would be if the car in front of you jammed on its brakes. When you get home you decide to do the calculation. You measure your reaction time to be 0.8 seconds from the time you see the car's brake lights until you apply your own brakes. Your owner's manual says that your car slows down at a rate of 6 m/s^2 when the brakes are applied.

Step 1: Focus on the Problem

Draw a picture, identifying the useful quantities.



Question: What distance did the car travel from when the brake light is seen to when it stopped?

Approach: Use the definitions of velocity and acceleration.

The velocity is constant until the brakes are applied. In this time interval, the average velocity is equal to the instantaneous velocity.

The acceleration is constant after the brakes are applied.

In this time interval, the average velocity is not equal to the instantaneous velocity. However, the average acceleration is equal to the instantaneous acceleration.

Figure 4.3 (continued). Example of an "ideal" student problem solution, with commentary

Commentary

The Problem. This problem emphasizes the basic definitions of velocity and acceleration in one dimension. It also emphasizes the difference between instantaneous kinematics quantities and their average values, a concept is very difficult for students. The problem gives the students practice in contrasting constant velocity motion with that at a constant acceleration. Students also need practice using the units of physical quantities. We give students problems with mixed sets of units so they will pay attention to them. Every specialty uses its own set of units for either historical reasons or convenience. From our questionnaire, we found that the faculties from departments that require their students to take our physics course want the physics course to show their students how to convert and manipulate units.

Picture. Students' difficulty with visualization can be immediately seen by their difficulty in drawing a useful picture. At the beginning of the course, many students need to draw very realistic looking objects such as cars while others are satisfied with more expert-like drawings using a simple symbol, such as a rectangle, to represent a car. Almost all beginning students have difficulty drawing multiple images on a single picture to represent an object at multiple positions of interest. They have difficulty making a decision, as to where those interesting positions might be. Many can verbalize these features before they can indicate them on a drawing. Students also have difficulty associating their pictorial representation with quantities that represent the object's motion. We emphasize to our students that once they have drawn the picture, then they should not have to read the problem again. This makes for better time efficiency in solving the problem.

Tip

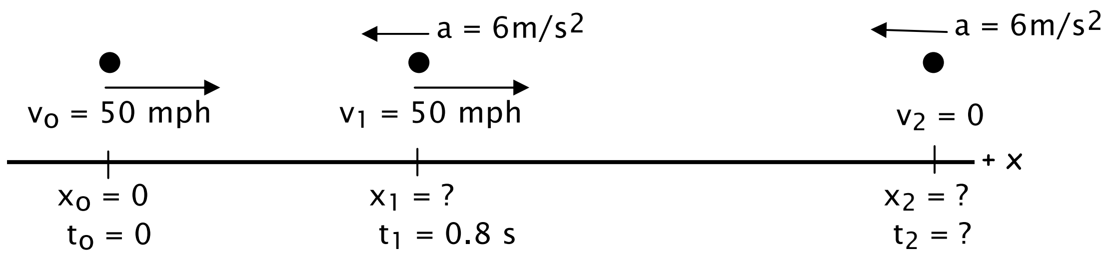
Question. Writing the question in their own words helps prevent students from solving for some quantity that is not desired. Here the student must decide on a reasonable definition of "stopping distance."

Approach. Writing an approach helps students concentrate on the important physics in the problem.

Figure 4.3 (continued). Example of an "ideal" student problem solution, with commentary

Step 2: Describe the Physics

Make a diagram of the situation, defining the quantities that physics uses to describe motion (velocity and acceleration at each interesting position and time on a coordinate system).



Target quantity: x_2

Possibly useful equations:

$$\bar{v}_x = \frac{\Delta x}{\Delta t}$$, for constant velocity $\bar{v}_x = v_x$

for constant acceleration $\bar{v}_x = \frac{v_i + v_f}{2}$

$$\bar{a}_x = \frac{\Delta v_x}{\Delta t}$$, for constant acceleration $\bar{a}_x = a_x$

Figure 4.3 (continued). Example of an "ideal" student problem solution, with commentary

Commentary

Make a diagram of the situation. Students have a great deal of difficulty associating an object with a specific acceleration and velocity at a specific position and time. This diagram helps them because it requires the drawing of an idealized object, now a point, at each interesting position. The diagram also begins the process of getting students to define an appropriate coordinate system for a situation. It gives the students practice in the process of going from real objects to idealized objects. For experts there is no significant difference between the picture and the diagram, but for beginning students there is. The picture gives practice in visualizing the behavior of real objects. The diagram, on the other hand, gives students practice with the visual relationships of physical quantities. As time goes on in the course, more and more students become more expert-like and combine these two types of images. At the beginning of the course, most of our students are not ready to combine these two types of visualization. Those who try to combine the picture and the diagram tend to become more and more confused as situations become more complex.

Target quantity. Writing down this quantity gives students a focus for their mathematics. When doing mathematical manipulations many students lose track of the quantity they want.

Possibly useful equations. This is where the student explicitly connects the conceptual approach to the problem with mathematics. It represents a gathering of mental resources or a "toolbox" for the quantitative solution to the problem. Even in the beginning chapter on kinematics in most textbooks, the number of equations students believe they will need to know can overwhelm them. Here they must decide to limit those equations to only those that are independent and might be useful. At this stage, it is OK if there are some extra equations since, the student does not yet know how to solve the problem.

To compel students to concentrate their efforts on the basic concepts and discourage the student behavior of formula memorization, we only allow our students to use equations chosen by the instructor (see Chapter 3, pages 31 - 33). The choice of equations depends on the course and the emphasis of the instructor. For example, the equations used in this solution would be given to the students in the algebra based physics course. In the calculus-based course, we replace the equation

$$\bar{v}_x = \frac{v_{xi} + v_{xf}}{2} \qquad \text{with} \qquad x_f = \frac{1}{2}a_x\Delta t^2 + v_{ox}\Delta t + x_o,$$

an equation directly connected to the calculus expression defining acceleration.

Figure 4.3 (continued). Example of an "ideal" student problem solution, with commentary

Step 3: Plan the Solution

Construct the chain of equations giving a solution. Begin with an equation containing the target quantity. Keep track of any additional unknown quantities that are introduced.

		Unknowns
Find x_2		x_2
$\bar{v}_{1,2} = \dfrac{x_2 - x_1}{t_2 - t_1}$	①	$\bar{v}_{1,2}$, x_1 , t_2
Find $\bar{v}_{1,2}$		
$\bar{v}_{1,2} = \dfrac{v_1 + v_2}{2} = \dfrac{v_1}{2}$	②	
Find x_1		
$v_1 = \dfrac{x_1 - x_o}{t_1 - t_o} = \dfrac{x_1}{t_1}$	③	
Find t_2		
$a_1 = \dfrac{v_2 - v_1}{t_2 - t_1} = \dfrac{-v_1}{t_2 - t_1}$	④	

Check for sufficiency:
Yes: 4 unknowns and 4 equations

Outline the solution steps. Work from the last equation to the first equation that contains the target quantity.

Solve ④ for t_2 and put it into ①.

Solve ③ for x_1 and put it into ①.

Solve ③ for $\bar{v}_{1,2}$ and put it into ①.

Solve ① for x_2.

Figure 4.3 (continued). Example of an "ideal" student problem solution, with commentary

Commentary

Construct the chain of equations giving a solution. We wish to convince students that solving a problem relies more on understanding the concepts of physics than on mathematical techniques. Students seem to trust mathematical manipulation to give them an answer and make "magical" mistakes to get one. This procedure restrains that manipulation by requiring that sufficient equations to solve the problem have been assembled first. It does not always yield the most elegant solution, but it is straightforward, easy to understand, and very general.

Students always know where to begin because they always start with an equation that contains the target quantity. The unknowns in that equation give the student a way to decide on next equation to be used and so on. This procedure gives students a logical way to build a chain of equations that connects the target quantity to quantities that are known. If the process reaches a "dead-end", the student explicitly see how to revise one of their decisions of which equation is used to determine an unknown quantity. This procedure depends on having linearly independent equations, which is another reason for the instructor to control the equations that can be used (see Chapter 3, pages 31 - 33). Although we emphasize paper and pencil solutions, constructing this chain of equations is very useful for computer or calculator algebra.

Remember that working backwards from the goal is a general thinking tool (heuristic) used by experts solving real problems. See the reference in Endnote 1 for a description of working backwards and other heuristics.

Check for sufficiency. Matching the number of equations with the number of unknowns gives the students an easy way to determine if they need more information to solve the problem. After a few weeks, we also point out how you can solve the problem if there are fewer equations than unknowns provided an unknown cancels out. We give instruction on how to detect such cases and the physics implication of these cases.

Outline the solution steps. Actually writing down an outline seems to be necessary for most of our algebra-based physics students, but not for the calculus-based students. Students who do not trust their mathematics background can become very confused when doing algebra unless they have a written plan. All introductory students often get into infinite algebraic loops when solving equations with more than two unknowns. Here when an unknown is determined, even in terms of other unknowns, it is immediately substituted in every upstream occurrence. This procedure assures that such algebraic traps are avoided.

Figure 4.3 (continued). Example of an "ideal" student problem solution, with commentary

Step 4: Execute the Plan

Follow the outline from Step 3.

Solve ④ for t_2:

$$(t_2 - t_1)a_1 = -v_1$$
$$t_2 - t_1 = \frac{-v_1}{a_1}$$
$$t_2 = \frac{-v_1}{a_1} + t_1$$

Solve ③ for x_1:

$$v_1 = \frac{x_1}{t_1}$$
$$v_1 t_1 = x_1$$

Solve ② for $\bar{v}_{1,2}$:

$$\bar{v}_{1,2} = \frac{v_1}{2}$$

Put into ① and solve for x_2:

$$\bar{v}_{1,2} = \frac{x_2 - x_1}{t_2 - t_1}$$
$$\frac{v_1}{2} = \frac{x_2 - v_1 t_1}{\frac{-v_1}{a_1} + t_1 - t_1}$$
$$\frac{v_1}{2}\left(\frac{-v_1}{a_1}\right) = x_2 - v_1 t_1$$
$$v_1 t_1 - \frac{v_1^2}{2a_1} = x_2$$

Calculate the value of the target quantity.

$$x_2 = \left(50\frac{mi}{hr}\right)(0.8s) - \frac{\left(50\frac{mi}{hr}\right)^2}{2\left(-6\frac{m}{s^2}\right)}$$

$$x_2 = \left(50\frac{mi}{hr}\right)\left(\frac{1609m}{mi}\right)\left(\frac{hr}{60\,min}\right)\left(\frac{min}{60s}\right)(0.8s) + \frac{\left(50\frac{mi}{hr}\left(\frac{1609m}{mi}\right)\left(\frac{hr}{60\,min}\right)\left(\frac{min}{60s}\right)\right)^2}{2\left(6\frac{m}{s^2}\right)}$$

$$x_2 = 18m + 42m = 60m$$

Step 5: Evaluate the Answer

x_2 is the distance traveled by the car from when brake light is seen to stopping. The question is answered.
The answer is in meters, a correct unit of distance.
A car is about 6 meters long so 10 car lengths is not an unreasonable distance to stop a car going that fast.

Figure 4.3 (continued). Example of an "ideal" student problem solution, with commentary

Commentary

Step 4 - Execute the Plan.

This is the only part of the problem containing mathematical manipulation. The mathematical solution follows the verbal outline. Here the student begins with quantities that are known and proceeds backwards through the chain of equations to the target quantity. The student can concentrate on the mathematics because they are assured of a solution when they follow the plan.

Step 5 - Evaluate the Answer

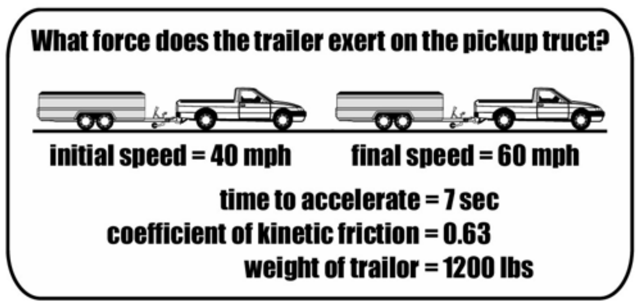
This step reinforces the connection of the physics used in the problem solution to the student's reality. It is the step most characteristic of expert problem solving and is the most difficult part of problem solving for introductory students.

Problem Solving as A Series of Translations

It is often helpful to have more than one way to think about something. Any problem-solving framework, including the Competent Problem-solving Framework, can be viewed as a series of actions to help students make increasingly abstract mental *translations* (steps) from the situation to an answer. In our framework, the first actions in each step define the mental translation. The last action in each step is a "bridge" to prepare students for the translation in the next step. The series of mental translations is outlined below.

Step 1. Focus on the Problem

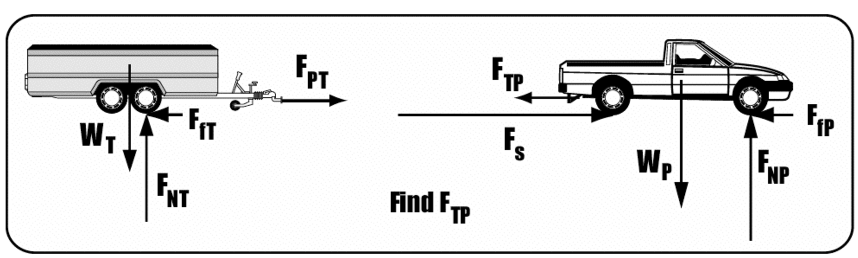
Translate from a situation to an image of the situation, including important information given in the problem.



Prepare for the next step by identifying the approach to the problem.

Step 2. Describe the Physics

Translate from the image of the situation to a diagrammatic representation of the situation.



Define symbols of the quantities used by physics to describe a situation.

Prepare for the next step by identifying the equations useful in describing the situation.

$$\bar{a} = \frac{v_f - v_i}{\Delta t} \qquad F_{PT} = F_{TP} = P \qquad \Sigma F_y = m\,a_y = 0$$

$$W = mg \qquad \Sigma F_x = m\,a_x = ma$$

$$\bar{a}_P = \bar{a}_T = \bar{a} \qquad F_f = \mu F_N$$

Step 3. Plan a Solution

Translate from a physics representation to a mathematical representation of the situation

	unknowns
	Find P
❶ $\Sigma F_x = m\,a$	
$P - F_{fT} = m_T\,a$	F_{fT}, m_T, a
❷ $F_{fT} = \mu F_{NT}$	F_{NT}
❸ $W_T = m_T g$	
❹ $a = \dfrac{v_f - v_i}{\Delta t}$	
....	

Prepare for the next step by identifying the mathematical steps necessary to reach a solution.

....

Solve ❸ for m_T and put into ❶.

Solve ❷ for F_{fT} and put into ❶.

Step 4. Execute Your Plan

Translate from a mathematical representation of the situation to a set of mathematical actions that yield a solution.

Find F_{fT}

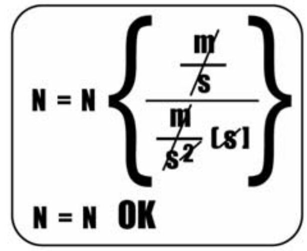
$$F_{fT} = \mu F_{NT}$$
$$= \mu W_T$$

$$P - F_{fT} = m_T\,a$$

$$P - \mu W_T = \frac{W_T}{g}\left\{\frac{v_f - v_i}{2\Delta t}\right\}$$

$$P = W_T\left\{\frac{v_f + v_i}{2g\Delta t} - \mu\right\}$$

Prepare for the next step by checking units during the mathematical process.



Step 5. Evaluate Your Solution

Translate from a mathematical solution to an answer connected to other knowledge.

Initial Objections to Teaching a Problem-solving Framework

As you were reading through the example of the Competent Problem-solving Framework, you may have had the following thoughts:

1. If students use this framework, they will write a lot in their problem solution. Won't this make them more difficult and time consuming to grade?

Actually our most common feedback from instructors using a framework similar to the example is that because students write more, student solutions are much easier and less time consuming to grade. This is because the solutions are more organized and easier to follow. Since student thinking is now explicit, it becomes more straightforward to determine where a student has gone wrong and grade accordingly. This also makes grading more useful to the instructor and allows that instructor to give more useful feedback to the student.

2. This framework is too long and complex. My students will see it as just adding busy work that slows their ability to solve a problem. They won't use it.

There is no getting around this. Expert thinking is a long and complex process when analyzed. However, with repetition, the mind combines many of the subprocesses together and automates them so that they happen very rapidly. To build toward expert-like problem solving, students need to practice these subprocesses. Because this is a fundamental change of their ideas about problem solving, students must begin by implementing each part of expert-like problem solving in a conscious manner. This will overload their short term memory which requires them to write down their thinking and, more importantly, use what they have written to solve the problem. As with learning any new skill, this does slow them down. This can be frustrating for students and cooperative groups [see Chapt. Xxx] and continuous modeling of the process provide important support to get over this hurdle. Working in cooperative groups, students quickly find that they can make progress toward solving problems that were previously well beyond their capability. After practicing such a framework for a month or two, both in groups and alone, their individual speed increases to that of their novice attempts at a solution with a much higher probability of success. In a single year, students will not achieve the mental automation of an expert but can achieve a level of competent problem solving. The key to making progress renovating mental structure is practicing that structure. As Vince Lombardi said, "Practice does not make perfect, only perfect practice makes perfect."

3. This framework reduces problem solving to following a recipe. It takes creative thinking out of problem solving.

The framework only points out the places in a problem solution where decisions need to be made. Since each decision can lead to a different path to a solution, one student's solution can be very different from another's. By explicitly showing the decision points, the framework emphasizes then need for creative thought in problem solving.

All research based problem-solving frameworks in physics are specific implementations of the Polya's general framework (Figure 4.1). They were, however, developed and tested for *different* populations of students. They divide the important actions into a different number of steps and sub-steps, describe the same actions in different ways, and emphasize different heuristics depending on the background and experiences of the intended population of students. The details of the framework that you decide to teach should be tailored to the needs and backgrounds of *your* students and to *you*r own approach to your introductory course. Chapter 14 provides some suggestions about how you could personalize a problem-solving framework, including some examples of different frameworks in physics textbooks.

Remember that breaking any activity down into its small but necessary steps, like starting a car in the winter, makes it seem very complex (see Figure 5.1, page 56). At first your personalized problem-solving framework will certainly seem so to your students. For students to use any such framework, the problems must be constructed so that novice strategies fail and support, such as cooperative groups, must be available to allow students to be successful from the beginning. This book contains suggestions for how to change the structure of your course, including Cooperative Problem Solving, so your students *will* use your personalized problem-solving framework.

We have already made two suggestions in Chapter 3, design context-rich problems for your students to solve, and limit the given equations students can use on an exam. In the next chapter we describe several course changes that make it easier for your students to use your personalized problem-solving framework rather than their own novice strategies. The chapters in Parts 2 and 3 describe how to structure and manage Cooperative Problem Solving.

Endnotes

1 Martinez, M. (1998). What is problem solving? *Phi Delta Kappan* 79, 605-609.

2 Some of the well known descriptions of the stages or "steps" in human problem solving are by: Wallis, G. (1926), *The art of thought*, NY, Harcourt, Brace & World; Dewey, J. (1933), *How we think*, NY, Heath; Kingsley H.L. & Garry R. (1957), *The nature and conditions of learning,* 2nd ed., NJ, Prentice-Hall; and Bransford, J.D. & Stein, B.S. (1984), *The ideal problem solver: A guide for improving thinking, learning, and creativity*, NY, W. H. Freeman and Company.

3 Polya, G. (1957). *How to solve it: A new aspect of mathematical method,* Princeton, NJ: Princeton University Press.

4 For an excellent discussion of the special knowledge used by experts, see Bereiter, C. & M. Scardamalia, M. (1993), *Surpassing ourselves: An inquiry into the nature and implications of expertise,* IL, Open Court, pages 25 –75.

5 For reviews of expert-novice research, see: Maloney, D.P. (1994), Research on problem solving in physics, in D.L. Gabel (ed.*), Handbook of research in science teaching andlearning*, (pp. 327-354), New York: Macmillan; and Hsu, L., Brewe, E., Foster, T. M., & Harper, K. A. (2004), Resource letter RPS-1: Research in problem solving. *American Journal of Physics*, *72*(9), 1147-1156.

6 See Clement, J. (1990), Non-formal reasoning in physics. The use of analogies and extreme cases, in J. Voss, D. N. Perkins, and J. Segal (Eds.), *Informal reasoning*, NJ, Erlbaum; and Johnson, P.E., Ahlgren, A., Blount, J.P., & Petit, N.J. (1980), Scientific reasoning: Garden paths and blind alleys, in J. Robinson (Ed.), *Research in Science Education: New questions, new directions*, Colorado Springs, CO, Biological Sciences Curriculum Study.

7 See, for example, Mettes, C.T.C.W., Pilot, A., & Roossink, H.J. (1981), Linking factual and procedural knowledge in solving science problems: A case study in a thermodynamics course, *Instructional Science*, **10**, 333–361; van Weeren, J.H.P., de Mul, F.F.M., Peters, M.J., Kramers-Pals H., & Roossink, H.J. (1982), Teaching problem-solving in physics: A course in electromagnetism," *American Journal of Physics*, **50**, 725–732; Heller, J.I. & Reif, F. (1984), Prescribing effective human problem-solving processes: Problem description in physics, *Cognitive Instruction*, **1**, 177–21: Bascones, J., &Novak, J.D. (1985), Alternative instructional systems and the development of problem-solving skills in physics, *European Journal of Science Education*, **7**, 253–261; Wright, D.S., & Williams, C.D. (1986), A WISE strategy for introductory physics, *Physics Teacher*, **24**, 211–216; Van Heuvelen, A. (1991), Overview, case study physics, *American Journal of Physics*, **59**(10), 898-907; Reif, F. (1995), Millikan Lecture 1994: Understanding and teaching important scientific thought processes, *American Journal of Physics*, **63**, 17–32

[8] Research indicates that curriculum innovations that incorporate one or more common features result in better problem-solving performance in students. These features are: (1) explicit teaching of problem-solving heuristics (similar to the Competent Problem Solving Framework); (2) modeling the use of the heuristics by the instructor, and (3) requiring students to use the heuristics explicitly when solving problems. See Hsu, L., Brewe, E., Foster, T. M., & Harper, K. A. (2004). Resource letter RPS-1: Research in problem solving. *American Journal of Physics*, **72**(9), 1147-1156, Section IV.

[9] For the stages between novice and expert, see Endnote 4.

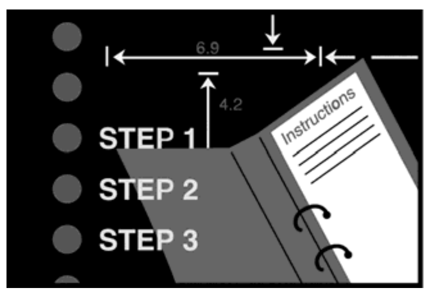
[10] Heller, K. & Heller, P. (2000). *The competent problem solver for introductory physics: Algebra*, McGraw-Hill Higher Education.

Chapter 5
Reinforcing Student Use of a Problem-solving Framework

In this chapter:

- ✓ Flow charts of problem-solving decisions.
- ✓ Answer sheets with cues from the problem-solving framework
- ✓ Example problem solutions worked out on the answer sheets

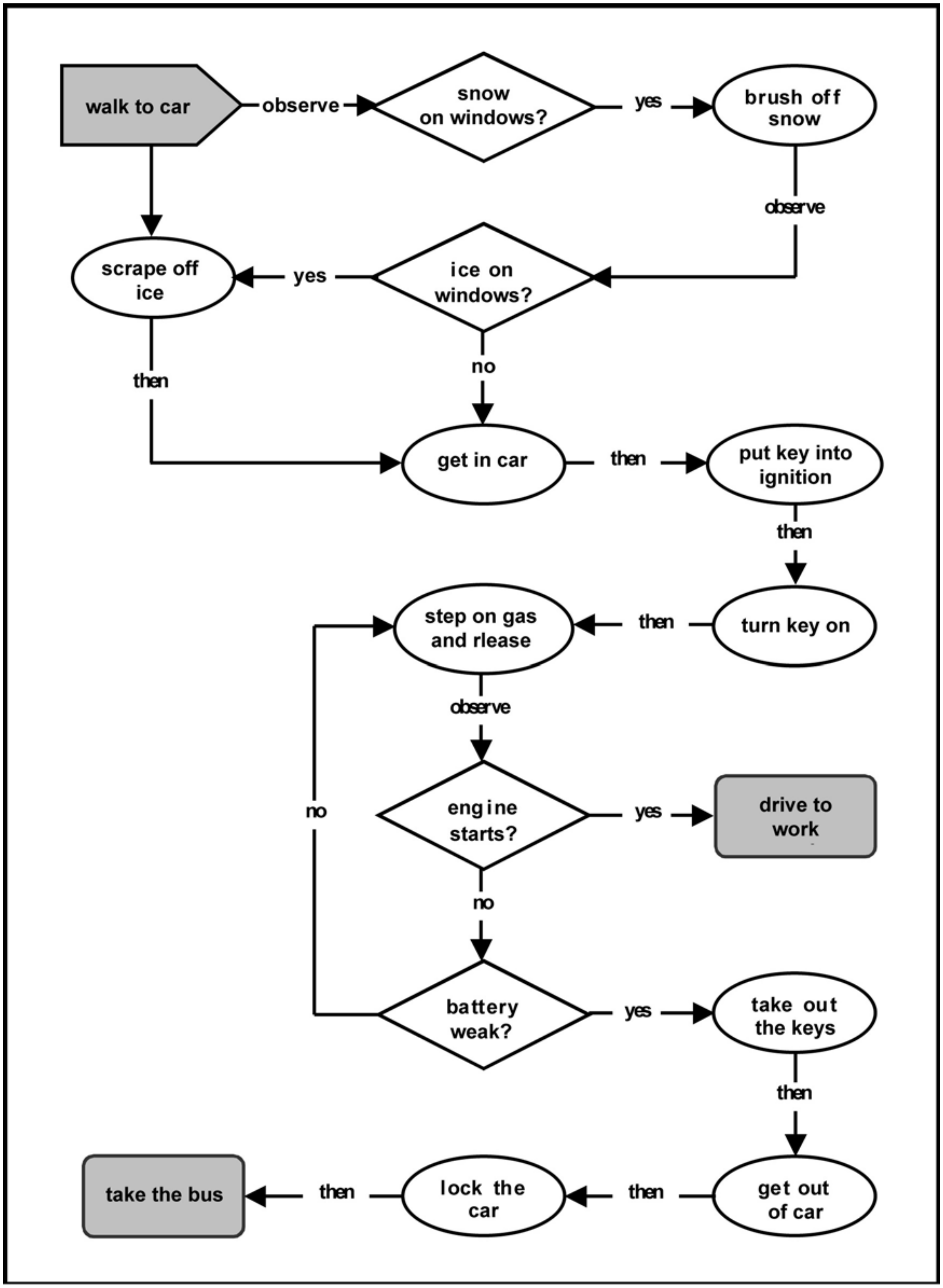
In Chapter 4 we described a general problem-solving framework used by experts in all fields, then gave an example of a problem-solving framework that we use in our introductory, algebra-based physics courses. A problem-solving framework is a logical and organized guide to the decisions needed to build a problem solution. It gets students started, guides them to what to consider next, organizes their mathematics, and helps them decide how to evaluate their answer. [See Chapter 14 for how to personalize a problem-solving framework for your students.]



Problem solving requires making many decisions. Breaking any activity down into the actions necessary to accomplish it, makes it seem very complex (see Figure 5.1). At first a problem-solving framework will seem too complex to your students. The Third Law of Education states: *Make it easier for students to do what you want them to do and more difficult to do what you don't want.* How can you make a problem-solving framework easier for your students to use?

One answer is to present and reinforce your problem-solving framework in a variety of ways. This helps students with different backgrounds and experiences build their own mental models of the framework. In Chapter 4 we showed one way to present a problem-solving framework. In this chapter we describe three additional ways to describe and present your problem-solving framework to help your students become more comfortable using it. In the next chapter (Chapter 6)

Figure 5.1. Steps for Starting a Car In Winter


walk to car
observe
snow on windows?
yes
brush off snow
observe
ice on windows?
yes
scrape off ice
then
no
get in car
then
put key into ignition
then
turn key on
then
step on gas and rlease
observe
engine starts?
yes
drive to work
no
battery weak?
no
yes
take out the keys
then
get out of car
then
lock the car
then
take the bus

we explain why these techniques, while necessary, are not sufficient for students to adopt a logical, organized problem solving framework. This is the rationale for Cooperative Problem Solving (CPS).

Flow Charts of Problem-solving Decisions

We commented earlier that, at first, a problem-solving framework seems very complex to your students. You need to reassure them that, once they get the hang of it, the framework is simpler and more natural than it appears. You might even show your students how a simple action, such as starting a car, looks complicated if you break it into steps. This example is shown in Figure 5.1. You might also remind them of the first time they drove a car. That it seemed like they had to check so many things and make so many decisions. Now they still have to check the same things and make the same decisions but it seems effortless.

Remember the 1st Law of Instruction: *Doing something once is not enough.* Demonstrating building a complete problem solution using a framework in your lectures at the beginning of the course is not enough. You must demonstrate the framework every time you give an example problem solution. This will be at least every time you introduce a new topic. Students also need to practice using the framework to solve appropriate problems with scaffolding for beginners. Scaffolding is structure that supports the learning of problem-solving skills. For example, we found that the one-page outline of the *Competent* problem-solving framework (Chapter 4, page 37) was not enough information for the majority of our students to understand and begin to use the framework by themselves. Many students needed to read an explanation of each step and sub-step in the framework.

We also found that these written explanations were also not enough for many students. We present each step of our Competent Problem-solving Framework as a flow chart of the actions in each step and sub-step, as illustrated in Figures 5.2a through 5.2e. Next to each action are some decisions, in question form, that students may need to make to complete each action successfully.

Notice that the flow charts are very simple, and limited to four or five actions. The last three steps include only one decision loop rather than the actual number of loops. This is intentional. Our research indicated that students in introductory physics courses find flow charts with more than one decision loop overwhelming and they reject them as too complex.

Problem-solving Answer Sheets

Another way to reinforce the use of a problem-solving framework is to provide worksheets for student solutions that include cues for the major steps of your problem-solving framework. Worksheets are another form of scaffolding or provide students with another “ladder” (scaffolding) up the learning mountain.

Figure 5.2a. Flow chart of Step 1 -- Focus on the Problem.

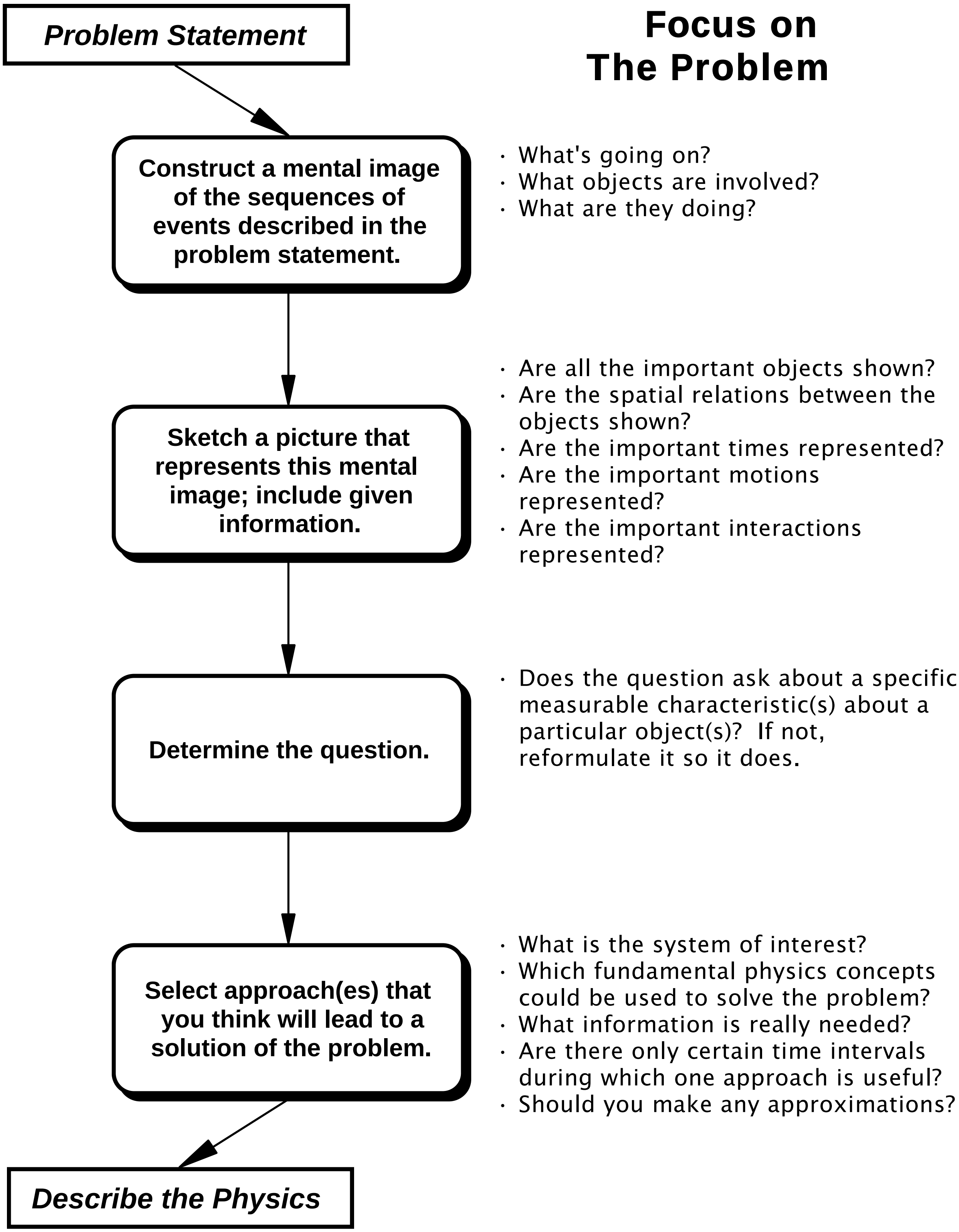
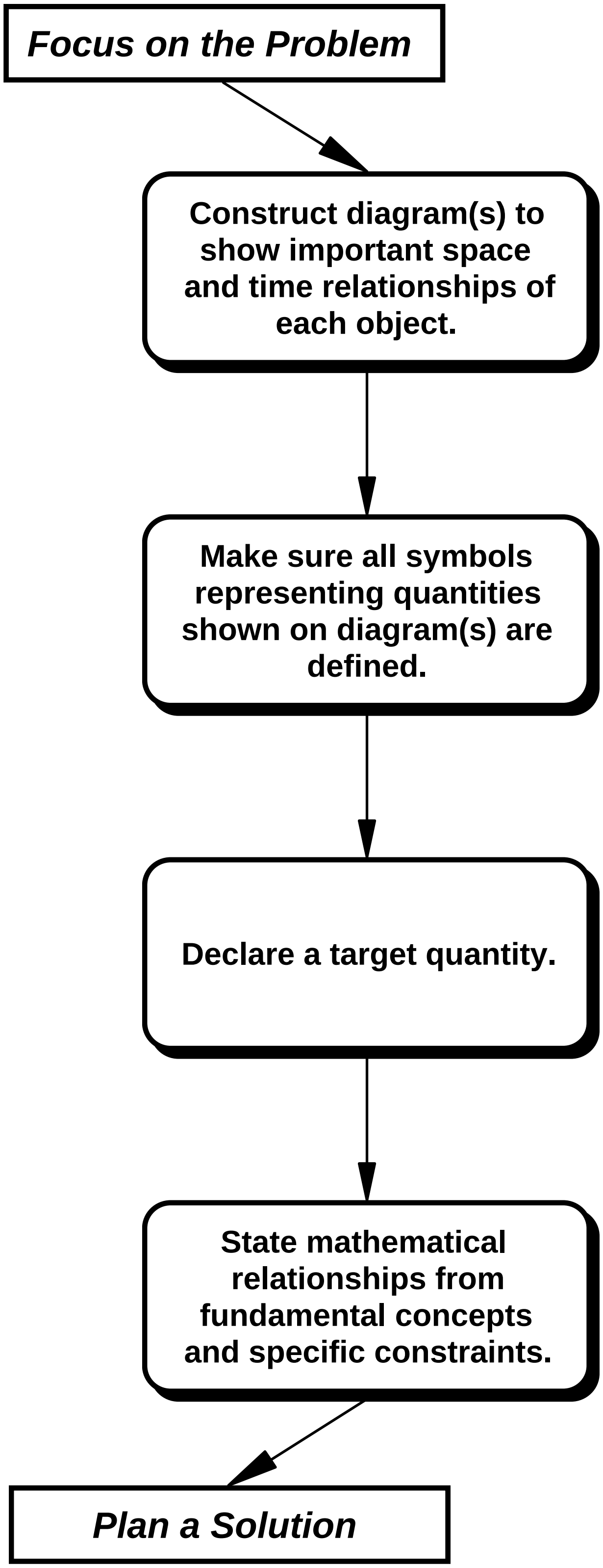


Figure 5.2b. Flow chart of Step 2 – Describe the Physics



Describe the Physics

- What coordinate axes are useful? Which direction should be positive?
- Relative to the coordinate axes, where is (are) the object(s) for each important time?
- Are other diagrams necessary to represent the interactions of each object or the time evolution of its state?

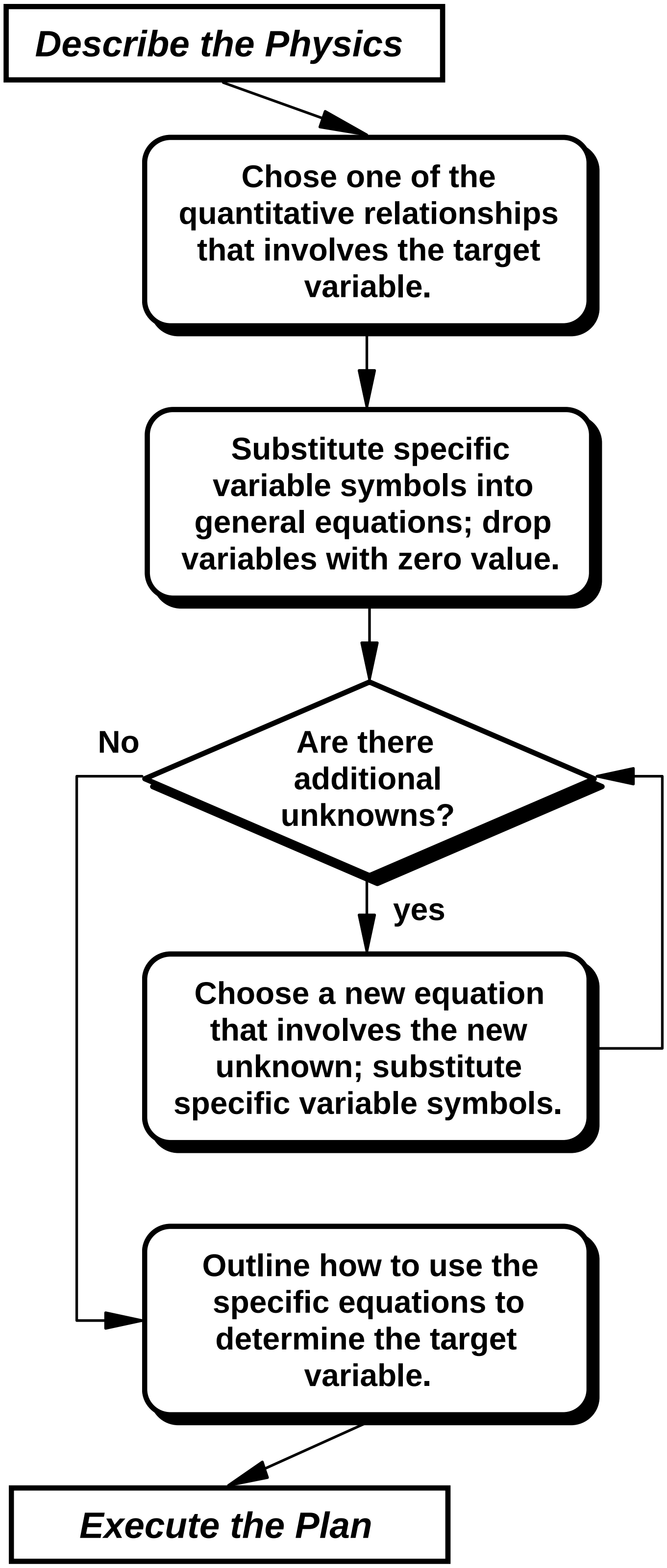
- What quantities are needed to define the problem mathematically using the approach chosen?
- Which symbols represent known quantities?
  Which symbols represent unknown quantities?
- Are all quantities having different values labeled with unique symbols?
- Do the diagrams have all of the essential information from the sketch?

- Which of the unknowns defined on the diagram(s) answers the question?

- What equations represent the fundamental concept(s) specified in our approach and relate the physics quantities defined in the diagram?
- During what time intervals are those relationships either true or useful?
- Are there any equations that represent special conditions that are true for some quantities in this problem?

Figure 5.2c. Flow chart of Step 3 – Plan a Solution

Plan a Solution



- Which quantitative relationship includes the target quantity?
- For what object does that equation apply?
- For what time interval does that equation apply?

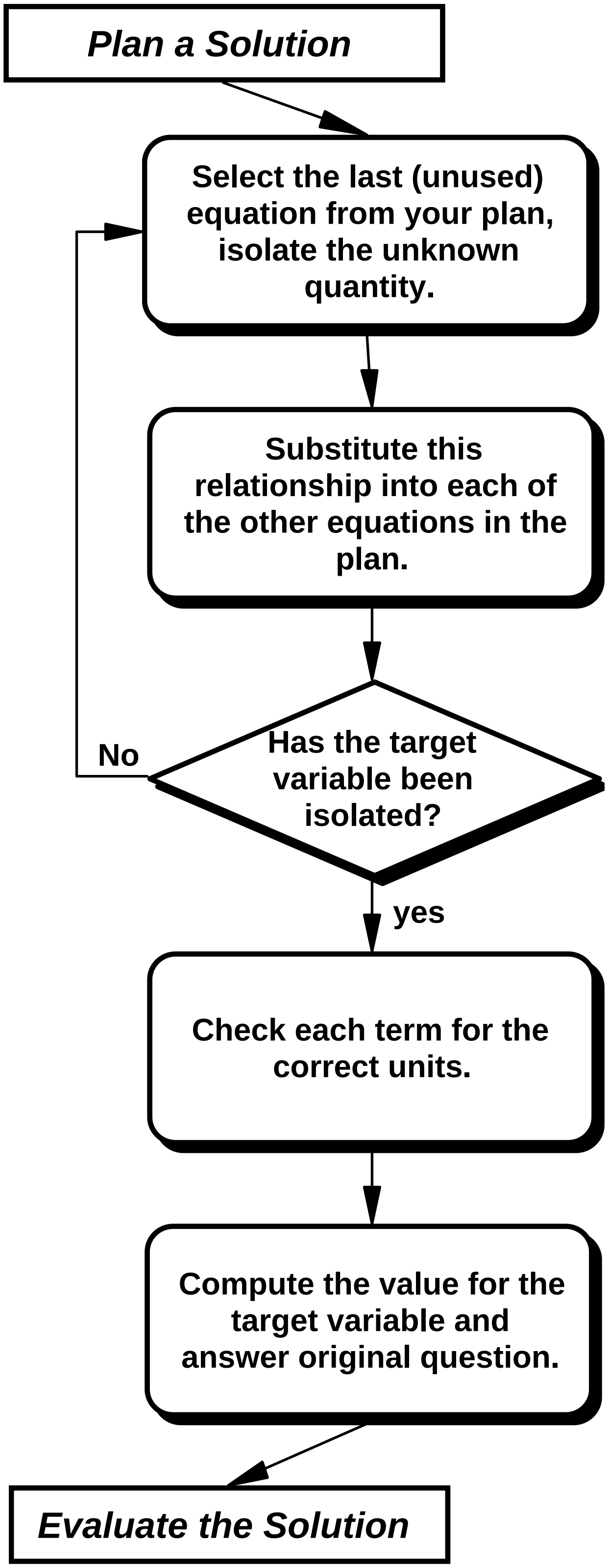
- Are there any unknowns in the equation other than the target quantity?
- Are there any unknowns that cancel out in the algebra?

- Which quantitative relationship includes the unknown quantity?
- For what object does that equation apply?
- For what time interval does that equation apply?
- Is this equation different from those already used in this solution?

- · Are there the same number of equations as there are unknown quantities? If not, will one of the unknowns cancel out in the algebra?
- In what order should the equations be combined in order to solve for the target quantity in terms of only known quantities?

Figure 5.2d. Flow chart of Step 4 – Execute the Plan

Execute the Plan



- Start with the first equation in your plan.
- The last equation used in this loop should contain the target quantity as the only unknown.

- What unknown is the target of this specific equation?
- Which the other unused equations in your plan have that unknown?
- Are there any quantities that cancel out in the algebra?

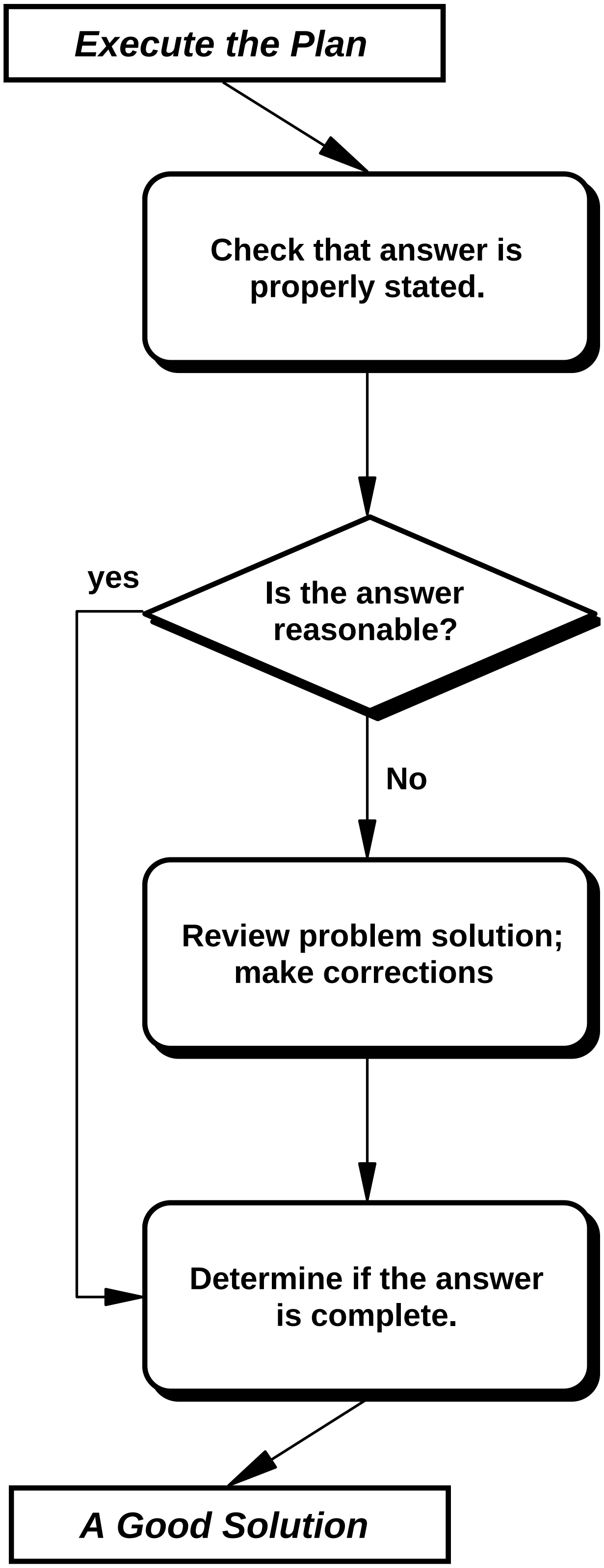
- Has each of the equations in the plan been used only once?
- Is each step of the mathematics legitimate?

- After all the substitution for unknowns, is the only unknown left the target quantity?
- Are the units the same on both sides of the equation?

- Which values (numbers with units) from the physics description should be put into the equation for the target quantity?
- Do you need to convert units?

Figure 5.2e. Flow Chart of Step 5 -- Evaluate the Solution

Evaluate the Solution



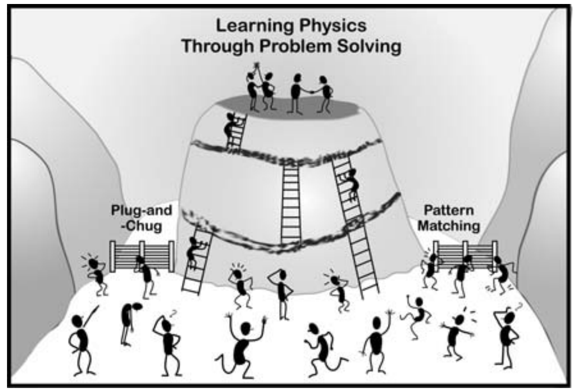
- Do the units make sense?
- Do vector quantities have both magnitude and direction?
- If someone else read just your answer, would they know what it meant?

- Does the answer fit with your picture of the situation?
- Is the answer the magnitude that you would expect in this situation?
- Do you have any knowledge of a similar situation that you can compare with to see if the answer is reasonable?
- Can you change the situation (and thus your equation for the target quantity) to describe a simpler problem to which you know the answer?

- Is your physics description complete?
- Are the definitions of your physics quantities unique?
- Do the signs of your physics quantities agree with your coordinate system?
- Can you justify all of the mathematical steps in your solution execution?
- Did you use units in a consistent manner in your execution?
- Is there a calculation mistake in the execution?

- Have you answered the question from the Focus the Problem step?
- Could someone else read and follow the solution plan?
- Are you sure you can justify each mathematical step by referring to your plan?

An example worksheet for the Competent Problem-solving Framework for an algebra-based course is shown in Figure 5.3.



We require our students to solve all individual and group test problems and group practice problems on these sheets for the first 4 - 6 weeks of the course.[1] Students can make copies of these sheets for solving their homework problems to help them practice correctly.

As students become more comfortable with the problem-solving framework, the worksheets are dropped -- students write their solutions on blank paper. However, the steps are given on all test Information sheets, as illustrated in Chapter 3, page 32.

Instructor Problem Solutions

Posting your problem solutions to homework or test problems using the framework also reinforces the student use. Figure 5.4 shows an example of a problem solution for a typical textbook homework problem. The problem is suitable for use at the end of the treatment of dynamics for students in an introductory algebra- or calculus-based physics class. The details within a solution depend on the specific goals of the physics class. In this case the solution was for an algebra-based class.

Putting Together the Scaffolding for Students: A Resource Book

Tip

We found it useful (and saves departmental funds) to put all the scaffolding for students into one resource booklet that students buy with their textbook or is available for download on the class web page.[1] This booklet includes the following items.

- ✓ An explanation of each step of the problem-solving framework.
- ✓ A one-page outline of the framework.
- ✓ Flow-charts for each step in the framework.
- ✓ A blank worksheet with cues from the framework (Figure 5.3). Students can photocopy the worksheet for practice.

The remainder of the booklet is divided into three parts, one for kinematics, one for dynamics using Newton's second law, and one for the conservation of energy. Each

Figure 5.3. Answer Sheets for Competent Problem-solving Framework
Calculus-based Course

FOCUS on the PROBLEM
Picture and Given Information

Question(s)

Approach

DESCRIBE the PHYSICS
Diagram(s) and Define Quantities

Target Quantity(ies)

Quantitative Relationships

PLAN the SOLUTION

Construct Specific Equations

Check for Sufficiency

Outline the Math Solution

EXECUTE the PLAN

Follow the Plan

Check Units

Calculate Target Quantity(s)

EVALUATE the ANSWER

Is Answer Properly Stated?

Is Answer Unreasonable?

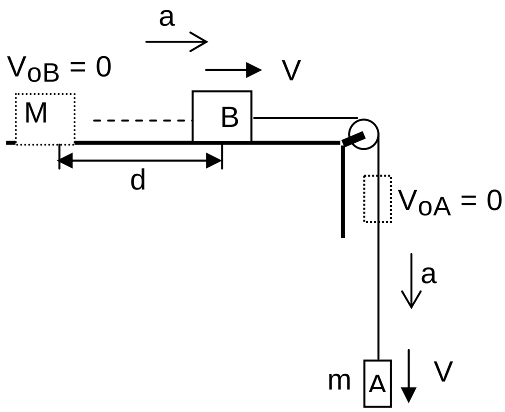
Is Answer Complete?

Figure 5.4. Example of a textbook problem solution on a worksheet.

Problem: A 20-Kg block is pulled along a horizontal table by a light cord that extends horizontally from the block over a pulley attached to the end of the table, and then down to a hanging 10-Kg block. The coefficient of friction between the 20-Kg block and the table surface is 0.40. Determine the speed of the blocks after moving 2 meters. They start at rest.

FOCUS on the PROBLEM

Picture and Given Information



$M = 20$ kg $\quad v_{oA} = v_{oB} = 0$

$m = 10$ kg $\quad v_{fA} = v_{fB} = v$

$d = 2$ m $\quad a_A = a_B =$ constant

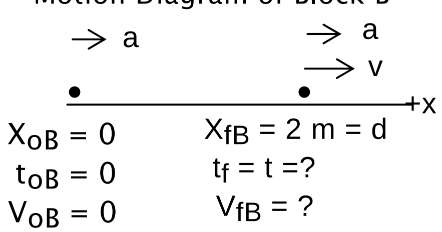
Question: Find speed of blocks after moving 2 m from rest.

Approach: Assume a massless string and a massless and frictionless pulley. Since the blocks move together, they always have the same speed and magnitude of acceleration. Use kinematics to relate final speeds to acceleration. Use Newton's Laws to find the constant acceleration.

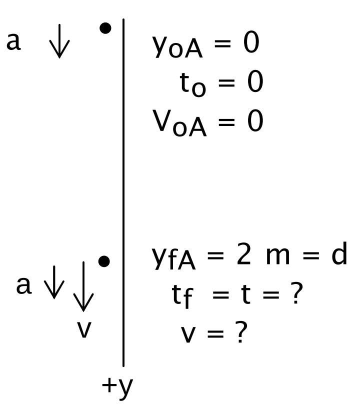
DESCRIBE the PHYSICS

Diagram and Define Variables

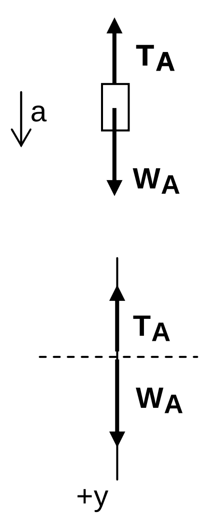
Motion Diagram of Block B



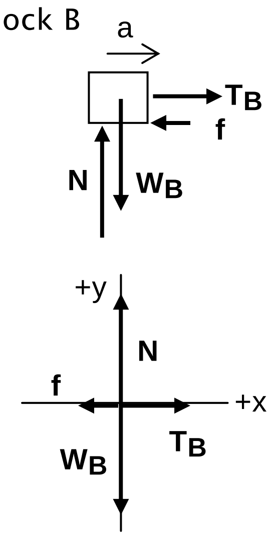
Motion Diagram of Block A



Block A



Block B



Constraints

Quantitative Relationships:

$$\bar{a}_r = \frac{v_{fr} - v_{or}}{t_f - t_o} \qquad \sum F_r = ma_r \qquad a_A = a_B = a$$

$$\bar{v}_r = \frac{r_f - r_o}{t_f - t_o} \qquad f = \mu N \qquad v_{fA} = v_{fB} = v$$

$$\bar{v}_r = \frac{v_{fr} + v_{or}}{2} \qquad W = mg \qquad T_A = T_B = T$$

Target Variable(s): $v_{fA} = v_{fB} = V$

PLAN the SOLUTION

	Unknown
Find v: for Block B	v
$\bar{v}_B = \frac{v + v_{oB}}{2}$	
① $\bar{v}_B = \frac{v}{2}$	$\bar{v}_B$
Find $\bar{v}_B$: for Block B	
$\bar{v}_B = \frac{x_{fB} - x_{oB}}{t_{fB} - t_{oB}}$	
② $\bar{v}_B = \frac{d}{t}$	t
Find t: for Block B	
$a_B = \frac{v_{fB} - v_{oB}}{t_{fB} - t_{oB}}$	
③ $a = \frac{v}{t}$	a
Find a: for Block A	
$\sum F_A = ma_A$	
$W_A - T_A = ma$	
④ $mg - T = ma$	T
Find T: for Block B (x direction)	
$\sum F_{xB} = Ma_{xB}$	
⑤ $T - f = Ma$	f
Find f	
⑥ $f = \mu N$	N
Find N: for Block B (y direction)	
$\sum F_{yB} = Ma_{yB}$	
⑦ $N - Mg = 0$	

Check for Sufficiency:

Yes -- 7 equations and 7 unknowns; solve problem working backwards from ⑦ to ①

EXECUTE the PLAN

Solve ⑦ for N and put into ⑥;

$$N - Mg = 0$$
$$N = Mg$$
$$f = \mu Mg$$

Put ⑥ into ⑤ and solve for T:

$$T - \mu Mg = Ma$$
$$T = Ma + \mu Mg$$

Put into ④ and solve for a:

$$mg - (Ma + \mu Mg) = ma$$
$$g(m - \mu M) = a(m + M)$$
$$g\frac{m - \mu M}{m + M} = a$$

Put into ③ and solve for t: Put into ②:

$$a = \frac{v}{t}$$
$$t = \frac{v}{a} = \frac{v(m+M)}{g(m - \mu M)}$$
$$\bar{v}_B = \frac{d}{t} = \frac{dg(m - \mu M)}{v(m+M)}$$

Put ② into ① and solve for v:

$$\bar{v}_B = \frac{v}{2} = \frac{dg(m - \mu M)}{v(m+M)}$$
$$v^2 = \frac{2dg(m - \mu M)}{m + M}$$
$$= \frac{2 \bullet 2m \bullet 9.8m/s^2 \bullet (10kg\ - 0.40kg)}{10kg + 20kg}$$
$$= 2.6\ m^2/s^2$$

$$\boxed{v = 1.6\ m/s}$$

EVALUATE the ANSWER

Does the mathematical result answer the question asked? *YES, the speed of the blocks after moving 2 meters from rest is 1.6 m/s.*

Is the result properly stated with appropriate units? *Yes, the units of v are m/s.*

Is the result unreasonable? *NO. v should be less than for free fall (6.3 m/s), and it is.*

section starts with an explanation of how to draw the appropriate physics diagrams: motion diagrams, force diagrams, and energy tables. Each section also includes the following.

- ✓ Several textbook like problems with solutions on the worksheets.
- ✓ Several context-rich problems for students to practice solving using the problem-solving framework.

Endnotes

1 For example, see Heller, K. & Heller, P. (2000), *The competent problem solver for introductory physics: Algebra*, McGraw-Hill Higher Education; and Heller, K. & Heller, P. (2000), *The competent problem solver for introductory physics: Calculus*, McGraw-Hill Higher Education

Chapter 6
Why Cooperative Problem Solving: Best Practices

In this chapter:

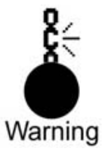
- ✓ Why use cooperative problem solving – the solution to a dilemma.
- ✓ A theoretical framework,.
- ✓ Establishing an environment of expert practice
- ✓ The relationship between course structures and cognitive apprenticeship

Chapter 2 addressed why many students in introductory courses fail to learn physics through problem solving. In Chapters 3 and 4 we claimed that students need to solve complex questions, such as context-rich problems, using a problem-solving framework that emphasizes making decisions based on physics concepts. Unfortunately, introducing suitable problems together with an expert-like problem solving framework, as outlined in Chapter 5, is not enough for students to be successful in learning physics through problem solving. In this chapter we describe the dilemma that led us to adopt Cooperative Problem Solving (CPS).

For many of us it is useful to have a theoretical framework to guide our actions, whether in science or in teaching. The final section of this chapter describes the theoretical framework we found useful implementing a coherent instructional strategy for introductory courses that leads to the improvement of students' conceptual understanding, problem solving performance, and more expert-like attitudes towards physics and learning physics. To be fair, we developed much of this pedagogy empirically before this theory was available based on good teaching practice and observation of student behavior. Nevertheless, once we were aware of the theory, the role played by the features of that pedagogy and what additional features should be added.

The Dilemma and a Solution

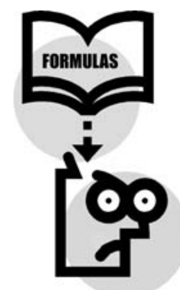
To learn physics, students need to examine their personal physics ideas and how they apply them in different situations. Problem solving, requires students to do just that if the questions they are addressing are complex, sophisticated and straightforward enough enough, such as context-rich problems, that requires using a framework that emphasizes making decisions using physics concepts. This problem-solving framework is initially unfamiliar, complex, and unnatural for beginning students. Even after clear and careful instruction, most students will fail in their initial attempts to use it. As a result they become frustrated, and revert to their familiar novice strategies (see Chapter 2 for novice strategies) which while they may not succeed are at least familiar.



It is reasonable to think that it might begin having them practice this problem solving framework with simpler, more straightforward problems such as those found in many textbooks. We did. We thought students could practice the problem-solving framework on those problems until the framework become more familiar, then they could successfully go on to context-rich problems. Unfortunately those straightforward problems yield, or seem to yield, to the students' novice strategies. Students see no reason to apply an unfamiliar problem-solving framework. They did not get the practice. That gave a dilemma:

- If students are given context-rich problems to solve using a competent problem-solving framework, then they are initially unsuccessful. There is no reason to change from their novice strategy that is also unsuccessful, but at least familiar.

- If students are given "easy" problems to solve using a competent problem-solving framework, they are initially successful using a novice strategy. Again, there is no motivation to change.



A solution to this dilemma is to use cooperative groups to provide support, so that students can be initially successful solving context-rich problems using an expert-like framework that has been explicitly demonstrated in class. The practice with cooperative groups provides the coaching that students need to move toward to an expert-like framework when solving problems individually. How this is done is discussed in Parts 2 and 3.

A Framework for Teaching and Learning Introductory Physics: Cognitive Apprenticeship

A useful instructional theory should help make connections between as many different aspects of teaching and learning as possible. Of course, it should agree with the currently available data and have some predictive power. As in science, it is not necessary that a theory be *true*, in some absolute sense, for it to be useful. The theory that we find useful in teaching introductory physics is called cognitive apprenticeship (or situated learning).[1] It supports out 1st Law of Instruction: Doing something once is not enough.

In a very real sense, the cognitive apprenticeship theory provides a description and justification of the very familiar educational scheme that is the basis for our graduate education. The theory also provides very practical guidance for teaching classes.

Cognitive Apprentice is based on two observations.

- Learning something like physics is a complex process that depends on students' existing knowledge and how they use that knowledge. Learning depends on the unique background of each learner.
- Apprenticeship is the most effective type of instruction that humans have devised for complex learning.

The goal of the inventers of cognitive apprenticeship was to identify the essential features that make apprenticeship so powerful and apply them to the classroom situation. One key feature of apprenticeship is that learning is directly connected to a situation that is *meaningful* to the student. Another essential feature is that the student must observe the action of experts, and the results of that action, from beginning to end. The beginning of an action must have a motivation meaningful to students (i.e., who cares?). The end of an action is a conclusion that students perceive as useful (i.e., what good is it?).

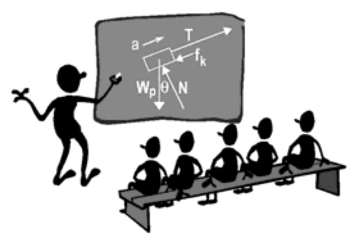
Within this framework, students must immediately engage in tasks that, while at a beginning level, they perceive as fitting into a more complex expert-like task. As students get such experience in a variety of contexts they build neural pathways that connect the new knowledge to their existing experiences. This integration makes new knowledge accessible to students, and is more useful than information learned in isolation. A rich context forces students to examine and perhaps modify their existing knowledge,, expressed as neural connections, when achieving a desired result.

Students begin with a knowledge structure that is a complex and unknown network of neural connections (see Chapter 2, page //). They can only explore these networks while in the act of using (activating) them and determining if they have achieved the desired results. Successful learning requires the establishment of new links in and between existing networks, establishing new networks, and eliminating or weakening old links in or between existing networks. This learning can be facilitated by instruction that is rich enough in context so that it is individualized, even in a large class. For humans, this construction of individual knowledge is fostered in large part, by interactions with other people.[2]

In an apprenticeship system, teaching has four essential functions: modeling, scaffolding, coaching, and fading. That is, first show students in detail what you want them to do (model or demonstrate). Provide structure or support for students to practice using what you want them to learn (scaffolding). Have students practice in their own way what you have demonstrated with corrective feedback as they are doing it (coach). Have them practice performing the task themselves while withdrawing some of the scaffolding (fade). This is not a sequential process in which each step is performed only once. Effective learning requires looping through these functions as needed by the individual student. Each of these functions is described below.

Modeling

Learning to accomplish a complex task is most efficient if the entire task is first shown to the students. Organizing the information from that demonstration of the process is aided if the learner knows the motivation for the task and major subtasks that comprise that task. For example, the first step in learning how to play baseball is to see a baseball game and to have the motivation for the major actions, such as hitting or running the bases, explained. Think how difficult it would be for people to learn how to play a game of baseball if they were first taught the vocabulary (e.g., base, ball, home run, double), then coached in specific skills (e.g., batting, pitching, sliding, bunting), all ***before*** they saw a game of baseball or were given the opportunity to play the game. Of course, having seen one or even many baseball games does not mean that you can play baseball.



For a student to learn an intellectual process, such as using the fundamental concepts of physics to solve problems, they must first see how the game is played. Someone must show them problems solved from their beginning to their end while making sure that *all* of the internal intellectual subtasks are explicitly shown to the student. Because people's observations are processed through their existing knowledge structure, such demonstrations of the same process must be repeated as the student gains knowledge. An experienced baseball player's observations of a baseball game are quite different from those who have never seen the game before, but both learn from the experience.

Even after a task is demonstrated in a clear and detailed manner, most people cannot accomplish that task on their own. They need to *practice*. However, practice is only beneficial if the correct procedures are practiced. A novice will usually practice an idiosyncratic blend of the new thing they were expected to learn and their existing knowledge base. This type of practice can actually be damaging to their progress. As a famous and successful football coach remarked, "Practice does not make perfect, only perfect practice makes perfect." [reference Vince Lombardy] Every learner needs scaffolding and an opportunity to practice while getting immediate feedback. A coach provides this feedback. Modeling is a necessary but not sufficient condition for learning -- scaffolding and coaching are essential.[3]

Scaffolding

In architecture, scaffolding is a *temporary* structure workers use during the construction of a building. Scaffolding lets workers reach places that would otherwise be inaccessible. In education, scaffolding is structure that supports learning. Scaffolding can include helpful instructor comments, a compelling task or problem, templates such as worksheets for problem solving, practice problems, grading rubrics, and collections of useful resources.[4] An example of scaffolding is training wheels for learning to ride a bicycle, that is taken away over time to assure that the learner is able to perform the skill on their own.

We have provided many examples of scaffolding in this book.

① Context rich problems (Chapters 3, 8, 15, and Appendices B and C)

② Verbal description of a problem-solving framework (Chapters 4 and 5)

③ Problem-solving flow charts (Chapter 5)

④ Information sheets for tests (Chapters 3 and 11)

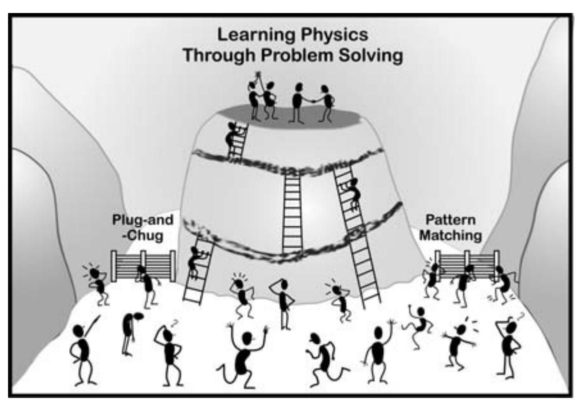
⑤ Worksheets with problem-solving cues (Chapters 5)

⑥ Instructor solutions (Chapters 5 and 11)

⑦ Description of roles for cooperative problem solving (Chapter 8)

⑧ Forms for groups to evaluate how their functioning while solving a group problem (Chapters 8 and 11)

In our introductory physics courses, the first six examples of scaffolding are also provided in a resource book for students.



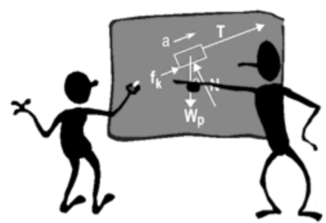
Scaffolding and the Third Law of Education. You may have noticed that scaffolding is related to our Third Law of Education: *Make it easier for students to do what you want them to do (ladders), and more difficult to do what you don't want them to do (fences).* Scaffolding comprises the ladders up the learning mountain.

Coaching

People learn by doing, but they require guidance to correct faults when they attempt to become skillful at a task that has been demonstrated. Without this guidance, practice can reinforce bad habits. “Coaching” consists of helpful instructor clues, reminders, or any type of constructive feedback given to students *while they are practicing a complex task such as problem solving in their own way.* Coaching gives students immediate feedback when they are most aware of the interconnections of their problem-solving actions to their knowledge base.

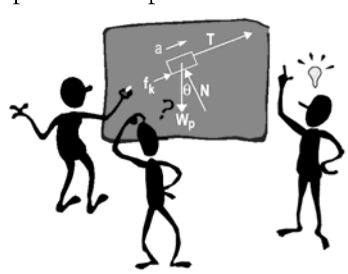
Coaching begins from an individual student’s approach. It is not a repeated illustration of the instructors “correct way.”. Coaching can only happen individually or in a small group because each person has a different background with different experiences, learning styles and talents (see Chapter 13 for how to coach in CPS). An experienced coach can diagnose a student’s difficulties with a task, probe the student’s knowledge network as applied to accomplishing that task, decide on a treatment for those difficulties, and guide the student in resolving the difficulties. One important part of coaching is to guide students’ actions by structuring the situation in which they practice.

In higher education, three different kinds of coaching can usually be provided to students: coaching by an expert (instructor), coaching by a person more experienced than the student (graduate or undergraduate teaching assistants), and peer coaching by fellow students at the same level.



`Expert Coaching.` An experienced coach can observe a student’s actions and evaluate what that student is attempting to do, why they are probably having difficulty, and how the student might make progress based on expert procedures and the current ability of the student. This coach can select from among several techniques guide the student to one that is appropriate. This coach can also construct artificial situations to point out inconsistencies in the student’s thinking or constrain the student to not practice a subtask incorrectly.

`Experienced Coaching.` A person with more experience than the student in accomplishing that task can diagnose a student’s ineffective or incorrect procedures. This coach can point out inconsistencies in the student’s thinking and demonstrate a correct procedure usually based on personal experience.



`Peer Coaching.` Peers can question whether an action is being done correctly or ineffectively and show how they do the action. In addition, to pointing out inconsistancies in the knowledge structure of others, the act of explaining is itself one of the most effective methods for learning.

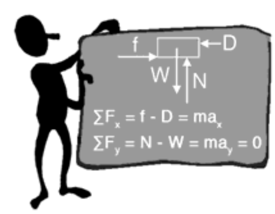
Each type of coach has advantages in an instructional situation. Clearly, the peer coach is more closely attuned to the vocabulary, meaningful examples, and ways of thinking of the other students and is less intimidating. Students are more likely to try out their own ideas with a peer than with a professor, or even with a more advanced student. Peer coaching is very efficient because everyone is in the process is learning. Every instructor knows that the best way to learn something is to teach it. However, peer coaches often do not have sufficient knowledge to recognize an error, or the experience to suggest how to make an idea plausible to others.

On the other hand, the expert coach can more easily recognize students' errors and suggest lines of thought that may be more aligned with those of the student.. Because the expert is usually also an authority figure, this coach can suppress a student's necessary exploration of their own ideas. With an expert coach, the student almost never gets the learning advantage of being the teacher.

Experienced coaches are a compromise between expert and peer coaches. They bring more expertise about the subject matter and different pathways to a solution than a peer coach. They can also be less of an authority figure than the expert coach. As a practical matter, there are usually more peer coaches available than either expert or experienced coaches. For this reason alone, peer coaches are useful to give the instant feedback so necessary in the coaching process.

A multi-level coaching system seems to work best in situations with a wide variety of students. The expert coach sets the practice task and designs the learning situation under which the task is carried out. The peer coaches explore their own, and each other's, way of accomplishing the practice task guided by their collective impressions of the correct outcome from the previous demonstrations of the process. The experienced coach corrects mistakes not caught in the peer coaching process and guides the students to be more effective peer coaches. In a traditional apprenticeship the master, the journeyman, and the other apprentices accomplish these three levels of coaching. In a research group the professor, the post doc and advanced graduate students, and the other graduate students at the same level accomplish the three levels of coaching.

Fading



The goal of instruction is for students to be able to accomplish a task on their own and in situations different from that of the instruction. Fading is the act of slowly removing as much of the instructional guidance as possible. Not only must the teacher set up the instructional framework to facilitate the coaching function, but must also remove much of this framework before the end of the course. For example, if worksheets are introduced to guide students through a problem-solving framework, they need to be replaced by plain paper before the student leaves the course. Another example is that students need to solve problems on their own in addition to solving them in a group.

Consistency

Research indicates that instruction that incorporates one or more common features contributes to better problem-solving performance in students.[5] These features are: (1) explicit teaching of problem-solving heuristics (similar to the Competent Problem Solving Framework); (2) modeling (demonstrating) the use of the heuristics by the instructor, and (3) requiring students to use the heuristics explicitly when solving problems. These are three features of cognitive apprenticeship. [what about coaching???]

Establishing an Environment of Expert Practice

In the learning model called cognitive apprenticeship the actions described above must be carried out in what is called an environment of expert practice. An environment of expert practice exists when each student knows what the instruction is trying to accomplish and why it is important to the student. In other words, every student should be able to answer the following three questions at any time in the course:

1. Why is what we are learning now important to me?
2. How is it related to what I already know?
3. How might I use this knowledge?

Constructing the environment of expert practice in a classroom requires that the instructor begin a lesson with a motivation within a context that is perceived as meaningful and useful to the student. It must continually and explicitly point out how the parts of the lesson link to students' previous knowledge, including their experiences. Because students do not have the same motivations or experiences, establishing an environment of expert practice means using a variety of contexts, examples, and motivations throughout the lesson. A lesson should not be structured as a mystery story with a surprising or interesting reveal at the end.

Course Structures and Cognitive Apprenticeship

Within a traditional course structure, modeling can be done in the lecture part of the class and coaching in the discussion and/or laboratory sections with the help of cooperative groups. In a studio or laboratory based course, modeling and coaching of problem solving can be interweaved as necessary in a very effective manner. Fading occurs in all venues, but is particularly apparent in individual assignments such as laboratory reports and on tests.

Lectures and Demonstrating the Problem-solving Process

The lecture is an effective method for demonstrating problem solving by showing, explaining, and motivating the details of each step of the solution process. One can introduce new physics topics by attempting to solve that needs the development of a new concept for its successful resolution. Demonstrating the problem-solving process during lectures can actively engage students. It is always important to allow the students several minutes to read the problem and begin their own solution before the demonstration of the process begins. This process serves as an advanced organizer so that students begin to access that part of their knowledge network that is relevant. To incorporate some peer coaching, students can be encouraged to compare their start of a solution with those of their neighbors and try to resolve any differences.

Good educational practice has always suggested pauses in a lecture to ask students a simple question and allow them several minutes to answer in writing, or now electronically. That question may be an elaboration of a step in the problem-solving process, a simple application of a point just demonstrated, or the logical next step in the solution process. Again peer coaching can be introduced by have students compare their results with their neighbors even in a large lecture class. To get good student participation, remember the Zeroth Law of Instruction (*If you don't grade it, students don't do it*) and collect at least some answers for grading. Electronic techniques using "clickers" makes this easy to do.

Demonstrating every decision in a problem-solving framework takes time. You will not be able to go through many problems in a class period. However, when you demonstrate problem solving that begins with the fundamental principles (e.g. Newton's second law, conservation of energy), every problem solved to illustrate one concept also reinforces problem solving for other concepts. Remember that the purpose of demonstrating is to illustrate the problem-solving process. Students will still need coaching to actually be able to do it.

Lectures and Coaching

It is possible to have some peer coaching interweaved into lectures. When pausing a lecture to ask questions, and after each student has written an individual answer, ask them to compare their answers with their neighbors. This time honored technique of peer coaching is especially effective just before an answer is to be submitted for grading.[6] The few minutes necessary for this technique is remarkably effective in keeping students involved in the process when the lecturer demonstrates the construction of a problem solution. See Chapter 9 page 102 for an outline of steps for demonstrating and coaching the use of a problem-solving framework for solving problems.

Sections and Coaching

The most effective coaching occurs in a small classroom situation that is physically configured to facilitate students working together with both peer and experienced coaching (see Chapter 9, pages 103 - 106). Cooperative grouping[7] is a very successful technique to structure the coaching process that has been used from elementary schools to business settings. It has been applied in many subject areas in the university. Describing cooperative grouping and its advantages for coaching problem solving in introductory physics is discussed in Part 3 (Chapters 11, 12, and 13).

Endnotes

[1] See, for example: Collins, A., Brown, J.S. & Newman, S.E. (1989), Cognitive apprenticeship: Teaching the crafts of reading, writing and mathematics, in *Knowing, learning, and instruction: Essays in honor of Robert Glaser* edited by L.B. Resnick, Hillsdale NJ, Lawrence Erlbaum, pp. 453–494; and Brown, J. S., Collins, A., & Duguid, P. (1989), Situated cognition and the culture of learning, *Educational Researcher*, **18**(1), 32-42.

[2] See, for example, the research summary in Bransford, J., Brown, A., & Cocking, R. (Eds), (2000), *How people learn: Brain, mind, experience, and school*, Washington, DC: National Academy Press. Cognitive Apprenticeship is often associated with the social learning theory of Lev Vygotsy. See, for example, Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes.* Cambridge, MA: Harvard University Press.

[3] Many researchers refer to two types of scaffolding, soft or contingent scaffolding (such as cooperative grouping with feedback) and hard scaffolding (e.g., problem solving flowcharts. See, Saye, J.W. and Brush, T. (2002), Scaffolding critical reasoning about history and social issues in multimedia-supported learning environments, *Educational Technology Research and Development*, **50**(3), 77-96

[4] Scaffolding has been defined and discussed theoretically in a variety of different ways. See, for example, Hartman, H. (2002). Scaffolding and cooperative learning, *Human Learning and Instruction*. New York: City College of City University of New York; Zydner, J. M. (2008). Cognitive tools for scaffolding students defining an ill-structured problem, *Journal of Educational Computing Research,* **38**(4), 353-385; Saye, J.W. and Brush, T. (2002), Scaffolding critical reasoning about history and social issues in multimedia-supported learning environments, *Educational Technology Research and Developmen*t, **50**(3), 77-96; Jonassen, D. (1997), Instructional design models for well-structured and ill-structured problem-solving learning outcomes. *Educational Technology Research and Development*, **45**(1), 65-94; Holton, D. and Clark, D. (2006), Scaffolding and metacognition, *International Journal of Mathematical Education in Science and Technology*, **37**, 127-143.

[5] Taconis, R., Ferguson-Hessler, M.G.M., &. Broekkamp, H. (2001), Teaching science problem-solving, *Journal of Research in Science Teaching*, **38**, 442–468. The author did a meta-analysis of 22 previously published articles on teaching problem solving in science classes. They also found that having students work in groups did not improve problem solving unless the group work was combined with the teaching of problem-solving heuristics, modeling the use of the heuristics by the instructor, and/or requiring students to use the heuristics explicitly when solving problems.

[6] McKeachie, J.W. (1996), *Teaching tips: strategies, research, and theory for college and university teachers,* Lexington, MA, D.C. Heath. For an application in physics see Mazur, E. (1996), *Peer Instruction: A User's Manual*, Prentice-Hall.

[7] Johnson, D.W, Johnson, R.T. & Smith, K.A. (2006), *Active learning: Cooperation in the college classroom*, *3rd Ed.* Edina MN, Interaction Book Company.

Part 2

Using Cooperative Problem Solving

In this part . . .

There is an old cliché that fits here, "If it ain't broke, don't fix it!" We are assuming that while your introductory physics course may not be completely broken, you are dissatisfied with your students' problem-solving performance and are looking for a way to "fix it." There are many physics education reforms from which to choose. In the chapters of Part 2 we provide information to help you decide whether you want to adopt Cooperative Problem Solving.

This part of the book attempts to answer two questions to help you determine if cooperative problem solving might be useful in your course. The first question is: What is cooperative problem-solving (CPS)? In answering this question, *Chapter 7* describes the differences between students doing group work and students working in a cooperative group.

Chapters 8 and 9 give practical information about the second question: What course changes are needed for optimal implementation of CPS? *Chapter 8* discusses appropriate problems and how to structure and manage cooperative groups for optimal implementation of CPS. *Chapter 9* discusses how to structure a course for CPS, including scheduling and other resources (personnel and space), as well as appropriate grading practices for the course and for grading students' problem solutions. Both chapters convey "what to shoot for," and *not* where you can make a reasonable beginning. The last section Chapter 9 outlines a way to get started in lecture with informal groups.

Chapter 10 describes the research results for improvement in problem solving skills and conceptual understanding of physics with Cooperative Problem Solving (CPS), both for partial and full implementation.

Chapter 7
Cooperative Problem Solving: Not Just Working in Groups

In this chapter:

- ✓ The differences between traditional and cooperative groups.
- ✓ The five elements of cooperative problem solving.
- ✓ The differences in achievement in traditional and cooperative settings.

Just what is cooperative problem solving? The answer to this question is *not* having students spend some time each week solving problems in a group. We have often met instructors who have had students solve problems in groups, with no improvement in individual problem-solving performance. In fact the research to date indicates that group work, by itself, does not raise the achievement of students. As is often the case, to understand what cooperative-group problem solving is, it is helpful to know what it isn't. In this chapter we describe some of the differences between traditional-group and cooperative-group problem solving.

Two Examples

Imagine that you observed two small classes in which students work in groups to solve a problem. Each class consists of 15 – 18 students, and they are solving a kinematics problem with two-dimensional motion. The first class is what we will call "traditional-group problem solving." In the second class, students are engaged in cooperative-group problem solving. Below is a description of what you might observe in each class.

Example of Traditional-group Problem Solving

Before class begins, the students are sitting quietly in rows facing the instructor. The instructor begins the class by talking for about 15 minutes, reviewing the projectile motion equations, then tells the students to get into groups and solve Problem #5 at the end of Chapter 2 in their textbook. The instructor sits at a desk in front of class.

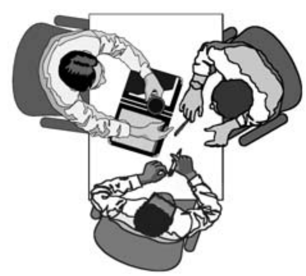
The students get into groups, usually with their neighbors. The class is very quiet, and *most students are working independently*. They flip through Chapter 2 for a while, then settle on a page what seems to have a relevant example solution to a similar problem. Each student starts writing a solution. Occasionally they talk with each other, asking questions about what page they are on or what equation to use.

After about 20 minutes, some students turn towards each other and start comparing their solutions. They start by comparing their numerical answers. In the groups where members got the same answer, the conversation usually turns social. In the groups with different individual answers, students compare the equations they used and the numerical answers for different intermediate quantities. In some groups, a student will recognize or agree that they did something wrong, and decide to change their solution. In other groups, however, students do not come to an agreement about how to solve the problem.

The instructor goes from group to group, answering the individual questions of students. Shortly before the end of class, the instructor collects the students' individual solutions and shows them on the wall board how to solve the problem.

Example of Cooperative Problem Solving

Just before class students are clustered in groups of three or four, facing each other and talking. The instructor begins class by talking about 5 minutes -- reminding students that they have just started two-dimensional motion, that the problem they will solve today was designed to help them understand the relationship between one-dimensional and two-dimensional motion, and they will have 35 minutes to solve the problem. The instructor gives one person in each group (the recorder) and answer sheet, and provides all students access to the problem and all the fundamental equations they have studied in class to this point in time (either projected or on individual sheets).



The class is quiet for a few minutes as they read the problem, then there is a buzz of talking for the next 30 minutes. No textbooks or notes are open. Group members are talking and listening to each other, and only one member of each group is writing a problem solution. They mostly talk about how the objects are moving, what they know and don't know, what they need to assume, the meaning and application of the equations that they want to use, and the next steps they should take in their solution.

The instructor circulates slowly around the room, observing and listening, occasionally interacting with the groups that the instructor judges need the most help. This pattern of circulating and intervention continues for about 30 minutes. At the end of that time, the instructor assigns one member from each group (usually not the member who recorded the solution) to draw a motion diagram and write the equations they used to solve the problem on a wall board. That member can ask for help from the remaining group members as necessary. The instructor then tells the class to examine the board for a few minutes to determine the similarities and differences of the group solutions. Based on what is on the wall board, the instructor then leads a class discussion focused on the confusion between the perpendicular components of two-dimensional motion.

These examples illustrate several differences between traditional group problem solving and cooperative problem solving. These differences are summarized in Figure 7.1. Cooperation is **not** having students sit side-by-side and talk with each other as they do their individual assignments. Cooperation is **not** assigning a problem (or lab report) to a group of students where one student does all the work and others put their names on it. Cooperation is **not** having students solve problems individually with the instruction that the ones who finish first are to help the slower students. Cooperative Problem Solving is much more than being physically near other students, discussing the problem with other students, helping other students solve the problem, or sharing procedures, although each of these is important.

Elements of Cooperative Problem Solving

Johnson, Johnson, and Smith (2006)[1] characterize cooperative learning through five basic elements. These elements are described briefly below.

Positive Interdependence exists when each student in a group believes they are linked such that they cannot succeed unless everyone contributes. In other words, they believe that they "sink or swim together."

In a problem-solving session, positive interdependence is structured by group members: (1) agreeing on the answer and solution strategies (goal interdependence); and (2) fulfilling assigned role responsibilities (role interdependence). The instructor must assess the group problem solutions and give the results back to the groups. Occasionally the result of the assessment is a grade, with each group member getting the same grade (reward interdependence). See Chapters 8 and 9 for more details about role assignment and grading.

Figure 7.1. Differences between traditional group and cooperative group problem solving

Traditional Groups	Cooperative Groups
Students are assigned to work together, and accept that they have to do so (or at worst, have no interest in doing so).	Students are assigned to work together and recognize that it is useful to do so.
The group problems require very little joint work -- they can be solved as easily by an individual as by a group.	The group problems require a level of physics knowledge and decision-making that make them difficult for even the best students' to solve individually.
Best Case: Students believe that they will be evaluated and rewarded as *individuals*, not as group members. So they interact primarily to compare their procedures and solutions. There is no motivation to teach or learn from each other. Worst Case: Students believe they are competing for the best grades. So when they interact, they view each other as rivals who must be defeated or "dummies" whose questions steal time from their own learning.	Students believe that their success depends on the efforts of all group members. They interact by co-constructing a single problem solution. They help each other by clarifying, explaining, and justifying ideas and procedures as they solve the problem.
When groups are not functioning well, individuals either disengage or blame their other group members for not being prepared. Students who are not doing as well as they would like blame the other group members for “dragging them down.”	Students hold each other accountable for doing high quality work while they solve the problem together. All members take responsibility for providing leadership and resolving conflicts.
After the review, the instructor does nothing except answer factual questions (or at worst, spends all the time helping a few students get the right answer).	The instructor constantly monitors groups, coaching groups that need help with specific points of physics and groups that are not functioning well. This is followed by a whole-class discussion.

Face-to-Face Promotive Interaction exists when each student orally gives a provisional approach to a task, discusses with other members of the group the nature of the physics concepts and problem-solving techniques being used, teaches their knowledge to the other group members, and discusses the connections between present and past learning. This face-to-face interaction is called "promotive" because students promote (i.e., help, assist, encourage, and support) each other's efforts to learn.

Individual Accountability/Personal Responsibility requires the instructor to ensure that the performance of each individual student is assessed and the results given back to the individual as well as the group. Each student needs to know that they are ultimately responsible for their own learning. No one can "hitch-hike" on the work of others. Common ways of structuring individual accountability include giving individual exams, choosing a group spokesman at random when the instructor interacts with a single group and when the group interacts with the class (see Chapter 12, page //).

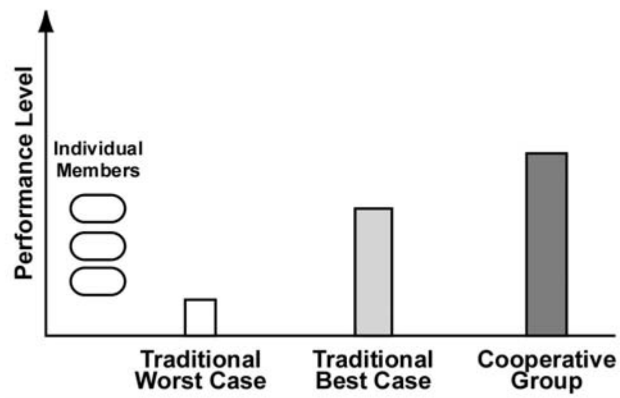
Collaborative Skills are necessary for group functioning. Students must have and use the needed leadership, decision-making, trust-building, communication, and conflict-management skills. Explicit attention must be paid to the development of these skills, especially with students who have never worked cooperatively in learning situations. A common way to reinforce collaborative skills is through role assignment (see Chapter 8, pages //-//) and coaching in these skills when intervening with dysfunctional groups (see Chapter 13).

Group Processing requires each group to discuss among its members how effectively they worked together and maintained an effective working relationship. For example, at the end of their working period the groups might discuss their functioning by answering two questions:

1. What is something each member did that was helpful for the group?
2. What is something each member could do to make the group more effective next time?

Such processing is a quick method of diffusing resentments, improving collaborative skills, ensuring timely feedback, and reminding students that everyone must contribute for the group to be efficient (see Chapter 8, pages // to //).

Figure 7.2. A representation of the performance level of traditional and cooperative groups: a summary of research across different age groups and subject areas.



The next five chapters explain in detail how these elements of cooperative learning can be implemented in Cooperative Problem Solving (CPS).

Achievement for Traditional Groups and Cooperative Groups: Summary

The major difference between traditional-group and cooperative-group problem solving is *how* students go about solving the problem. In traditional-group problem solving, students tend to solve the problem individually, typically with the help of a textbook or class notes, then compare their answers in groups. In cooperative group problem solving, students are actively engaged in the ***co-construction*** of a problem solution, based on what they know at the time.

There are several learning advantages to the collaborative co-construction of a problem solution.[2] During this process, students actively examine their own conceptual and procedural knowledge they request explanations and justifications from each other. This constant process of explanation and justification helps clarify *each* member's thinking about the physics concepts to be used, and how these concepts should be applied to the particular problem.[3] Moreover, each member practices and observes the thinking strategies of others. This introduces differences that they can incorporate into their problem solutions.

Nearly 600 experiments and over 100 correlation studies have been conduced during the last 100 years comparing the effectiveness of cooperative, competitive, and individual learning environments. Cooperation among students typically results in higher achievement and greater productivity for different age groups, in different subject areas, and in different settings.[4] A summary of the studies conducted at the higher education level may be found in Johnson, Johnson, and Smith (2006).[1]

From this extensive research, the difference in achievement of traditional-group and cooperative-group problem solving is summarized in Figure 7.2. At the worst (when students compete for grades), group performance is less than the potential of the individual members. The average class performance is typically lower than in classes with no group work. At the best (when students are evaluated against an absolute standard), the group performance is better than that of some of the members but the more hard-working and conscientious members would perform better if they worked alone. The average class performance on individual tests is about the same or slightly higher than in classes with no group work. With CPS, the group is more that the sum of its parts, and **all** students (even the best students) perform better on both group and individual problems than if there were no group component to the instruction. The average class performance is higher than in classes with no group work.[5]

Endnotes

[1] Johnson, D.W, Johnson, R.T. & Smith, K.A. (2006). *Active learning: Cooperation in the college classroom*, *3rd Ed.* Edina MN, Interaction Book Company.

[2] See, for example: Schoenfeld, A.H. (1983), Beyond the purely cognitive: Belief systems, social cognitions, and meta-cognitions as driving forces in intellectual performance," *Cognitive Science*, **8**, 173-190; Brown, A. L. & Palincsar, A. S. (1989), Guided, cooperative learning and individual knowledge acquisition, in *Knowing, learning, and instruction* edited by L. B. Resnick, Hillsdale NJ, Lawrence Erlbaum, pp. 393-451; and Collins, A., Brown, J. S. & Newman, S. E. (1989), Cognitive apprenticeship: Teaching the crafts of reading, writing and mathematics," in *Knowing, learning, and instruction: Essays in honor of Robert Glaser,* edited by L. B. Resnick, Hillsdale NJ, Lawrence Erlbaum, pp. 453 – 494.

[3] Hollabaugh, M. (1995), *Physics problem-solving in cooperative learning groups*, Ph.D. Thesis, University of Minnesota

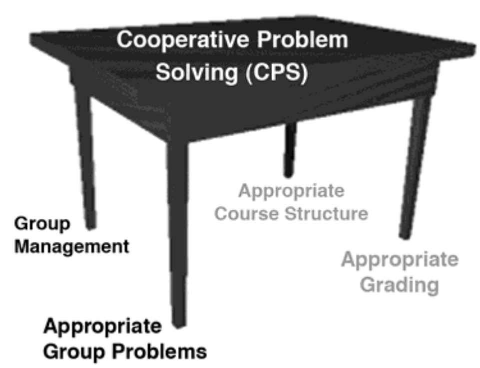
[4] See Brown, A. L. & Palincsar, A. S. (1989), Guided, cooperative learning and individual knowledge acquisition, in *Knowing, learning, and instruction* edited by L. B. Resnick, Hillsdale NJ, Lawrence Erlbaum, pp. 393-451; Johnson, D.W., Johnson, R.T., & Smith, K.A. (1991), *Cooperative learning: Increasing college faculty instructional productivity*. ASHE-ERIC Higher Education Report 1991-4, Washington, DC: The George Washington University, School of Education and Human Development; and Johnson, D.W., Johnson, R.T., and Smith, K.A. (2007), The state of cooperative learning in postsecondary and professional settings. *Educational Psychology Review,* **19** (1), 15-29.

[5] For data supporting these two statements, see Heller, P., Keith, R., & Anderson, S. (1992), Teaching problem solving through cooperative grouping. Part 1: Groups versus individual problem solving, *American Journal of Physics* **60**(7), 627-636.

Chapter 8
Managing Groups and Appropriate Group Problems

In this chapter:
- ✓ Managing groups: group size, assignment, changing groups, role assignment and rotation, and group processing.
- ✓ Why textbook problems are ineffective group problems.
- ✓ What are the criteria for effective problems?

Like a table, Cooperative Problem Soving (CPS) is supported by four legs:
- ✓ Managing groups for optimal learning.
- ✓ Appropriate group problems
- ✓ Appropriate grading
- ✓ Appropriate course structure for cooperative problem solving



If one or more of the legs is short, then the table is unbalanced. The learning gains will be less than with optimal implementation of CPS. How close to the optimal you can get depends on your constraints and experience (see Chapters 9 and 11).

This chapter contains suggestions and recommendations for and structuring and managing groups and appropriate group problems. The following chapter provides recommendations for structuring the introductory physics course and appropriate grading.

Group Structure and Management

There are several aspects of group structuring that affect learning, such as group size, group composition, how long groups stay together, and the roles of individual students in the groups. Our recommended structures and their rationale are described in this section. The figures contain a brief description of our research that supports each structure.[1]

Figure 8.1. Why three-member groups are better than pairs or four-member groups?

For the co-construction of a physics problem solution by students in introductory courses, we found the "optimal" group size to be **three** members. A three-member group is large enough for the generation of diverse ideas and approaches, but small enough to be manageable so that all students can contribute to the problem solution.

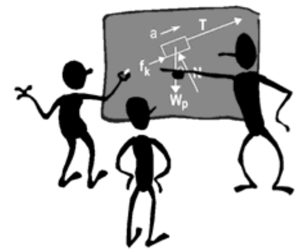
An examination of written group problem solutions indicated that three- and four-member groups generate a more logical and organized solution with fewer conceptual mistakes than pairs. About 60 - 80% of pairs make conceptual errors in their solution (e.g., an incorrect force or energy), whereas only about 10 - 30% of three-or four member groups make these same errors. Observations of group interactions suggested several possible causes for the lower performance of pairs. Groups of two did not seem to have the "critical mass" of conceptual and procedural knowledge for successful completion of context-rich problems. They tended to go off track or get stuck with a single approach to a problem, which was often incorrect.

With larger groups, the contributions of the additional student(s) allowed the group to jump to another track when it seemed to be following an unfruitful path. In some groups of two, one student often dominated the problem solving process, so the pair did not function as a cooperative group. A pair usually had no mechanism for deciding between two strongly held viewpoints except the constant domination of one member, who was not always the most knowledgeable student. This behavior was especially prevalent in male-female pairs. In larger groups, one student often functioned as a mediator between students with opposing viewpoints. The issue was resolved based on physics rather than the personality trait of a particular student.

In groups of four students, either one person was invariably left out of the problem solving process or the group split into two pairs. Sometimes the person left out was the more timid student who was reticent to ask for clarification. At other times the person left out was the most knowledgeable student who appeared to tire of continually trying to convince the three other group members to try an approach, and resorted to solving the problem alone. To verify these observations, we counted the number of contributions each group member made to a constant-acceleration kinematics problem from the videotapes of a typical three-member and four-member group. Each member of the group of three made 38%, 36%, and 26% of the contributions to the solution. For the group of four, each member made 37%, 32%, 23%, and 8% of the contributions to the solution. The only contribution of the least involved student (8%) was to check the numerical calculations.

Group Size and Assignment

We found that the optimal group size is three for students not experienced in effective group work (see Figure 8.1). In retrospect, the reason is almost obvious. Groups of two have no simple mechanism for deciding between two strongly held opinions. Within a group of three, each of the proponents must explain their idea to a third person. Also a group of two introductory students often lacks some physics knowledge necessary to attack a problem. With a group of four it is difficult for each student to contribute to the problem solution in the time they have. Our research indicated that one member is usually quiet, although they may be an effective group member in well-functioning groups. Of course, if your class is not divisible by three, then you will have some pairs or four-member groups. We found that four-member groups generally work better than pairs. [The exception to this rule-of-thumb is if students are working with computers -- then pairs are preferable to groups of four.]



We recommend assigning students to ***mixed-achievement groups*** based on past problem-solving performance on exams, rather than letting students form their own groups (see Figure 8.2). Below are the advantages of group assignment.

Optimal Learning. The **most important** reason to assign students to groups is because past research in cooperative group learning (including our own research) indicates that students learn more when they work in mixed-performance groups than when they work in homogeneous-performance groups. We do not, however, want students to label the high, medium and lower-performance students in their groups, so we do not tell them how we assign group membership (see also Chapter 11, page //).

Law

Attitude Advantage. It is much easier to set and enforce rules in the beginning of a class and loosen the enforcement later, than to do the opposite. No matter what you do, you will have to change at least some group members to avoid dysfunctional groups. If you assign groups at the beginning, you will have fewer disgruntled students. You can loosen the rule of assigned groups later in the course if your students get to know each other and become experienced in effective teamwork. This is an example of the 2nd Law of Instruction: *Don't change course in midstream; structure early then gradually reduce the structure.*

Practical Advantage. There are practical reasons for assigning students to groups. For example if you teach in a large commuter school, most students do not know each other at the beginning of class. They would feel very uncomfortable being told simply to "form your own groups." If you teach at a small residential college, students may know each other but have established behavior patterns that are not based on learning physics, and often not conducive to it. Assigning groups allows the natural breakup of existing social interaction patterns.

Changing Groups

There are both optimal-learning and practical reasons for changing groups.


Warning

Avoid Homogeneous Groups. One reason to change groups is that you are likely to start with many homogeneous-achievement groups, which is not optimal for student learning. Normally you do not know the problem-solving performance of your students at the beginning of class. With a small number of students, there can be large random fluctuations in the achievement-mix of your groups.

Avoid Role Patterns. In groups, the necessity to verbalize the procedures, doubts, justifications and explanations helps clarify the thinking of all group members. Students both practice and observe others perform these roles, so they become better individual problem solvers. If students stay in the same group too long, they

Figure 8.2. Why it better to assign students to mixed-achievement groups

In our research, we examined the written problem solutions of both homogeneous and mixed-achievement groups (based on past problem-solving test performances). The mixed-performance groups (i.e., a high, medium and lower performing student) consistently performed as well as high performance groups, and better than medium and low performance groups. For example, our algebra-based class was given a group problem that asked for the light energy emitted when an electron moves from a larger to a smaller Bohr orbit. 75 percent of the mixed-performance groups solved the problem correctly, while only 45% of the homogeneous groups reached a solution.

Observations of group interactions indicated several possible explanations for the better performance of heterogeneous groups. For example, on the Bohr-orbit problem the homogeneous groups of low-and medium-performance students had difficulty identifying energy terms consistent with the defined system. They did not appear to have a sufficient reservoir of correct procedural knowledge to get very far on context-rich problems. Most of the homogeneous high performance groups included the gravitational potential energy as well as the electric potential energy in the conservation of energy equation, even though an order-of-magnitude calculation of the ratio of the electric to gravitational potential energy had been done in the lectures. These groups tended to make the problem more complicated than necessary or overlooked the obvious. They were usually able to correct their mistake, but only after carrying the inefficient or incorrect solution further than necessary. For example, in the heterogeneous (mixed-performance) groups, it was usually the medium or lower performance student who pointed out that the gravitational potential energy term was not needed. ["But remember from lecture, the electric potential energy was lots and lots bigger than the gravitational potential energy. Can't we leave it out?"] Although the higher performance student typically supplied the leadership in generating new ideas or approaches to the problem, the low or medium performance student often kept the group on track by pointing out obvious and simple ideas.

In heterogeneous groups, the low- or medium-performance student also frequently asked for clarification of the physics concept or procedure under discussion. While explaining or elaborating, the higher-performance student often recognized a mistake, such as overlooking a contributing variable or making the problem more complicated than necessary. For example, a group was observed while solving a problem in which a car traveling up a hill slides to a stop after the brakes are applied. The problem statement included the coefficient of both static and kinetic friction. It was the higher performance student who first thought that both static and kinetic frictional forces were needed to solve the problem. When the lower-performance student in the group asked for an explanation, the higher-performance student started to push her pencil up an inclined notebook to explain what she meant. In the process of justifying her position she realized that only the kinetic frictional force was needed.

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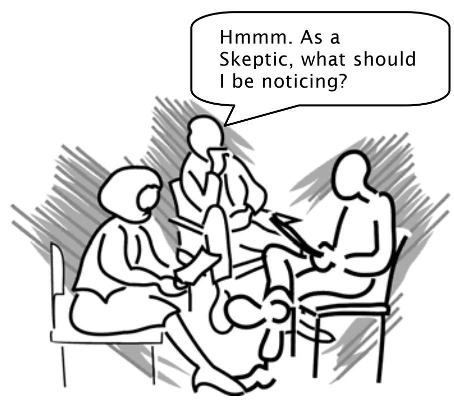
tend to fall into role patterns. The result is that they do not rehearse the different roles they need to perform on individual problems, and consequently do achieve optimal learning gains.

Difficult Students. A third, practical reason for changing groups is that your first group assignments may include some dysfunctional groups (because of personality conflicts). Students find it miserable to contemplate working a whole semester with someone who isn't compatible, and may disengage. However, most will accept the challenge of working together if they know that it is for a limited time. After you get to know the students better, you can place the "difficult" students in a better group. Strategies for dealing with difficult group members are discussed in Chapter 13.

Individual Responsibility. Finally, one of the most important reasons to change groups is to reinforce the importance of the individual in cooperative problem solving. The most difficult point in the course for group management is the first time you change groups. By that time most groups have been reasonably successful, and students are convinced they are in a "magic" group. Changing groups elicits many complaints, but is necessary for students to learn that success depends on individual effort and not on a particular group.

Frequency of Group Change. Students need to work in the same group long enough to experience some ***success***. The frequency of changing groups can decrease over the course as students become more confident and comfortable with CPS. For example, we change groups about 3 - 4 times in the first semester, but fewer times in the second semester. Since students are very sensitive to grades (Zeroth Law of Instruction), we change groups only after a class exam. The information from that exam also provides useful input for assigning groups (see Chapter 11).

Group Role Assignment and Rotation



There are many different roles that can be assigned for different types of tasks. For problem solving, we assign planning and monitoring roles that students have to assume when they solve challenging problems individually -- Manager, Checker/Recorder, and Skeptic/Summarizer. When an expert solves a problem, they continually organize and modify a plan of action, making sure they don't loose track of where they are and what they need to do next. These are the internal management functions. At the same time, they function as *recorder* continually *checking* their solution to make sure it follows a logical and organized path. Finally, the expert is continually *skeptical*, asking questions about each step -- "What other possibilities are there? Should I apply a different principle to solve this problem?"

Most students do not exhibit these reflective (metacognitive) practices when solving a problem so the group roles we give them allows them to practice this behavior. A description of the group roles we give to students is shown in Figure 8.4. Since the roles emphasize the reflective skills that students need when solving a problem individually, we do not assign socially useful roles such as leader, facilitator, or conciliator. These roles are assumed naturally by members of the group.

Figure 8.3. Example of definition of group roles.

In your discussion section for this course, you will be working in **cooperative** groups to solve written problems. To help you learn the material and work together effectively, each group member will be assigned a specific role. Your responsibilities for each role are defined on the chart below.

ACTIONS	WHAT IT SOUNDS LIKE*
MANAGER DIRECT THE SEQUENCE OF STEPS. KEEP YOUR GROUP "ON-TRACK." MAKE SURE EVERYONE IN YOUR GROUP PARTICIPATES. WATCH THE TIME SPENT ON EACH STEP.	*"First, we need to draw a picture of the situation."* *"Let's come back to this later if we have time."* *"Chris, what do you think about this idea?"* *"We only have 5 minutes left. Let's finish the algebraic solution.*
RECORDER/CHECKER ACT AS A SCRIBE FOR YOUR GROUP. CHECK FOR UNDERSTANDING OF ALL MEMBERS. MAKE SURE ALL MEMBERS OF YOUR GROUP AGREE WITH EACH THINK YOU WRITE. MAKE SURE NAMES ARE ON SOLUTION.	*"Do we all understand this diagram I just finished?"* *"Explain why you think that . . . ."* *"Are we in agreement on this?"* *"Here, sign the problem we just finished!"*
SKEPTIC/SUMMARIZER HELP YOUR GROUP AVOID COMING TO AGREEMENT TOO QUICKLY. MAKE SURE ALL POSSIBILITIES ARE EXPLORED. SUGGEST ALTERNATIVE IDEAS. SUMMARIZE (RESTATE) YOUR GROUP'S DISCUSSION AND CONCLUSIONS. KEEP TRACK OF DIFFERENT POSITIONS OF GROUP MEMBERS AND SUMMARIZE BEFORE DECIDING.	*"What other possibilities are there for ...?"* *"I'm not sure we're on the right track here. Let's try to look at this another way. . . ."* *"Why?"* *"What about using . . . . instead of . . . . ?* *"So here's what we've decided so far."* *"Chris thinks we should . . . . , while Pat thinks we should . . . ."*

Figure 8.4. Research on assigning and rotating group roles.

Our observations of group interactions after we assigned roles indicated that the number of dysfunctional groups (e.g., one student dominates; students cannot resolve a difference of opinion) at any given time decreased from about 40% (2 in 5 groups) to about 10 - 20% (less than 1 out of every 5 groups). With fewer dysfunctional groups, an instructor has more time for appropriate and timely intervention to coach physics, optimizing the learning of all students. Our interviews confirmed that students in groups with assigned and rotated roles became more comfortable with their group's interactions, particularly at the beginning of the course.

In *well functioning groups*, members share the roles of manager, checker, skeptic and summarizer, and role assumption usually fluctuates over time. As long as the reflective problem solving functions are being displayed and practiced, students do not need to be reminded to "stick to their roles." Unfortunately, most of these roles are not in evidence, especially at the beginning of the course, so we have to assign them.

We also continue to assign group roles because they are an efficient technique to reduce the number of dysfunctional groups (see Figure 8.4). Students in dysfunctional groups are not learning. The roles help reduce the number of dysfunctional groups in several ways described in Chapter 13.

Individual Responsibility. At the beginning of an introductory class, many students have never participated in cooperative problem solving and do not know what they are supposed to do. The roles remind them of appropriate individual actions in a group.

Optimal Learning. Assigning roles allows students to practice behavior that may not be natural or even socially acceptable. For example, "I don't want to be bossy, but I am the manager. Let's move on to . . .." In addition, we initially had some students who were too polite to disagree openly with the ideas of other group members. The role of "Skeptic" allowed these students a socially acceptable way to disagree.


Remember

Coaching Groups. The group roles provide an efficient and effective way for the instructor to coach groups that are having difficulty applying physics concepts and principles to solve the problem. These techniques are discussed in Chapter 13. Remember the 2nd Law of Instruction: *Don't change course in midstream; structure early then gradually reduce the structure.* This means it is very difficult to assign roles when you finally discover you need them. As students become more comfortable and competent with CPS, the group roles slowly and naturally "fade" away from students' minds, except when you intervene with an occasional dysfunctional group.

Figure 8.5. An example of a Group-Functioning Evaluation Sheet

Date: ________________ **Group #:** _______

1. Use the following grid to rate yourself on your participation and contributions in your group's problem solving. Also, agree on a group rating. **0 = Poor, 1 = Fair, 2 = Good, 3 = Excellent.**

	Name	**Name**	**Name**	**Name**	**Group**
a. Participation in solving problem.					
b. Contribution of ideas to thorough analysis of the problem *before* generating appropriate equations.					
c. Contribution of ideas to planning a math solution (*before* the numerical solution).					
d. Overall use of a logical, organized approach to solving the problem.					

2. What are two **specific** actions we did today that helped us work together towards a successful solution?

3. What is a **specific** action that would help us do even better next time?

Group Signatures: **Manager**: ______________________________.
Skeptic: ______________________________.
Recorder/Checker: ______________________________.
Summarizer: ______________________________.

Group Processing

One of the elements that distinguish cooperative groups from traditional groups is structuring occasional opportunities for students to discuss how well they are solving the problems together and how well they are maintaining effective working relationships among members. For this purpose, we use the *Group Functioning Evaluation* form shown in Figure 8.5. After the group has discussed and completed the evaluation, the instructor spends a few minutes in a class discussion of the answers to Question 6, so students can consider a wider range of ways groups could function better. Common answers include: "Come better prepared; Listen better to what people say; Make better use of our roles (e.g., "Be sure the Manager watches the time so we can finish the problem." or "Be sure the Skeptic doesn't let us decide too quickly.").

In our studies, we found that when students were given a chance to discuss their group's functioning, their attitude about group problem solving improved. There was also a sharp decrease in the number of students who visited instructors during office hours to complain about their group assignment. In addition, groups that were not functioning well improved their subsequent effectiveness following these discussions. For example, in groups with a dominant student, the other group members were more willing to say things like: Hey, remember what we said last week. Listen to Kerry. She's trying to explain why we don't need all this information about the Lunar Lander's descent." In groups that suffered from conflict avoidance, there were comments like: "Oops! I forgot to be the skeptic. Let's see. Are we sure friction is in this direction. I mean, how do we know it's not in the opposite direction?" As usual, this result was consistent with the research on pre-college students.[2]

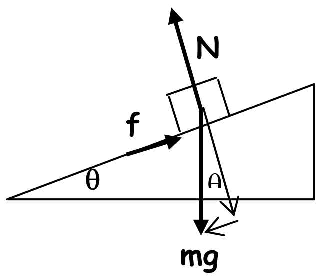
Appropriate Group Problems

With appropriate grading, course structure, and group management in place, we found that even the more difficult, end-of-chapter textbook problems were usually not effective learning tools when used for either individual exams or for group problems (see Chapter 3). For example, one semester we gave students all the relevant equations from their textbook and the group problem shown in Figure 8.6.

Although this problem has a minimal context, group discussions tended to revolve around "what formulas should we use," rather than what physics concepts should be applied to this problem." An illustration of a typical group solution for this problem is shown In Figure 8.6. The students in this group did not begin with a discussion and analysis of the forces acting on the carton in this situation. Instead, they attempted to recall the force diagram from their text, which were for a block sliding *down* an inclined plane. Consequently, their solution has the frictional force in the wrong direction and the force equation has a sign error. The students began by

Figure 8.6. A Typical Group Solution to a Textbook Problem. The arrows have been added to show the progression of the mathematical solution.[3]

Textbook Problem. A 5.0-kg carton slides 0.5 m up a ramp to a stop. The ramp is at an angle of 20° to the ground, and the coefficient of kinetic friction between the carton and the ramp surface is 0.60. What is the initial velocity of the carton?



$$f = \mu N$$
$$= 0.6\ N$$

$$N = mg \cos \theta$$
$$= 5 \bullet 9.8 \bullet 0.93$$
$$= 46$$

$$f = 0.6 \bullet 46$$
$$= 27.6$$

$$f - mg \sin \theta = ma$$
$$27.6 - (5 \bullet 9.8 \bullet 0.34) = 5a$$
$$\frac{27.6 - 16.8}{5} = a$$
$$2.17 = a$$

$$v = at$$
$$= 2.17\ t$$

$$x = vt$$
$$0.5 = vt$$
$$\frac{0.5}{t} = v$$

$$\frac{0.5}{t} = 2.17\ t$$
$$\frac{0.5}{2.17} = t^2$$
$$0.48 = t$$

$$v = at$$
$$= 2.17 \bullet 0.48$$
$$\boxed{v = 1.04\ \text{m/s}}$$

haphazardly plugging numbers into formulas until they had calculated a numerical answer. Their conversation concerned finding additional formulas that contained the same symbols as the unknown variables: “Can’t we use this distance formula [x = vt]? It has v and t in it.” They did not discuss the meaning of the symbols or formulas, and they incorrectly combined an equation containing an instantaneous velocity (v = at) with a one containing an average velocity ($x = v_{ave}t$) to calculate the initial velocity of the block. From observations, interview data, and the examination of group problem solutions, we estimated that about two-thirds of the groups used this “formulaic” problem-solving framework rather then the logical, organized problem-solving framework modeled during lecture. This solution is a combination of novice strategies described in Chapter 2 (pages // - //), pattern matching and plug-and-chug.

Figure 8.7. The Traffic Ticket problem

You are driving up a steep hill when suddenly a small boy runs out in the street. You slam on your brakes and skid to a stop, leaving a 50-foot skid mark on the street. As you are recovering from the shock, a watching policeman comes over and gives you a ticket for exceeding the 25 mph speed limit.

You wonder whether you might fight the traffic ticket in court, so before you leave the scene you collect some information. You determine that the street makes an angle of 20° with the horizontal. The coefficient of static friction between your tires and the street is 0.80, and the coefficient of kinetic friction is 0.60. Your car's information book tells you that the mass of your car is 1570 kg. You weigh 130 lbs, and a witness tells you that the boy had a weight of 60 lbs and took about 3 seconds to cross the 15-foot wide street. Should you fight the traffic ticket in court?

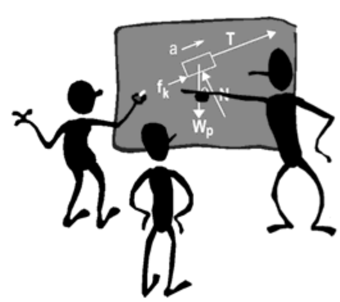
Effective group problems follow the 3rd Law of Instruction: *Make it easier for students to do what you want them to do and more difficult to do what you don't want.* That is, group problems should be complex enough so there is a ***real*** advantage to discussing both the problem situation and applicable physics before plowing ahead. We will use the traffic-ticket problem in Figure 8.7, which is a modification of the textbook problem in Figure 8.6, to illustrate some of the criteria for a good group problem.

Criteria 1. A group problem must be designed so that:

- There is something to discuss *initially* so that *everyone* (even the weakest member) can contribute to the discussion. For example in the traffic-ticket problem, students spend time initially determining how the car is moving and drawing a picture of the situation. Early successful contributions encourage students to make larger contributions later.
- There are several decisions to make in solving the problem. For example in the traffic-ticket problem, students have to decide what quantity to calculate and which information is relevant to the solution.

Criteria 2. A group problem must be *challenging* enough so that:

- Even the best student in the group cannot immediately see how to solve the problem.
- Knowledge of basic physics concepts is *necessary* to interpret the problem. The traffic-ticket problem discourages the algorithmic use of formulas in several ways. For example, students must decide whether the 3-second time the boy took to cross the street is relevant to finding the initial velocity of the car just before the brakes were applied.

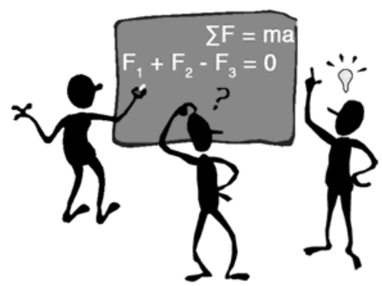


- Students' physics difficulties arise naturally and must be discussed. For example, many students confuse static friction and kinetic friction. To solve the traffic-ticket problem students must discuss the meaning of each, and which type of friction should be applied in this situation.

- Students feel good about their role in arriving at the solution. There are enough decisions to discuss in the traffic-ticket problem that all students contribute, even the quiet students. We found in our research that it is often the quiet student who requests an explanation or clarification that moves the group forward.[4]

Criteria 3. At the same time, the group problem must be *simple* enough so that:

- The mathematics is not excessive or complex.
- The solution path, once arrived at, can be understood, appreciated, and easily explained to all members of the group. For example, the traffic-ticket problem can be solved by the straight-forward application of kinematics concepts and Newton's Second Law. A majority of groups can reach a solution in the time allotted.



The traffic-ticket problem has many traits that make it a challenging problem, even for a group. Striking the right balance between complexity and simplicity for problems is difficult for the instructor. We have developed and tested a set of criteria for judging the difficulty of a problem to decide if they would be suitable for group practice or exams (see Chapter 16). It is probably not an accident that the criteria for designing a problem that encourages learning physics, given in Chapter 3, also results in a useful group problem.

Endnotes

1 Heller, P. & Hollabaugh, M. (1992). Teaching problem solving through cooperative grouping. Part 2: Designing problems and structuring groups, *American Journal of Physics*, **60**(7), 637-644.

2 See Johnson, D. W. and Johnson, R. T. (1989), *Cooperation and competition: Theory and research*, Edina MN, Interaction; and Johnson, D.W., Johnson, R.T., and Smith, K.A. (2007), The state of cooperative learning in postsecondary and professional settings. *Educational Psychology Review,* **19** (1), 15-29

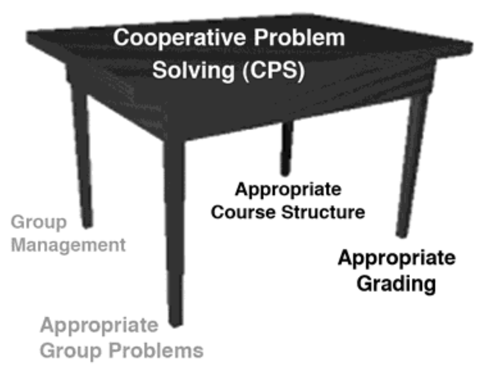
3 Heller, P., Keith, R., & Anderson, S. (1992). Teaching problem solving through cooperative grouping. Part 1: Groups versus individual problem solving, *American Journal of Physics,* **60**(7), 627-636.

4 Hollabaugh, M. (1995), *Physics problem-solving in cooperative learning groups*, Ph.D. Thesis, University of Minnesota

Chapter 9
Course Structure and Grading for Cooperative Problem Solving

In this chapter:

- ✓ Scheduling for continuity of group interactions and finding adequate assistance and rooms for CPS.
- ✓ What is the appropriate grading in a CPS course.
- ✓ How to give consistent grades and feedback for students' problem solutions.
- ✓ How to get started with informal groups in lecture.



The learning advantage of cooperative problem solving (CPS) sessions lies in the students' *co-construction* of a problem solution. There are several aspects of structuring a course for cooperative problem solving that make it *easier* for students to co-construct group solutions than to solve the group problems individually (3rd Law of Instruction). These structures include scheduling cooperative group work, adequate assistance, and adequate rooms. In addition, effective implementation of CPS sessions requires appropriate course grading, including overall grading for the course and consistent grading and feedback of students' problem solutions.

These structures are described in this chapter. The last section contains suggestions for how to get started with informal groups in your lecture.

Remember, the success of CPS depends on effective demonstrations of the use of a research-based, problems-solving framework to solve problems during your classes.

Figure 9.1. Outline for demonstrating the use of a research-based problem-solving framework when solving problems in class

1. Adapt a research-based problem-solving framework that emphasizes the application of fundamental physics principles (e.g., kinematics, Newton's second law, conservation of energy) to solve problems. A problem-solving framework is a logical and organized guide to arrive at a problem solution. It gets students started, guides them to what to consider next, organizes their mathematics, and helps them determine if their answer is correct. [See Chapter 14 for how to personalize a framework for the needs of your students.]

2. Demonstrate how to use the framework every time you solve a problem in class.*
   - Show every step, no matter how small, to arrive at a solution.
   - Explain all decisions necessary to solve the problem.
   - Always use the same framework, no matter what the topic.
   - Hand out or have on your website examples of complete solutions to problems emphasizing the physics decisions and showing every step.
   - After some time, allow each student to make their own reasonable variations of the framework for their solutions.

3. Demonstrating the problem solving process can actively engage students.
   - Always allow the students several minutes to read the problem and begin their solution before modeling begins.
   - Pause several times while demonstrating the problem solving framework to ask students a simple question and allow several minutes for students to:
     - write their answer or answer electronically;
     - turn to their neighbor and discuss the answers to the question; or
     - work in an informal group of 3 to answer the question.
   - The question may be an elaboration of a step in the problem-solving process, a simple application of the point just demonstrated, or the next step in the solution process.

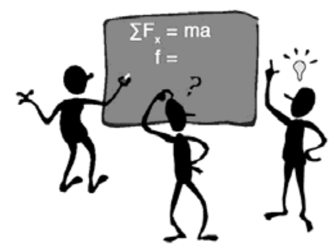
* It is difficult to put yourself in the minds of students and demonstrate the use of a problem-solving framework to solve a problem. Remember, the "problems" in introductory physics are not real problems for you. It is difficult to ask yourself continually: "What would I do next if I didn't know how to solve this problem already?"

Scheduling: Continuity of Group Interactions

To successfully co-construct a problem solution, students need to have time to discuss the physics and a procedure to solve the problem, and to be coached in problem solving by an instructor. This typically requires at least 30 minutes of continuous working time. Taken together with the brief introduction to the problem and a summary discussion, this means one group problem session takes about 50 minutes.

We found that to make a substantial impact on student learning, students also need to solve at least two problems per week while being coached in groups. The reason is mostly a matter of building trust in a group. If students work as a group only once a week, it takes them longer to get to know each other enough to establish trust, so they feel comfortable sharing their conceptual and procedural knowledge and requesting explanations and justifications. You can, however, achieve respectable improvement in students' performance when students solve one group problem each week ***if*** students work in the same groups in other aspects of your course, such as their lab.

If you have block scheduling (1.5 – 2 hours) or a studio format (combined lecture, recitation, and lab), you probably do not have a scheduling problem. If you do have a scheduling problem, there are many creative solutions. For example, we know one instructor (who teaches both the lecture and labs) who decided that students did not use their three-hour lab time effectively – they usually came late and finished early. So the lab time was broken into one hour of CPS and two hours to complete the (same) lab experiments.



At our very large university we had more severe scheduling problems. Many years ago we started with different courses for lecture and the lab, and no "recitation" sections. First we changed to one course that included the lab, then we changed the course registration procedure and included a discussion section. Now for each lecture section of the course, students also register for one section that meets at two scheduled times – a 50 minute "discussion" (CPS) and a two-hour laboratory. The same instructor coaches the same groups in both the discussion section and the laboratory. To better use the laboratory time to support learning physics through problem solving, we developed context-rich laboratory problems.[1]

What Resources Are Needed to Implement CPS?

Two additional and related resources (besides your own time) may be needed to implement CPS effectively:

- Adequate assistance (personnel); and
- Adequate rooms.

There are many tradeoffs between these resources, which are both in short supply at many institutions. Some of the tradeoffs are outlined in the sections below.

Adequate Assistance

The limiting factor in implementing CPS is the number of students per instructor. It is difficult for any instructor to teach 9 or more groups without assistance. Experienced instructors can teach comfortably 7 - 8 groups per class section. Inexperienced (first-year) graduate or undergraduate TAs, however, can handle only 5 groups per section.

If you are the only instructor of small classes, then no additional assistance is needed to implement CPS. If you are in this fortunate situation, you could skip the rest of this section. But if you do not have one experienced instructor per 20-28 students (or one first-year TA for every 15 – 18 students), people have found creative ways to find free assistance for CPS.

"Free" Undergraduate Assistance. We know one instructor who teaches an introductory physics course with 40 students at a small, liberal-arts college. He negotiated with the Education Department to offer course credit to physics undergraduate students in their secondary teacher-education program. These selected students assist the instructor during the course time set aside for CPS.

Change Responsibilities of Paid Assistants. If you have undergraduate or graduate students who spend all or part of their time grading homework problems and/or lab reports, consider reassigning all or some of this time for assistance with CPS. For example, we cut down slightly on the number of required lab reports, and devised a rubric for grading that allows TAs to do some lab evaluation during lab time. We also eliminated or sharply curtailed the grading of homework and redirected this effort to coaching in CPS sections.

Other colleges and universities have instituted computerized homework "grading", or give short multiple-choice tests on the computer each week. Both options allow you to check whether students can use appropriate mathematics in simple one-or two-step exercises. Of course, you can continue to assign some of the more difficult, end-of-chapter problems for homework, even when they are not graded.

Tip

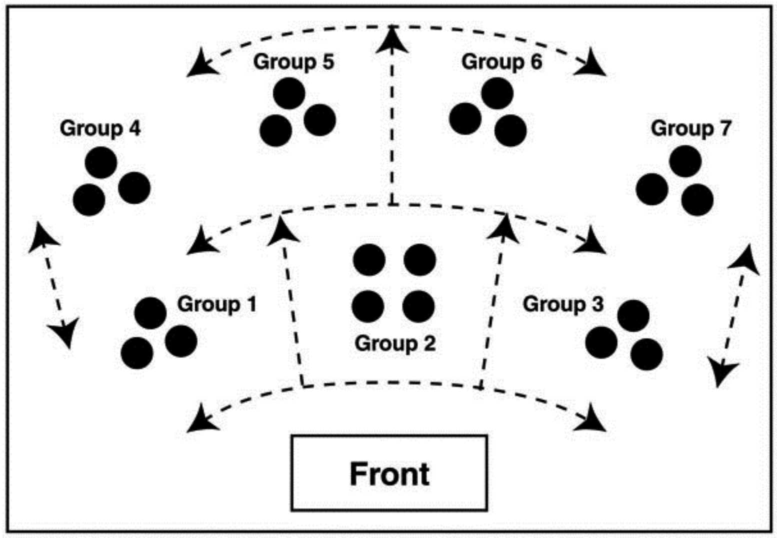
To get students to do homework, it is useful to make one problem on your quiz very close to a homework problem and tell students you will do so. After the quiz, point out which problem was very close to the homework problem. Because subtlety is lost on most students, it is useful to use similar objects and situations as the homework problem simply solving for a different variable. The most useful homework procedure that we have found is to post a sample quiz on your web site about 2 weeks before your next quiz. Do not post the answers or solutions until a few days before the quiz. Many more students will seriously work on problems and get help from the TAs in the tutorial room if you call something a sample quiz rather than homework.

What Kind of Room is Needed?

The *ideal* room for CPS (in most disciplines, not just physics) is a carpeted room with two walls of boards and small, round cocktail-style tables with moveable chairs that accommodate 3 to 4 students (see Chapter 8, page // for optimal group size). The room has adequate space between groups for the instructor to circulate easily among the groups. However, such rooms are rarely available. The minimum room requirements for CPS are:

1. The room must have sufficient wall space for one person from each group to write or post simultaneously parts of their group's problem solution.

Figure 9.2. Top diagram view of movable chairs (black circles) in room arranged for a CPS session. Dashed arrows indicate spaces for instructor to circulate to each group.



2. Students must be able to sit facing each other. In other words, for effective group interactions, students need to be able to look directly at each other, knee-to-knee (see Chapter 7, page //).
3. The room must be large enough so there is space between groups to allow groups to function independently and the instructor to circulate to each group.

No Boards (or not enough boards). The problem of enough board space can be solved in many ways. If you have storage space in the room, you can make "white boards" on which groups can write their diagrams and equations with dry-erase pens. White boards are actually preferable to writing on the blackboard because each group can decide what they want to put on the board together, rather than sending one person to write on the blackboard. Large sheets of whiteboard can be purchased at hardware/lumber yards and cut into 2 ft. x 3 ft. pieces.

If you have neither storage space for white boards nor sufficient board space, you can use sheets of white butcher paper for groups to write on. The disadvantage of butcher paper is the need for a large flat surface to write on (our students sometimes use the floor), and the necessity to tape the sheet to the wall and remove them at the end of class.

The minimum requirements for CPS eliminate rooms with stadium seating and strip tables with fixed seats. However, the following rooms meet the criteria.

Figure 9.3. Top diagram view of room with strip tables (rectangles) and movable chairs (black circles). Dashed arrows indicate spaces for instructor to circulate to each group.

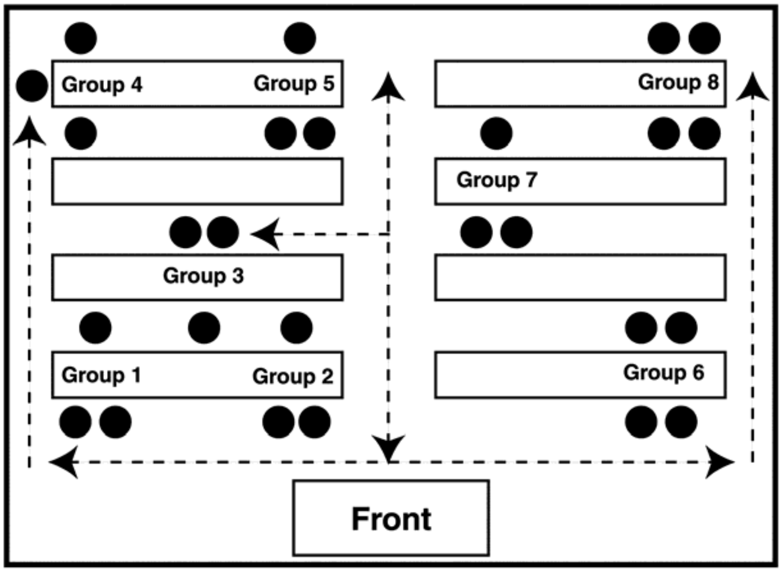
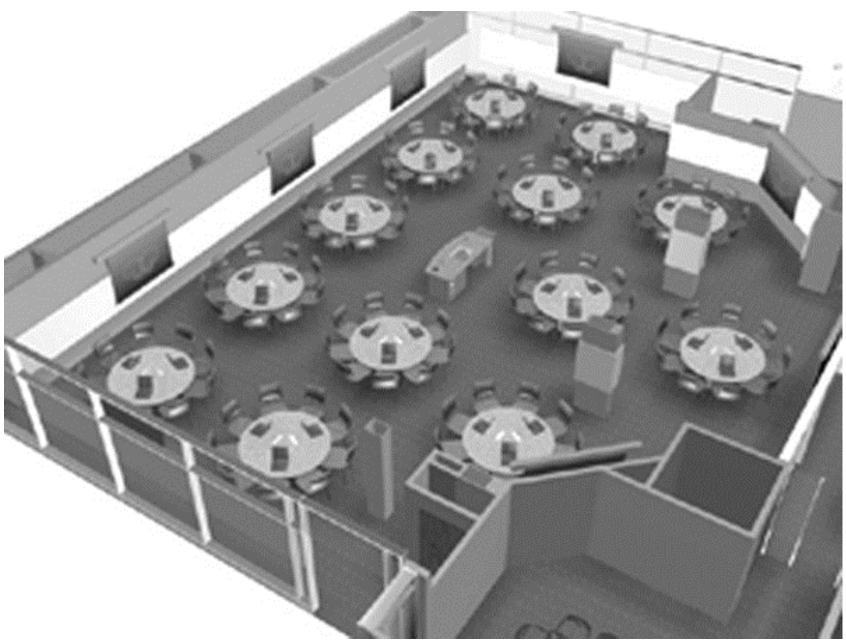


Figure 9.4. Diagram of studio format room at Massachusetts Institute of Technology (MIT)[2]



No Tables, Moveable Chairs. Minimally you need a room with *moveable* chairs that is large enough to accommodate your class size. The room does not have to be in the science or physics building. We schedule CPS discussion sections in small classrooms all over campus. The map in Figure 9.2 is an example how chairs can be arranged so the groups can function independently and the instructor can circulate to each group.

Strip Tables with Moveable Chairs. We do not recommend that you implement CPS in a room with rows of long, narrow tables fixed to the floor. But it can be done if the room is large enough and the chairs are moveable (or at least swivel). For example, the map on Figure 9.3 shows how 8 groups could be arranged in a room with eight strip tables. The groups are far enough apart, and the instructor can get to each group.

Lab Room. Lab rooms are not ideal, but if you have a *small* class, you can do cooperative problem solving in most lab rooms. You must, however, be able to arrange the groups so students can be sitting facing each other, for example at the end of tables. Of course, all the apparatus must be cleared out of the way.

Room with Large Tables. Some universities have replaced stadium seating in large lecture rooms with the studio format – large rooms with tables that seat 6 to 10 students, as illustrated in Figure 9.4. There is usually sufficient space for the group recorder, in the middle chair, to move his/her chair back away from the table. The other two group members on either side of the recorder can turn their chairs around to face the recorder.

Appropriate Course Grading

Remember the Zeroth Law of Instruction: *If you don't grade for it, students won't learn it!* Like it or not, grading is the single most important teaching action. This section describes some recommendations for CPS courses.

Absolute Grading Criteria

The minimal requirement is that your course grades are ***not*** be based on a curve -- a "you win, I loose" grading policy. For groups to work, students must know that when they help others, they are not *reducing* their chances of getting a good course grade. To establish absolute grading criteria, you must be able to specify course standards in a manner independent of a specific problem. All grading practices within the course must conform to those standards. Initially you can base your grading criteria on results from previous classes, so they represent realistic goals of student performance. To guard against an occasional test that is too hard or too long, one can normalize each exam's score to a criterion that does not depend on the performance of the entire class. We typically normalize to a score that is humanly possible as determined by the highest score on an exam.

Number of Questions on a Test

One corollary of the Zeroth Law of Instruction is that students ignore (and resent) an activity that they do not see as having a direct effect on their grade. In other words, students need an explicit match between the time spent on an activity and how they are graded. For most students to become better problem solvers, as defined in Chapter 4, they need time to demonstrate their learning on tests. This determines the maximum number of questions on a test. We have found that typical students need about 20 minutes to solve a context-rich problem on a test. This time factor makes some students feel rushed, but more time does not help the majority of students arrive at a better solution.[3] This limits 50-minute tests to two problems and some multiple-choice questions. To give adequate feedback to students, we give a test about once every three weeks.

Information Available on Tests.

For students to see a match between group work and how they are graded, individual test problems should be graded using the same criteria as group problems. Group problems, whether for practice or graded, should always look the same as individual test problems from the students' point of view.

To discourage memorization of formulas and problem specific algorithms, the basic information needed to solve problems, including constants and equations, should be *supplied in the same way* for the individual tests and all group problems (see page Chapter 3, pages //-//). The information sheet that we give to students is the same for the group and the individual part of the test.

Grading Group Solutions.

Another consequence of the Zeroth Law of Instruction is that if you want students to work effectively in cooperative groups, then you must, at least occasionally, grade the group product. That is, some small but significant part of student's total course grade should be for group problem solving. There are, of course, many ways to do this. You could, for example, assign 10 - 15% of each student's final grade to a fixed number of group problem solutions. That is, groups occasionally turn in one problem solution for grading, and each group member gets the same grade for the group solution. We have found that grading every group problem is counter productive. A constant grade pressure seems to inhibit group discussions that allow each student to explore their ideas about the physics. Under those conditions, student activity becomes exclusively directed to getting an answer.

When a group problem is graded, we have found that integrating it into a regular test situation makes it more meaningful to our students. For example, each of our tests has a group part and an individual part. The first part of the test is a group problem that students complete in their discussion sections. The following day students complete the individual part of the test, consisting of two problems and about ten multiple-choice questions, in the lecture room. No student believes that one problem out of three on a test is unimportant. When the final exam, which is entirely individual, is added, the group problems account for about 15% of their total test scores. Other parts of the grade (e.g., individual laboratory reports) reduce group work to about 10% of the students' course grade.

An advantage of this presentation of grading lies in the way students interpret their test scores. When groups are well managed (see Chapter 8), the highest score that students receive on a test is almost always for the group problem, which is also the most difficult problem on the test.[3] This reinforces the advantages of cooperative-group problem solving.

Consistent Grading of Problem Solutions

If your goal is to teach physics through problem solving, then the grading of both individual and group problem solutions should reflect the behavior you value in problem solving. For example, if you value a careful description and analysis of a problem *before* the mathematical manipulation of formulas, you should not grade only for the appearance of equations leading to a correct answer. Remember the Zeroth Law of Instruction: *If you don't grade for it, students don't do it.* From a student's perspective, grading only for the equations leading to a correct answer requires them to do what you do ***not*** want them to do -- solve problems disconnected from physics by manipulating formulas or trying to match a memorized mathematical solution pattern (see Chapter 3, pages //-//).

The consistent grading of problem solutions can provide your students with rewards for learning physics through problem solving, and place barriers to novice plug-and-chug and pattern-matching approaches. Below are some recommendations for this type of grading.

Grading Based on Problem-solving Framework

Adopt a grading scheme based on the successful completion of *each step* of the problem solving strategy you teach. For example, suppose each problem solution is graded on a 25-point scale. The table on the next page shows the distribution of points for the successful completion of each step in the Competent Problem-solving Strategy for an algebra-based course.

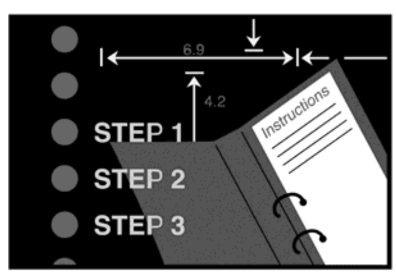
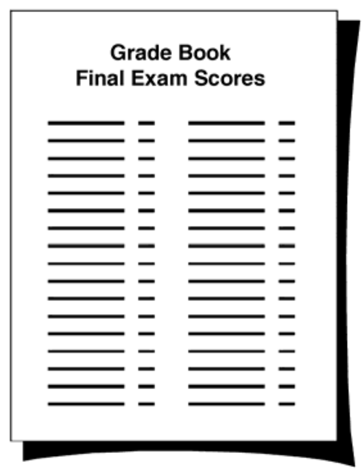


Figure 9.5. Points for grading written problem solutions on different tests

Steps	First 2 Tests	3rd Test	4th Test
Focus on the problem	7	5	5
Describe the Physics	8	7	6
Plan a Solution	5	7	6
Execute the Plan	3	4	5
Evaluate the Solution	2	2	3
TOTAL POINTS	**25**	**25**	**25**

This grading scheme is a formidable barrier for students who use the plug-and-chug or pattern-matching technique. For example, the solutions shown in Chapter 2, pages // and // would both get a low grade! On the first test, this is a big shock to those students who are accustomed to getting partial credit for calculating something, even if it makes no sense within physics.

Drop the Lowest Test Score.



In the first semester we give a test every 3 weeks, but drop the lowest test score for calculating the final course grade. This allows students to do poorly on the first test, and still get a good course grade -- *if* they start using a logical and organized problem-solving strategy. Unfortunately, this practice has a consequence that is detrimental to students who do well on all of the tests. They see no point in taking the last test in the course since they will drop one of their good scores. Following the Zeroth Law of Instruction, they don't attend to that material and do poorly on that part of the final exam. To avoid this behavior, our grading scheme gives a lower weight to the final exam if no test score is dropped. For the student, it is always worth doing well on a test. For the same reason, no student is allowed to completely drop the final examination.

Change the Point Distribution for Grading Problem Solutions.

Periodically changing the number of points allocated to each step in your problem-solving strategy (see Figure 9.5) can provide students with a ladder up the learning mountain (see Chapter 1, page //). For example, during the first weeks of the course, many of our students have difficulty carrying through an entire problem solution. To emphasize the importance of the initial qualitative steps in problem

Figure 9.6. Grading Feedback for Students' Problem Solutions

FOCUS ON THE PROBLEM AND DESCRIBE THE PHYSICS

1. **Picture or Diagram is misleading or inaccurate**
   a. missing important objects or interactions
   b. includes spurious objects or interactions
   c. other incorrect diagrammatic translations of problem information
2. **Relevant variables not assigned and clearly labeled**
   a. many important variables not defined
   b. defined variables not clearly distinguished from each other
3. **Approach invalid, too vague, or missing**
   a. application of principles inappropriate
   b. misunderstanding of fundamental principle
   c. simplifying approximations not stated or inappropriate
4. **Necessary fundamental principles missing**
5. **Incorrect or invalid statement of known values or assumptions**
6. **Incorrect assertion of general relationships between variables**
   a. application of principles to inappropriate parts of the problem
   b. incorrectly assumed relationship between unknown variables, such as $T_1=T_2$.
   c. overlooked important relationship between variables, such as $a_1=a_2$.
   d. misunderstanding of fundamental principle
7. **Incorrect statement of target variable or no target stated**
   a. target variable doesn't correspond to question in Approach
   b. does not explicitly state target variable
   c. wrong target
8. **Major misconception**

Plan the Solution

9. **Poor use of the physics description to generate a plan**
   a. physics description was not used to generate a plan
   b. inappropriate equation(s) was introduced
   c. undefined variables used in equations
10. **Improper construction of specific equations**
    a. inappropriate substitution of variables into general equations
    b. numerical values were substituted too soon
11. **Solution order is missing or unclear**
    a. there is no clear logical progression through the problem
    b. solution order can't be understood from what is written
12. **Plan can not be executed**
    a. there are not enough equations
    b. a relationship was counted more than once

Execute the Plan and Evaluate the Solution

13. **Execution is illogical**
    a. Incorrect physics was introduced to solve the problem
    b. unacceptable mathematical assumption was used
14. **Mistake in execution**
    a. algebra mistake
    b. used incorrect values for known variables
    c. used “math magic” (e.g., $m_1=m_2=1$, took square root of negative number)
15. **Did not check units and/or evaluate answer**

solving, we give students the most points for these steps in the first weeks of the course. As the course progresses and students become more knowledgeable and comfortable with all the problem-solving steps, the allocation of points is more evenly distributed, as illustrated in Figure 9.5.

Feedback on Students' Problem Solutions

Although it is helpful to give students extensive comments on each problem solution, such feedback is excessively time consuming for a large class. We developed a chart of common student mistakes, shown in Figure 9.6, to help grade problems consistently and to give feedback. If the chart is available to students, then as mistakes are encountered on problem solutions, the grader can indicate them by number (e.g., 2b, 5, 7a).

Getting Started

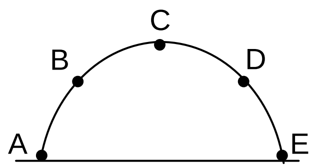
To adopt CPS, it is helpful to be in the frame of mind of an experimenter -- cut yourself some slack and ***take the long-term view***. Implementing a new teaching technique is similar to implementing a completely new measurement technique in a laboratory. When you start, it naturally takes more time and effort than the old, comfortable technique. More frustrating still, the first time you "turn it on," it either doesn't work at all, or you don't get the expected results. You have to tinker with it, make adjustments, and fine-tune the technique until you get the optimal results.

If you and your students have no experience with active learning techniques, we recommend that you start with informal groups in the lecture before you implement CPS. Informal groups involve asking a short question during a lecture, having your students turn to their neighbor(s) to discuss the answer and come to consensus. This time-honored technique for improving lectures has been given many names, such as the "one-minute paper," "peer instruction," as well as "informal" cooperative grouping." Some example questions for two-dimensional motion are shown on the following pages. After students discuss the answer for a few minutes, the instructor asks for a call of hands for each answer (or use an electronic system of clickers).

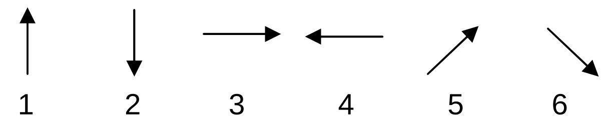
There are many times during a lecture you can stop and ask a short question of informal groups. For example, you can ask a question before you start a lecture to find out what students already know, and focus their thoughts on the lecture to come (see Figure 9.7a). You can ask a question after lecturing for some time to see if students have understood the main ideas of your lecture (see Figure 9.7b). When you are demonstrating how to solve a problem, you can ask students about the next step in the problem solution (see Figure 9.7c).

Figure 9.7. Examples of group questions to use in lecture.

Figure 9.7a. Example of group question to use before a new topic is introduced.



1. A ball is thrown into the air and follows the path shown at left.

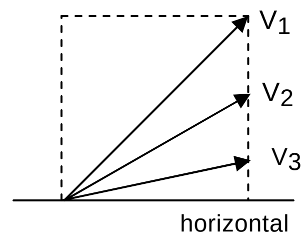


Which arrow best represents the direction of the ball's *acceleration* at point B?
At point C? At point D?

Which arrow best represents the direction of the ball's *velocity* at point B?
At point C? At point D?

Figure 9.7b. Example of a group question to check for understanding after the topic is introduced.

Three identical balls are simultaneously thrown with the three velocities shown by the vectors in the diagram at right.

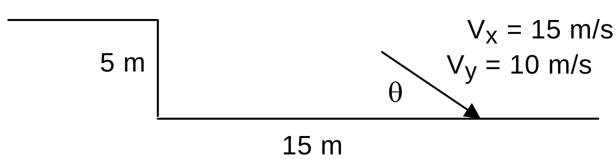


Ignoring air resistance, which of the following statements is true?

A. They all move through the air with the same speed.
B. They hit the ground at the same place.
C. They remain in flight for the same length of time.
D. While all three balls are in flight, they are always at the same vertical distance above the ground.
E. While all three balls are in flight, they are always the same horizontal distance from the starting point.

Figure 9.7c. Example of group question to use for moving on to the next step..

A ball rolls off a flat roof that is 5 meters high. One second later, the ball lands 15 meters from the house at an angle □ from the ground. When the ball lands, the horizontal component of its velocity (v_x) is 15 meters/second and the vertical component of its velocity (v_y) is 10 meters/second.



A. $\tan\theta = \dfrac{10\text{ m/s}}{15\text{ m/s}}$

B. $\tan\theta = \dfrac{15\text{ m/s}}{10\text{ m/s}}$

C. $\tan\theta = \dfrac{5\text{ m}}{15\text{ m}}$

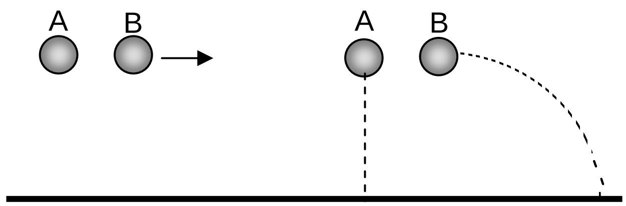
D. $\tan\theta = \dfrac{15\text{ m}}{5\text{ m}}$

E. $\tan\theta = \dfrac{10\text{ m/s}}{5\text{ m}}$

Figure 9.8. Examples of informal group questions for before and after a demonstration.

Figure 9.8a. Example prediction question before a demonstration

Two balls will be released from the same height at the same time. Ball A is dropped straight down, while B is given a horizontal kick. Which ball do you think will hit the floor first?



A. Ball A will hit the floor first.
B. Ball B will hit the floor first.
C. Both balls will hit the floor at the same time.
D. There is not enough information is given.
E. I don't have any idea.

Figure 9.8b. Example confirmation question after a demonstration.

Two balls were released from the same height at the same time. Ball A was dropped straight down while ball B was given a horizontal kick. Which ball hit the floor first?

A. Ball A hit the floor first.
B. Ball B hit the floor first.
C. Both balls will hit the floor at the same time.
D. I didn't hear when the balls hit the floor.

Finally, it is very helpful to ask questions before and after a demonstration. A prediction of what students think will happen in the demonstration (and why) helps focus student's attention on the purpose of the demonstration and provides you with information about your students' alternative conceptions (misconceptions). An example prediction question before a demonstration is shown in Figure 9.8a. Because some students' alternative conceptions are so strong that they "see" what they expect to see, also ask students what they observed right after the demonstration (see Figure 9.8b).

Several resources are available for conceptual questions that can be used for informal groups during lectures. In his book *Peer Instruction: A User's Manual*, Eric Mazur[4] provides many conceptual questions. There are many good questions in Lillian McDermott and Peter Schaffer's book, *Tutorials in Introductory Physics and Homework Package*.[5] In addition, Tom O'Kuma, David Maloney, and Curtis Hieggelke[6] have published ranking tasks, which make excellent conceptual questions for informal groups, in their book *Ranking Task Exercises in Physics: Student Edition*.

There are other advantages to starting slowly with informal groups in your lecture. First, it gives you a baseline from which to judge whether CPS is successful for you. Before you start CPS, you can collect some data from your course. For example, you could collect student answers to informal-group questions. You could also give the *Force Concept Inventory*[7] to your students as a pretest and posttest so you can compare your students' gains with national norms for this test at similar institutions.[8] You can also examine a sample of problem solutions from each test to determine the kinds of errors are your students making and how well they are expressing themselves.

Second, you may need time to review the available problem-solving frameworks, and modify these frameworks to match your preferences (see Chapter 14). It is be helpful to practice demonstrating your preferred framework when you solve problems in lecture. We have found professors often have difficulty putting themselves in the minds of students and demonstrating a competent framework. Remember, the "problems" in introductory physics are not real problems for you. It is difficult to ask yourself continually: "What would I do next if I didn't know how to solve this problem already?"


Remember

Remember the 2nd Law of Instruction. *Don't change course in midstream*. We do **not** recommend starting CPS in the middle of a course. The students have already set their behavior patterns and will resist any changes.

ENDNOTES

1. For examples of problem-solving labs, go to our website: http://groups.physics.umn.edu/physed/Research/PSL/pslintro.html.

2. For more information about Technology Enhanced Active Learning (TEAL) classrooms ant MIT, go to http://web.mit.edu/edtech/casestudies/teal.html.

3. For the data supporting this statement, see Heller, P., Keith, R., & Anderson, S. (1992). Teaching problem solving through cooperative grouping. Part 1: Groups versus individual problem solving, *American Journal of Physics,* **60**(7), 627-636.

4. Mazur, E. (1992). *Peer instruction: A user's manual*, Upper Sadler River NJ, Prentice Hall.

5. McDermott, L.C. and Shaffer, P.S., (2002). *Tutorials in introductory physics and homework package*, Upper Sadler River NJ; Prentice Hall

6. O'Kuma, T.L., Maloney, P.D., & Hieggelke, C.J. (2000). *Ranking task exercises in physics: Student edition*, Upper Sadler River NJ, Prentice Hall

7. Hestenes, D., Wells, M., & Swackhamer, G. (1992). Force concept inventory, *The Physics Teacher*, **30**, 141-158

[8] Hake, R. (1998). Interactive-engagement vs traditional methods: A six thousand student survey of mechanics test data for introductory physics courses. *American Journal of Physics*, **66**: 64-74.

Chapter 10
Results for Partial and Best-practice Implementation of CPS

In this chapter:
- ✓ Research investigations of students' improvement problem-solving skills with CPS
- ✓ Research investigations of students' improvement conceptual understanding of physics as measured by a multiple-choice test (FCI) and open-ended written questions
- ✓ Research investigations of the effect of partial and full implementation of CPS on students' conceptual understanding
- ✓ Research investigations of students' improvement in learning attitudes with CPS

Part 1 provided background about the unsuccessful problem-solving strategies of beginning students, how context-rich problems and a problem-solving framework help students engage in real problem solving, and why the best-practice implementation of Cooperative Problem Solving (CPS) is a useful tool for teaching physics through problem solving. Chapters 7 – 9 describe the foundations of Cooperative Problem Solving and provide detailed information about how to implement CPS for maximum effectiveness.

In this chapter we describe research results supporting the use of CPS to improve both students' problems-solving skills and their conceptual understanding of physics. A few results were published previously,[1] and many results were presented as contributed papers and American Association of Physics Teachers (AAPT) meetings. In this book, we have focused on the results. When necessary, comments about methodology appear in the endnotes.

Improvement in Problem-solving Performance

The best-practice implementation of Cooperative Problem Solving (CPS) includes teaching students an explicit problem-solving framework and having students practice implementing the framework by solving context-rich problems in cooperative groups. The full model follows all of the recommendations in this book, as outlined below.

1. Model (demonstrate) solving problems in lecture using of a research-based problem-solving framework which emphasizing the basic principles of physics (Chapters 4 and 14) and establishing a culture of expert practice (Chapter 6).
2. Use the scaffolding recommended in this book: worksheets with cues from the framework (Chapter 5); problem solutions on the worksheets (Chapter 5); and information sheets (Chapters 3 and 11). Students are required to solve problems on the worksheets for the first few exams.
3. Require students to buy a resource booklet that includes:
   - ✓ An explanation of each step of the problem-solving framework.
   - ✓ A one-page outline of the framework.
   - ✓ Flow-charts for each step in the framework.
   - ✓ A blank worksheet with cures from the framework (Figure 5.3). Students can photocopy the worksheet for practice.
   - ✓ Three sections (kinematics, dynamics using Newton's second law, and the conservation of energy). Each section includes an explanation of how to draw the appropriate physics diagrams (motion diagrams, force diagrams, and energy tables), several problems with problem solutions on the worksheets, and context-rich problems for students to practice solving using the problem-solving framework.
4. Work more closely with the teaching assistants to write appropriate context-rich problems (Chapters 3, 11, and 15) and help them with diagnosing, monitoring, and intervening with groups (Chapter 13).
5. Adopt grading practices that require students to use the problem-solving framework and get the most out of their CPS sessions (Chapter 9)
6. Gradually fade this structure over the course of the first semester:

The original research in the improvement in problem solving performance in best-practice CPS classes was done in the early 1990's with the algebra-based introductory course at the University of Minnesota. A change in students' problem-solving performance over time is very difficult to measure because conceptual understanding of the physics is a necessary but not sufficient condition for good problem solving performance.[2] In other words, a competent problem solver cannot solve a problem correctly if their understanding of the physics is imperfect.

Consequently, we developed and validated scales for six expert-like problem-solving skills for rating students' written problem solutions.

- *General Approach*: Does the physics description in the solution reveal a clear understanding of physics concepts and relations?
- *Usefulness of Description*. Does the physics description include the essential knowledge necessary for a solution
- *Specific Application of the Physics*: Starting from the physics they used, how well did the student apply this knowledge to the problem situation?
- *Reasonable plan*. Does the solution indicate that sufficient equations were assembled before the algebraic manipulation of equations was undertaken.

Figure 10.1a. Example of a student's problem solution at the beginning of an introductory course, showing a lack of logical progression in the solution.

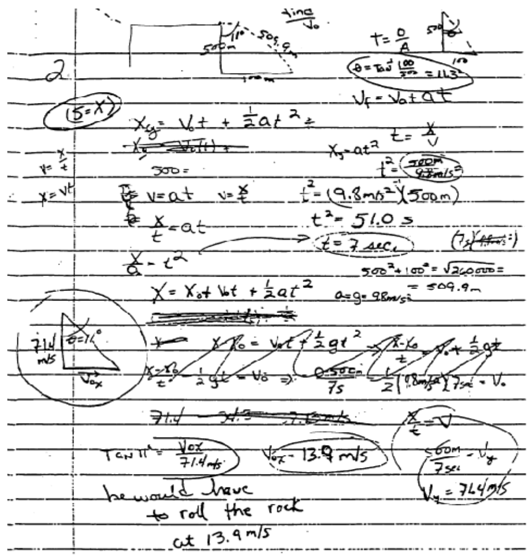


Figure 10.1b. Example of a student's problem solution showing improved logical progression near the end of an introductory course in which students are taught an explicit problem-solving framework and practice implementing the framework by solving context-rich problems in cooperative groups.

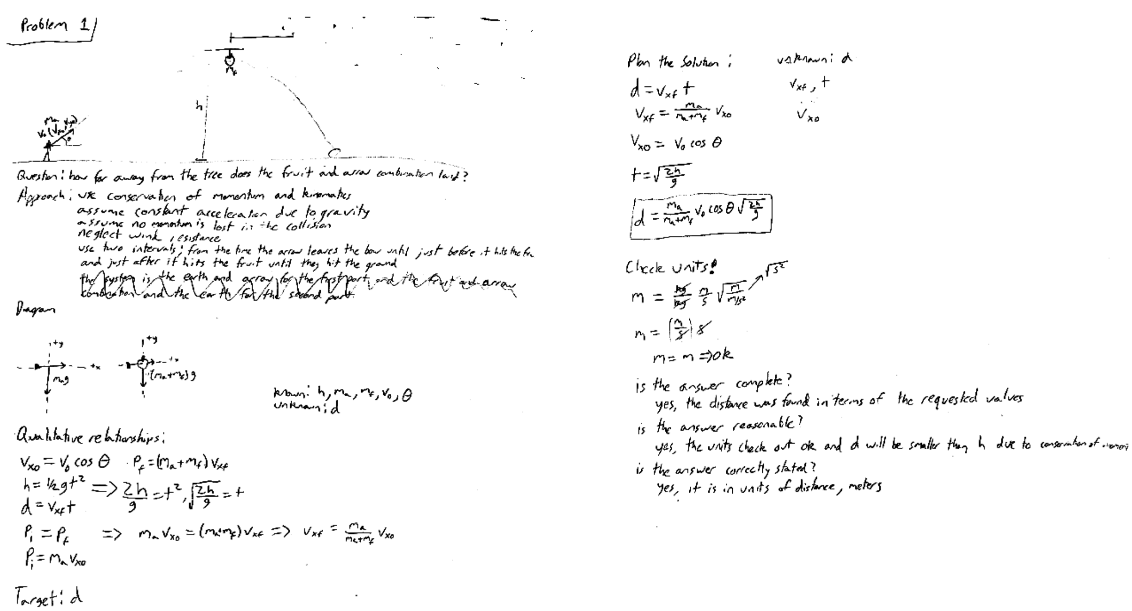
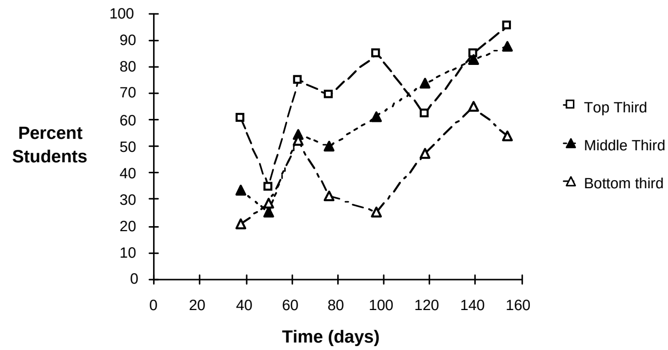


Figure 10.2. Percentage of the top third, middle third, and lower third of the class whose solutions followed a logical progression. The dashed lines are included for ease of reading the graphs.



- *Logical Progression*: Is the solution logically presented? (see Figure 10.1
- *Appropriate Mathematics*: Is the math correct and useful?

The scales for the last three skills are based on the physics that students used to solve the problem, regardless of whether the approach was correct, incomplete, or incorrect. The ratings for the six skills were equally weighted and normalized to yield an ordinal problem solving scale with a maximum score of 100.

Figure 10.2 shows the percent of students scoring in the top-third, middle third, and lower third of the logical progression scale on individual context-rich exam problems over the two quarters (22 weeks including final exam weeks) of the algebra-based course. The fluctuations in the graph are due to differences in the level of difficulty of the context-rich problems (see Chapter 16 for problem characteristics that affect the difficulty of a problem). The general trend, however, is improvement in the logical presentation of students' problem solutions. A visual inspection of Figure 10.1 shows the improvement in the logical progression of a student's solution at the beginning of the course and at the end of the course.

Similar improvement trends were found for the other problem-solving skills, with the exception of *General Approach* and *Appropriate Mathematics*. As expected, there was no improvement in the *General Approach* scores because each exam tested the new physics students were learning. We concluded that an instructional approach that combines the explicit teaching of a problem-solving framework with practice implementing the framework in cooperative groups is effective in improving the problem solving skills of all students on context-rich problems.

In addition, two traditional problems were included in the final exams of the experimental (cooperative problem solving) section and traditional section of the course. The results of the problem solving scores for the two sections are shown in Figure 10.3. We concluded that students who are taught an explicit problem-solving

Figure 10.3. Comparison of the median students' scores on two common final exam problems for CPS and a Traditional section of the algebra-based course.

	CPS Section: Median (N = 91)	Traditional Section: Median (N = 118)	Mann-Whitney Z Statistic
Problem #1	82	62	8.17*
Problem #2	71	50	4.20*

* p<0.0001

framework and practice implementing the framework in cooperative groups solve traditional problems in a more expert-like manner that than students who receive traditional instruction.

Replication Study

Only one replication of the best-practice CPS model has been reported in the research literature. Karen Cummings, Jeffery Marx, Ronald Thornton, and Dennis Kuhl[3] compared two innovations in studio physics,[4] Interactive Lecture Demonstrations (ILD)[5] and Cooperative Problem Solving (CPS). The gains in conceptual understanding measured by two tests were the same for both innovations, although ILD required little instructional time, while CPS required a sustained effort by the instructor. Students in CPS, however performed significantly better on the problem-solving section of the final course exam.

Related Studies

There is some evidence that constraining students to do a qualitative analysis of a problem before solving the problem with equations helps students both categorize problems in an expert-like manner and improve their problem-solving performance.[6] One study indicates that when students are *not* constrained to do a qualitative analysis first, the student groups show some progression towards expert-like behavior: earlier qualitative analysis and more selective requests for information as they gain more experience in solving context-rich physics problems. Approximately half of the groups, however, still complete the qualitative analysis task towards the end of the solution instead of earlier when it would be most useful to their work.[7]

Research also indicates that curricular innovations that are successful in improving students' problem-solving skill(s) have three common features: (1) explicit teaching of problem-solving heuristics; (2) modeling (demonstrating) the use of the heuristics by the instructor; and (3) requiring students to use the heuristics explicitly when solving problems.[8] The full CPS innovation has all of these features. Having students work in groups does not improve problem solving unless the group work is combined with the above features.[9] If you do a partial implementation of CPS, your students' conceptual understanding will improve as much as with other innovations, but you may not see a substantial improvement is your students' problem solving performance.

Improvement in Knowledge Organization

Original expert-novice research indicated that experts solve unusual physics problems by using the cues in the problem to decide what physics *principles* to apply (e.g., Newton's second law, conservation of energy). They use this principle(s) to qualitatively analyze the problem situation before beginning a mathematical solution.[10] An example of the hierarchical organization of physics knowledge by principles is shown in Chapter 2, page //. Novice students, on the other hand, are cued by the surface features of the problem (e.g., free fall, circular motion, inclined plane) and try to remember the formulas that apply in these cases. They do not qualitatively analyze the problem situation, but immediately start to manipulate equations. An example of the knowledge organization of a novice student is shown in Chapter 2, page //.

An early research method was to give experts and novices a set of problems to sort based on by how they would solve the problem. Our graduate student Ron Keith (now deceased) investigated whether the students who solve context-rich problems using the *Competent Problem-solving Framework*[11] move towards a more hierarchical organization of knowledge compared to students in a traditional algebra-based course.[12]

Interviewees sorted a set of 30 problems, written on cards, into groups based on how they would solve the problems. Each group was then labeled and described. A small sample of volunteer students from the traditional class, a sample of students in the CPS class who were identified as problem-framework users, and a sample of advanced physics graduate students and post-docs, participated in this interview task. Two examples of the problems are shown in Figure 10.3a.

An examination of the labels and descriptions resulted in the identification of two categories between novices and experts in how they decide to solve a problem, as described below:

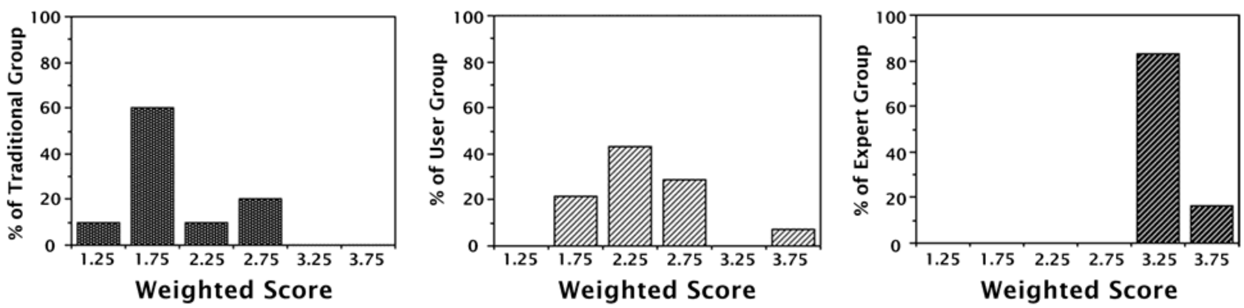
1. Surface Feature (Novice). No reference to how problem could be solved using physics principles (e.g., a "circular motion" label described as "they all have things going in circles").
2. Surface Feature. Refers to application of a specific equation or relationships, but not to physical principles (e.g., a "circular motion" label described as "use centripetal acceleration, $a = v^2/r$).
3. Surface Feature/Physics Principle. Refers to application of a physical law (e.g., a "collision" label described as "you could solve these problems either by conservation of momentum or energy").

Figure 10.4. (a) Two examples of sorting instrument problems. (b) Percentage of students receiving traditional receiving and their weighted categorization scores. (c)

Percentage of students who used the CPS problem-solving frameworks and their weighted categorization scores. (c) Percentage of experts and their weighted categorization scores.

(a) Energy Problem: While on a brief visit to the UMD campus, you park your 1000 kg car at the top of a 50 ft high, icy hill. It's not your lucky day. Your brakes fail and the car rolls down the hill. What is the speed of your car when it reaches the bottom of the hill?

Kinematics Problem: To reach her destination, a backpacker walked downhill with an average velocity of 3 mph. This average velocity resulted because she hiked for 4 miles downhill with an average velocity of 6 mph, then backtracked uphill with an average velocity of 1 mph. What distance did she walk uphill?



(b) (c) (d)

4. Physics Principle (Expert). Refers to application of physical law (e.g., "force laws," "use force diagrams," "energy conservation," "momentum conservation").

Most students sorted the 30 problems into a different number of groups based on different categories of how they would solve the problems. We created a weighted average score for each interviewee – the category value for each category (1, 2, 3, or 4), weighted by the number of problems in the category. Figures 10.3b, 10.3c, and 10.3d show the percentage of students with weighted average scores ranging from 1.0 to 4.0 in the traditional section, students in the CPS section identified as problem-framework users, and the experts.

A comparison of the graphs in Figures 10.4 b and 10.4c show that the students who use the problem-solving framework started to move towards expert-like categorization of problems in just 15 weeks of instruction. This is noteworthy considering that it takes several years for students to organize their physic knowledge hierarchically by basic principles.[13]

Improvement in Students' Conceptual Understanding of Motion and Forces as Measured by the Force Concept Inventory (FCI)

The Force Concept Inventory (FCI) is a 30-item multiple-choice test developed by David Hestenes, Malcolm Wells, and Greg Swackhamer[14] to probe students understanding of concepts in mechanics. The test questions are based on interviews with students by other researchers, the choices reflecting common student misconceptions. For decades, physics instructors have used the Force Concept Inventory (FCI) to measure the effectiveness of instructional strategies in improving students' conceptual understanding of kinematics and Newton's laws of motion.

A standard practice is to administer the multiple-choice test twice, once at the start of the semester and once at the end. Student achievement in conceptual understanding is then measured by improvement in the score from pre-instruction to post-instruction. In order to take into account the fact that a 10% improvement for a student with a 20% correct pre-test score is not necessarily the same as a 10% improvement for a student with a 90% correct pre-test score, a measure called the normalized gain was developed. The normalized gain in student conceptual understanding <g> is defined as follows:

$$< g >= \frac{\%\text{Correct}_{\text{post-instruction}} - \%\text{Correct}_{\text{pre-instruction}}}{100 - \%\text{Correct}_{\text{pre-instruction}}}$$

This expression is often referred to in the literature as the "*g*" or "Hake" factor[15], and is the ratio of the actual gain to the maximum possible gain.[16]

Improvement in FCI Scores with Practice Implementing CPS

Figure 10.5 shows the normalized gain in the Force Concept Inventory (FCI) for different faculty members in the first five years of implementation. The faculty at the University of Minnesota implemented a partial model of CPS. Cooperative problem solving replaced the traditional recitations, and problem-solving labs were under development. Each teaching assistant worked with the same students in both the new discussion sessions and in the labs, and participated in a two-week workshop to learn how to implement cooperative problem-solving techniques. For the most part, however, the faculty continued with their traditional lectures and approach to problem solving.

Faculty members A, B, C, D, and E taught the course more than one year. In each case, their FCI normalized gain scores increased considerably (about 1 – 2 standard deviations) from the first time they taught the course to the second (or third) time they taught the course. Faculty Members Z and C (Year 3) are interesting case studies. Faculty Member Z ignored the initial training of the TAs in CPS techniques. Instead, he met with a student focus group and implemented their suggestions in the lectures and discussion sections. The normalized gain in FCI scores was very low ($<g> = 0.10 \pm 0.7$) – below the national average of 0.20 ± 0.03 for traditional lecture courses. Faculty Member C misinterpreted the nature of group problems. Instead of context-rich problems, he gave difficult estimation

Figure 10.5. The FCI normalized gain sore for different faculty over the first five-year period of implementing partial CPS. The letters under each column represent different faculty members.

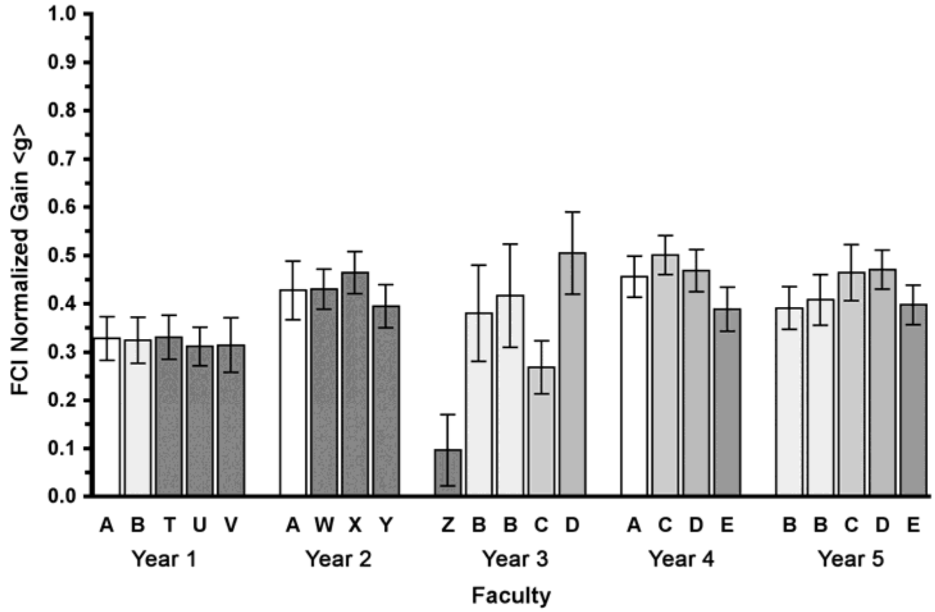


Figure 10.6. Average pre- and post- instruction Force Concept Inventory scores for the 41 faculty members who implemented CPS

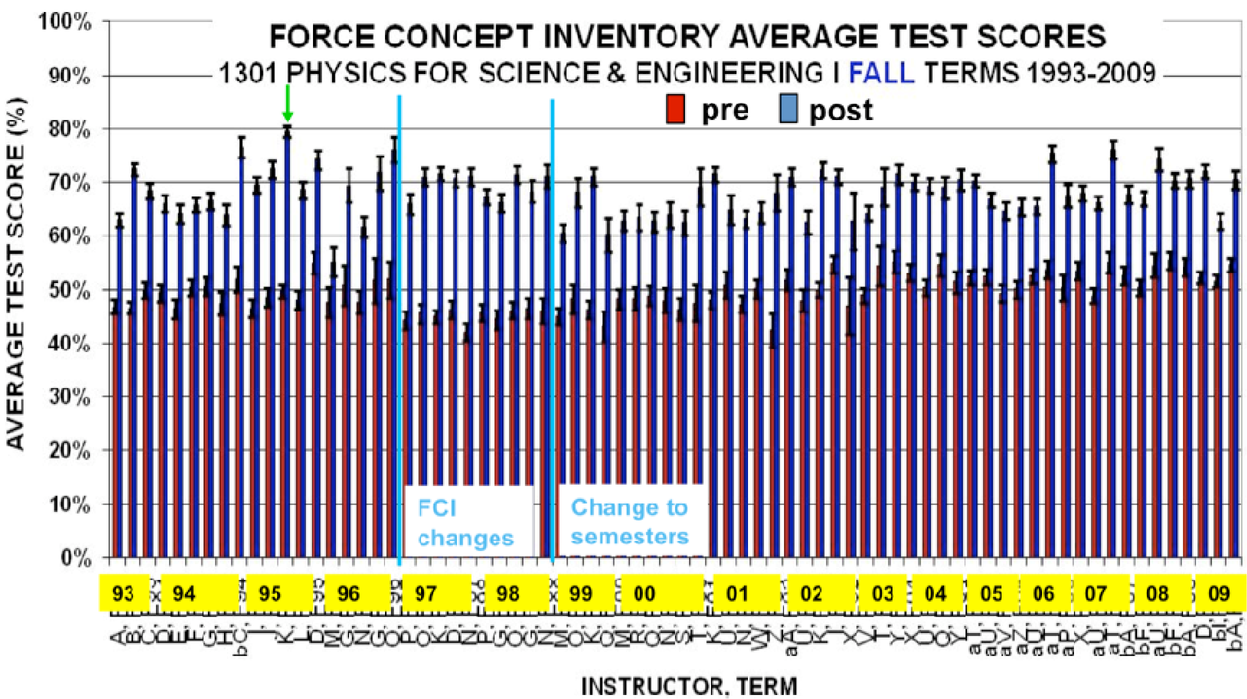
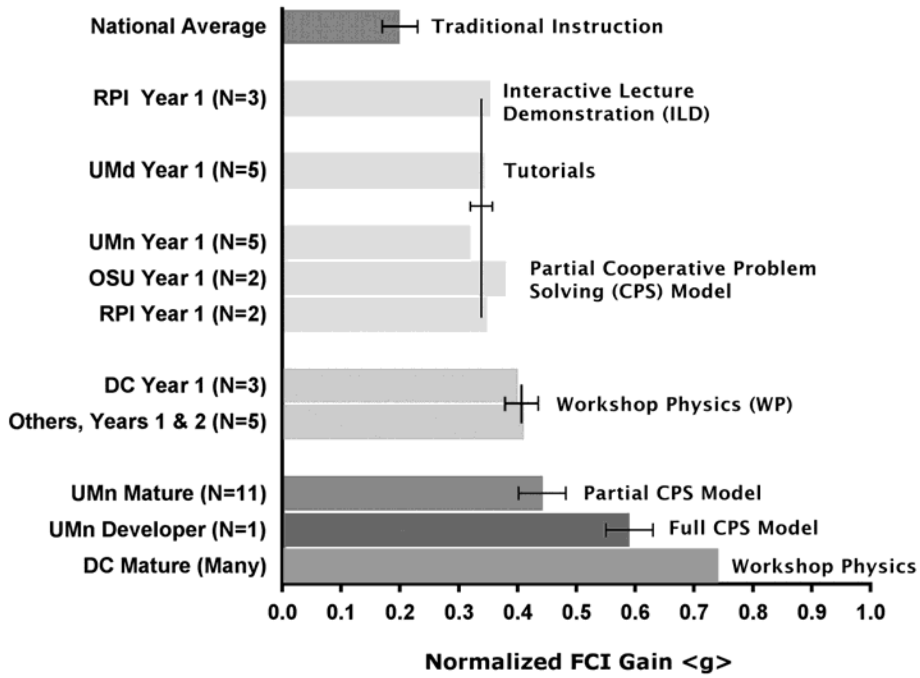


Figure 10.7 A comparison of the effect of first-year and mature implementation of an instructional innovation in a calculus-based introductory course on the FCI*



* The number in parentheses is the number of classes in the samples. The error bars are the standard deviation of the average of classes for N>1. The institutions in the left labels are: **RPI** - Rensselaer Polytechnic Institute;[2] **Umd** - University of Maryland;[17] **UMn** - University of Minnesota; **OSU** - Ohio State University;[18] **DC** - Dickinson College;[6,19] and **Others** - - two other small colleges.[6] The source of the data is in the endnotes.

problems, which most groups failed to solve. His normalized gain, $<g> = 0.27 \pm 0.5$, was slightly higher than the national average for traditional lecture courses.

Figure 10.6 shows the pretest and posttest FCI scores for the University of Minnesota faculty (41 faculty members) for the 17 years we have implemented CPS in the calculus-based course. Incoming student scores on the FCI are slowly rising, probably due to better high school preparation. The partial CPS implementation results in a post-instruction FCI score of approximately 67%. The best-practice implementation of CPS (indicated by arrow on figure) results in a post instruction FCI score of approximately 80%.

First Year and Mature Implementation of Different Instructional Innovations

Figure 10.7 shows the normalized gain on the Force Concept Inventory (FCI) for the implementation of four research-based instructional innovations in calculus-based classes: Interactive Lecture Demonstrations (ILD); Tutorials[20]; Workshop

Physics (WP)[21]; the partial implementation of Cooperative Problem Solving (CPS); and the best-practice implementation of CPS. For the other institutions in the Figure 10.4, the partial CPS implementations and Tutorials consist of replacing a standard recitation session with a CPS session or Tutorial session, while the lectures and labs remain the same. The best-practice CPS for calculus-based classes was implemented by one of the developers (author KH).

Figure 10.7 shows that the first-year implementation of Interactive Lecture Demonstrations (ILD), Tutorials, and the partial CPS implementations (<g> = 0.35 ± 0.3) improves students' conceptual understanding of forces by 15 % compared to traditional instruction (<g> = 0.20 ± 0.03). Mature implementation of partial CPS (<g> = 0.44 ± 0.03) improves students' conceptual understanding an additional 9%. Implementation of best-practice CPS (<g> = 0.59 ± 0.04) further improves FCI gains by an additional 15%.[22] This may be approaching the limit of what can be expected at a university that enrolls 150-200 students in each section with 5-7 teaching assistants.

Workshop Physics (WP) yielded the largest improvement in students understanding of forces for both first-year implementation (<g> = 0.41 ± 0.02) and for mature implementation (<g> = 0.74). The ability to integrate lecture, labs, and recitation into one classroom with an excellent curriculum and pedagogy accounts for this difference for colleges with small class sizes.

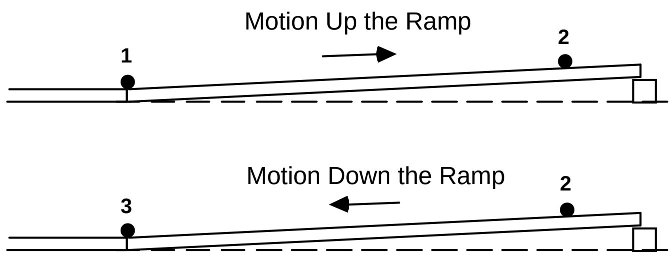
Improvement in Students' Conceptual Understanding of Motion and Forces as Measured by Open-Response Written Questions

The Ramp Problem

The open-response Ramp Problem (Figure10.8a) is a written adaptation of an interview question developed by Lillian McDermott and the Physic Education Group at the University of Washington.[23] The problem is designed to probe students' understanding of acceleration. It was administered pre- and post instruction to an algebra-based course the year before we implemented best-practice CPS. At the same time, it was administered to a traditional section of the calculus-based course. Our graduate student, Jennifer Blue, analyzed the results.[24]

Description of Categories. Standard qualitative research techniques were used to categorize the written responses.[25] Three major categories of responses emerged from the analysis: responses that include the accepted ideas about acceleration, responses that include alternative ideas (two sub-categories), and responses that could not be coded because too little was written. Figure 10.7b shows the percentage of students in each category, pre- and post-instruction, in the algebra-based partial CPS class and in the traditional calculus-based class. (Uncertainties in the results are estimates of the sampling error calculated as $(pq/N)^{1/2}$. The results are reported for students who responded to the problem both pre-instruction and post-instruction.

Figure 10.8a. The Ramp Question



A steel ball is launched with some initial velocity, slows down as it travels up a gentle incline, reverses direction, and then speeds up as it returns to its starting point. Assume friction is negligible.

a. Suppose we calculated the acceleration of the ball as it's moving *up* the ramp (from 1 to 2), and the acceleration as it's moving *down* the ramp (from 2 to 3). How would these two accelerations compare? (i.e., Are the accelerations the same size? The same direction?) Explain your reasoning.
b. Does the ball have an acceleration at its highest point on the incline (at position 2)? Explain your reasoning.

Figure 10.8b. Types of responses to question: How does the acceleration compare up and down the ramp?

	Algebra-based Partial CPS (N = 112)		Calculus-based Traditional (N = 138)	
Category of Response	**Pre (%)**	**Post (%)**	**Pre (%)**	**Post (%)**
1. Includes accepted idea of acceleration	6 ± 3	79 ± 4	19 ± 4	40 ± 5
2. Includes alternative ideas:				
a. confuse v and a, but believe motion up and down are the same	58 ± 5	16 ± 4	57 ± 5	51 ± 5
b. confuse v and a, but believe motion up and down are not the same	35 ± 5	2 ± 1	17 ± 4	6 ± 3
3. Response cannot be coded	1± 1	3 ± 2	7 ± 3	3 ± 2

Results. Figure 10.8b shows that twice as many students in the partial CPS algebra-based class (79%) had an understanding of acceleration at the end of the course than the students in the traditional calculus-based class (40%). 57 percent of the calculus-based students still confused velocity and acceleration at the end of the first quarter (10 weeks), compared to only 18% of the algebra-based students. The results for the traditional calculus-based students are consistent with the interview results by Trowbridge and McDermott[17] -- out of a sample of 36 interviewed students, 64% were successful after instruction. The results for the full CPS model are also consistent with the Force Concept Inventory (FCI) results in the last section.

The Accelerating Car Questions

We designed the Accelerating Car Questions, shown if Figure 10.9a, to probe students' understanding of the nature of forces and Newton's second law. The questions were administered to students in an algebra-based course the year before we implemented best-practice CPS (first data column in Figures 10.9b and 10.9c). Later, the same questions were administered to students in a traditional section of the calculus-based course (second data column). Later still, the questions were administered at the beginning of a second semester, when about half of the students came from other partial CPS models sections, and half from the full CPS model (third data columns). Jennifer Blue[18] completed the analysis for the algebra based course and Tom Foster completed the analysis for the calculus-based course.[26]

Description of Categories for Nature of Forces. Again, the students' responses were examined using standard qualitative research techniques.[19] Four categories emerged for the nature of forces. In the first category only Newtonian forces were identified and labeled, although other parts of the responses could be wrong. The second response category included Newtonian forces, but at least one was a 3rd-law pair force on the wrong object. For example, a student drew "the force of the car seat on the passenger" on their free-body diagram of the car. The third category included incorrect "pseudoforces," such as "the force of acceleration of the car" and "force of the car's engine that turns the tires." The fifth category contains student responses with no forces drawn.

Results for the Nature of Forces. Figure 10.9b shows that on the pretest, the same percentage of students in the algebra-based partial CPS course drew Newtonian forces (12%) as in the calculus-based traditional course (14%). The algebra-based partial CPS students made some progress in their understanding of the nature of forces; on the post-test 63% of the students drew Newtonian forces, compared with 12% on the pretest. 54 percent of the students in the traditional calculus-based course drew Newtonian forces on the post-test.

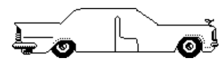
For students in a calculus-based course, participation in a partial CPS implementation results in 8% more students drawing Newtonian forces at the end of the course (62%) compared to the traditional course (54%). Participation if best-practice CPS results in an additional 13% of students drawing Newtonian forces (75%). This may be near the limit of what can be expected in large enrollment courses.

Description of Categories for Newton's Second Law. The analysis of student understanding of Newton's Second Law (Figure 10.9c) was separate from the analysis of student understanding of forces, so some student responses in higher categories may contain "pseudoforces." The first category contains responses that include accepted ideas about the sum of forces, although other parts of the responses could be wrong.

Figure 10.9a. The problem about an accelerating car

You are a passenger in a car that is traveling on a straight road while increasing speed from 30 mph to 55 mph. You wonder what forces cause you and the car to accelerate. When you pull over to eat, you decide to figure it out.
…

a. On the picture, draw and label arrows (vectors) representing all the forces acting on the *car* while it is accelerating. The length of the arrows should indicate the relative sizes of the forces (i.e., a larger force should be represented by a clearly longer arrow, equal forces by arrows of equal length). Below the picture, describe in words each force shown.



b. Which force(s) cause the car to accelerate? Explain your reasoning.

Figure 10.9b. Types of responses for the nature of the forces on the car

Category of Response	Algebra Course: Partial CPS (N = 112) pre (%)	post (%)	Calculus Course: Baseline: Traditional (N = 100) pre (%)	post (%)	Partial CPS (N=85) post (%)	Best-Practice CPS (N=71) post (%)
1. Only Newtonian forces	10 ± 3	51 ± 5	10 ± 3	39 ± 5	58 ± 5	73 ± 4
2. Newtonian forces, but some are 3rd-Law pair forces drawn on wrong object	2 ± 1	12 ± 3	4 ± 2	15 ± 4	4 ± 2	2 ± 1
3. Include non-Newtonian or "pseudoforces" (e.g., force of acceleration; force of the engine)	78 ± 4	37 ± 5	73 ± 4	39 ± 5	38 ± 4	25 ± 4
4. Cannot be coded	10 ± 3	0	8 ± 3	1 ± 1	0	0

Figure 10.9c. Types of responses to the question: Which force(s) cause the car to accelerate?

Category of Response	Algebra Course: Partial CPS (N = 112) pre (%)	post (%)	Calculus Course: Baseline Traditional (N = 100) pre (%)	post (%)	Partial. CPS (N=85) post (%)	Best-Practice CPS (N=71) post (%)
1. Includes correct ideas about summing Newtonian forces	3 ± 2	27 ± 5	7 ± 3	20 ± 4	42 ± 5	58 ± 4
2. Vague or incorrect summing	24 ± 5	9 ± 3	25 ± 4	29 ± 5	24 ± 4	21 ± 3
3. Includes Alternative Ideas						
a. Attributes acceleration to only one force	13 ± 4	56 ± 5	4 ± 2	23 ± 4	21 ± 4	19 ± 3
b. Attributes acceleration to something other than a force on the diagram	32 ± 5	0	48 ± 5	23 ± 4	7 ± 3	0
4. Can not be coded	27 ± 5	8 ± 3	16 ± 4	7 ± 3	6 ± 2	4 ± 2

The second category includes responses that contain vague or incorrect ideas about a "sum of something." Some responses in this category list forces without saying, "sum." Other responses in that category list things that are not forces, nor are they on the free-body force diagrams, like coefficients of friction. The third category includes responses that do not sum; they incorrectly cite only one force as the cause of acceleration. Most students in this category had drawn unbalanced forces on their drawings. The fourth category includes responses that cite things other than forces on the diagrams as the cause of acceleration. Some responses in this category attribute the car's acceleration to its engine, and others cite things like "motion" and "momentum" as the cause of acceleration. Responses in the fifth category could not be coded.

Results for the Newton's Second Law. Figure 10.9c shows that students in both the algebra-based partial CPS course and the traditional course showed very little improvement in their understanding of Newton's second law from pretest to posttest (23% and 13% respectively). For students in a calculus based course, participation in a the partial CPS course results in 22% more students with the correct idea of summing forces at the end of the course (42%) compared to the traditional course (20%). Participation if the best-practice CPS course results in an additional 16% of students with the correct idea of summing forces (58%). These results indicate that there is room for further improvement.

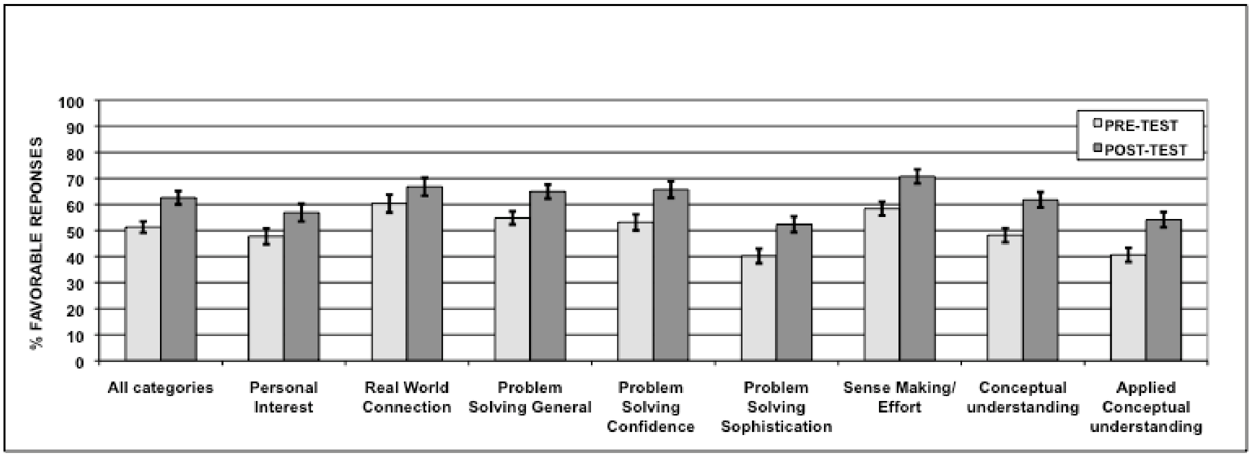
Summary for FCI and Open-Ended Questions

The results for the improvement in students' conceptual understanding of motion and forces as measured by the Force Concept Inventory (FCI) are consistent with the results for the open-response written questions. For the calculus-based course, partial implementation of CPS improved students conceptual understanding of motion and forces for the FCI ($\langle g\rangle = 0.44 \pm 0.03$ versus $\langle g\rangle = 0.20 \pm 0.03$ for traditional courses), students' understanding of the nature of forces (gain of 8% of students drawing Newtonian forces compared to traditional), and students' understanding of Newton's second law (gain of 22% of students with correct idea of summing forces compared to traditional). Implementation of best-practice CPS resulted in the most improvement on the FCI ($\langle g\rangle = 0.59 \pm 0.04$ versus $\langle g\rangle = 0.20 \pm 0.03$ for traditional courses), conceptual understanding of the nature of forces (gain of 21%), and understanding of Newton's second law (gain of 38%).

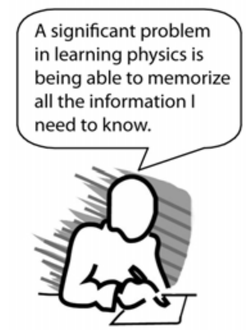
Improvement in Students' Learning Attitudes

Students enter our courses with sets of attitudes and beliefs about learning science. How we conduct our class sends messages about how, why, and by whom science is learned. Such messages are being studied with the goal of developing more expert-like views on the nature and practice of science in our students.[27][

Figure 10.10. Learning attitudes for a best-practice implementation of CPS



Over the last two decades, physics education researchers have developed several survey instruments to measure these attitudes and beliefs and to distinguish the beliefs of experts from the beliefs of novices.[28] For example, expert physicists see physics as a coherent framework of concepts that describe nature and are established by experiment. Novices see physics as isolated pieces of information with no connection to the real world, which are handed down by authority (e.g. teacher) and must be memorized.



Data have shown that, traditionally, student beliefs become more novice-like over the course of a semester.[29] Even in courses using reformed classroom practices that are successful at improving student conceptual learning of physics, student beliefs tend not to improve. Some success has been achieved, however, in courses specifically designed to attend to student attitudes and beliefs. [30]

At the University of Minnesota, the Colorado Learning Attitudes about Science Survey (CLASS Version 3) was used in 2009 to measure student beliefs at the start (pre) and end (post) of an introductory physics courses for pre-medicine and biology majors. This course implemented the best-practice CPS. The 'All Categories' favorable score is measured as the average percentage of the 42 survey statements to which the students answer in the favorable sense (e.g. as an expert physicist would).

The survey is also used to measure specific belief categories by looking at subsets of statements. Included are measurements of the following categories: 'Personal Interest' (I think about physics in my life); 'Real World Connections' (physics describes the world); "Problem Solving General" (equations represent concepts); Problem Solving Confidence' (I can usually figure out a way to solve problems); Problem Solving Sophistication (When I get stuck, I can figure out a different method); 'Sense Making/Effort' (I put in the effort to make sense of physics ideas); 'Conceptual Understanding' (physics is based on a conceptual framework); and "Applied Conceptual Understanding' (After I study a topic, I can apply it).

The results, shown in Figure 10.9, indicate significant improvement in all categories of students' attitude about learning physics. The overall improvement was 11% of favorable responses. This improvement is larger than in courses reported to attend to student attitudes and beliefs. We concluded that establishing a culture of expert practice (see Chapter 6), demonstrating a research-based problem-solving framework when solving problems in lectures, requiring students to practice using the framework for solving context-rich problems in cooperative groups (and for homework), appropriate course grading, and providing appropriate scaffolding, improves students' attitudes towards learning physics.

Endnotes

[1] Heller, P., Keith, R., & Anderson, S. (1992). Teaching problem solving through cooperative grouping. Part 1: Groups versus individual problem solving, *American Journal of Physics*, **60**(7), 627-636.

[2] For example, if students have a persistent misconception about a principle or concept (e.g., Newton's second law or acceleration), they would not typically be able to solve a complex problem correctly. On the other hand, students may be able to answer conceptual or qualitative questions about a principle or concept, but not be able to apply the concept correctly to a quantitative problem situation. They may not know the conditions of applicability of the principle or concept. Or they may have disconnected qualitative and quantitative knowledge, and apply the plug-and-chug or pattern-matching strategy to all quantitative problems, without regard to the meaning of principles and concepts.

[3] Cummings, K., Marx, J., Thorton, R., & Huhl, D. (1999). Evaluating innovation in studio physics, *American Journal of Physics*, Physics Education Research Suppl., **67**(7), S38-S44.

[4] The defining characteristics of Studio Physics are integrated lecture/laboratory sessions, small classes of 30–45 students, extensive use of computers, collaborative group work, and a high level of faculty–student interaction. Each section of the course is led by a professor or experienced instructor, with help from one or two teaching assistants. The teaching assistants' roles are to circulate throughout the classroom while the students are engaged in group work.

[5] Interactive Lecture Demonstrations (ILD) was developed by David Sokoloff and Ron Thornton to guide students through a three-part learning sequence of predict, confront, and resolve. The instructor shows and explains a situation designed to conflict with their predictions, gathers the students' predictions, and demonstrates and takes data as students copy the result. The students are then asked for an explanation (confront), and the instructor makes the bridge to similar situations (resolve). See Sokoloff, D.R., and Thorton, R.K. (1997), Using interactive lecture demonstration to create an active learning environment, *Physics Teacher*, **35**, 340-347; and Sokoloff, D.R., and Thorton, R.K. (2001), *Interactive Lecture demonstrations*, NY: Wiley

[6] See Dufresne, R.J., Gerace, W.J., Hardiman, P.T., & Mestre, J.P. (1992), Constraining novices to perform expert-like problem analyses: Effects on schema acquisition, *Journal of Learning Science,* **2**, 307-321; and Leonard, W.J., Dufresne, R.J., and Mestre, J.P. (1996), Using qualitative problem-solving strategies to highlight the role of conceptual knowledge in solving problems, *American Journal of Physics*, **64**, 1495.

[7] See Antonenko, P.D., Jackman, J., Kumsaikaew, P., Marathe, R.R., Niederhauser, D.S., Ogilvie, C. A., and Ryan, S.M., (submitted for publication), Understanding student pathways in context-rich problems, retrieved November 24, 2010, http://www.public.iastate.edu/~cogilvie/edresearch.html.

[8] For a brief discussion of these curricular innovations, see Hsu, L., Brewe, E., Foster, T. M., & Harper, K. A. (2004), Resource letter RPS-1: Research in problem solving. *American Journal of Physics*, *72*(9), Section IV

[9] See Taconis, R., Ferguson-Hessler, M.G.M., &. Broekkamp, H. (2001), Teaching science problem solving, *Journal of Research in Science Teaching*, **38**, 442–468. The authors did a meta-analysis of 22 previously published articles on teaching problem solving in science classes.

[10] See Chi, M. T. H., Feltovich, P. J., & Glaser, R. (1981), Categorizations and representation of physics problems by experts and novices, *Cognitive Science*, **5**, 121-152; Larkin, J., McDermott, J., Simon, D., & Simon, H. A. (1980), Expert and novice performance in solving physics problems, *Science*, **208**, 1335-1342; and Hardiman, P.T., Dufresne, R., and Mestre, J.P. (1989), The relation between problem categorization and problem solving among novices and experts, *Memory and Cognition*, **17**, 627.

[11] We developed criteria for selecting problem-framework users and non-users by examining the solutions of all the students for three context-rich problems one from the third exam, and two from the final (after 10 weeks of the quarter). The criteria for including students in user group did not include solving problems correctly, but on the qualitative analysis of the problem and planning a solution.

[12] Keith, R. (1993), Correlation between the consistent use of a general problem-solving strategy and the organization of physics knowledge. Unpublished PhD thesis: University of Minnesota.

[13] The opposite has also been shown. That is, novices who classify problems in a more expert-like manner are more proficient in solving problems. See Hardiman, P.T., Dufresne, R., &Mestre, J.P. (1989). The relationship between problem categorization and problem solving among experts and novices, *Memory and Cognition*, **17**, 627–638.

[14] Hestenes, D., Wells, M., & Swackhammer, G. (1992). Force concept inventory, *The Physics Teacher*, **30**(3), 141-158

[15] Hake, R. (1998). Interactive-engagement vs traditional methods: A six thousand student survey of mechanics test data for introductory physics courses. *American Journal of Physics*, **66**: 64-74

[16] There is a question about whether relative gain is the best measure of improvement, especially in a class that has a high pretest average. A common practice today is to leave out the top 10-15 percent of the students on the pretest, and report the absolute gain for those students in the class who can improve their FCI scores significantly.

[17] Redish, E.F., Saul, J.M., & Steinberg, R.N. (1997). On the effectiveness, of active engagement microcomputer-based laboratories, *American Journal of Physics*, **65** (1), 45-54

[18] Communication from Jeff Saul, 1996.

[19] The <g> for Dickinson College, mature implementation is from Redish, E.F. (2003). *Teaching physics with the physics suite*, MA: Wiley

[20] Tutorials were developed by Lillian McDermott and the Physics Education Group at the University of Washington. This supplementary curriculum is designed to be used in small group sessions in which three or four students work together collaboratively. Worksheets guide students through a three-part learning sequence of predict, confront, and resolve. See McDermott, L.C. and Shaffer, P.S., (2002). *Tutorials in introductory physics and homework package*, Upper Sadler River NJ; Prentice Hall

[21] Workshop Physics instruction is based on a four-part learning sequence. Students in small classes of 10 – 20 students make predictions about a phenomenon, reflect on their observations and try to reconcile any differences; they develop definitions and equations from theoretical considerations; they perform experiments to verify predictions based on theory; they apply their understanding in solving problems. See Laws, P.W. (1991), Calculus-based physics without lectures, *Physics Today*, **44**(12), 24-31; Laws, P.W. (1997). *Workshop physics activity guide,* NY, Wiley; and Laws, P. (1997), Millikan lecture 1996: Promoting active learning based on physics education research in introductory physics courses, *American Journal of Physics*, **65**, 14-21.

[22] The full CPS model was only implemented once in the calculus-based course because different sections of the course (~ 1000 students) are taught each semester. Students can enroll in any section, and take a common final exam. Therefore, the sections cannot be very different from each other in content or pedagogy.

[23] Trowbridge, D.E. & McDermott, L. C. (1981). Investigation of student understanding of the concept of acceleration in one dimension, *American Journal of Physics*, **49**(3), 242-253.

[24] Blue, J. (1993). Sex differences in open-ended written response questions about motion and forces, Unpublished Master's Degree Paper, University of Minnesota.

[25] See Strauss, A.L. (1987). *Qualitative analysis for social scientists*, Cambridge: Cambridge University Press; and Strauss, A.L. and Corbin, J. (1990). *Basics of qualitative research: grounded theory procedures and techniques*, Newbury Park, CA: Sage.

[26] Foster, T. & Heller, P. (1996). Does continuing in an introductory Physics sequence help students understand the basics? A peek into Pandora's Box, *AAPT Announcer*, **26**(2): 61

[27] See Hammer, D. (2000), Student resources for learning introductory physics. *American Journal of Physics*, Physics Education Research Supplement, **68**(S1), S52-S59; Elby, A. (2001), Helping physics students learn how to learn, *American Journal of Physics*, *Physics Education Research Supplement* **69**, S54-S64 (2001); and Redish, E.F, (2003), *Teaching Physics with Physics Suite*, Wiley.

[28] See Redish, E.F., Saul, J.M., & Steinberg, R.N. (1998), Student expectations in introductory physics, *American Journal of Phys*ics, **66**, 212-224; Halloun, I. (1996). Views about science and physics achievement, The VASS story, in *Proceedings of the International Conference on Undergraduate Physics Education,* American Institute of Physics Press, College Park, MD; Elby, A. (2001), Helping physics students learn how to learn, *American Journal of Physics*, *Physics Education Research Supplement* **69**, S54-S64; and Adams, W.K., Perkins, K.K., Dubson, M., Finkelstein, N.D., and Wieman, C.E. (2005), The design and validation of the Colorado Learning Attitudes about Science Survey, *Proceedings of the 2004 Physics Education Research Conference*, Rochester, NY: PERC Publishing.

[29] Redish, E.F., Saul, J.M., & Steinberg, R.N. (1998). Student expectations in introductory physics, *American Journal of Phys*ics, **66**, 212-224

[30] See Redish, E.F. (2003), *Teaching Physics with Physics Suite*, Wiley; Hammer, D. (2000), Student resources for learning introductory physics. *American Journal of Physics*, *Physics Education Research Supplement*, **68**(S1), S52-S59; and Perkins, K. Adams, W. K., Finkelstein, N. D., Pollock, J. S., & Wieman, C. E. (2005), Correlating student attitudes with student learning using the Colorado Learning Attitudes about Science Survey, *Proceedings of the 2004 Physics Education Research Conference*, Rochester, NY: PERC Publishing.

Part 3

Teaching a CPS Session

In this part . . .

Another cliché is that "the devil is in the details." We are assuming you have made the decision to teach a CPS session with your students. The chapters in this part provide detailed descriptions or how to prepare for a CPS session, teach the session, and monitor, diagnose, and intervene with groups.

Chapter 11 provides a description of how to prepare for a CPS session, including assigning students to groups with roles or rotating roles, preparing a group problem and information sheet, writing the problem solution, preparing an answer sheet (optional), and preparing a group function evaluation sheet (as necessary).

Chapter 12 explains the steps in teaching a CPS session, including opening moves, the middle game, and the end game.

Chapter 13 provides information about how to monitor, diagnose, and intervene in groups, particularly dysfunctional groups and groups having difficulty with physics.

Chapter 11
Preparation for CPS Sessions

In this chapter:

- ✓ Overview of the routine for a CPS session.
- ✓ How to assign your students to groups.
- ✓ Preparing a group *Problem & Information* sheet and deciding on the part of the solution you want groups to discuss.
- ✓ Preparing an *Answer Sheet* (optional).
- ✓ Preparing a *Group Functioning Evaluation* form (optional)

The usual Cooperative Problem Solving (CPS) routine, like a game of chess, has three parts -- Opening Moves, a Middle Game, and an End Game. As in chess, both the opening moves and the end game can be planned in detail. The middle game - collaborative problem solving -- has many possible variations.

Opening Moves (~ 5 minutes). Opening moves determine the mind set that students should have during the Middle Game -- the collaborative solving of a problem. The purpose of the opening moves is to answer the following questions for students.

- ♦ Why has this particular problem been chosen?
- ♦ What should we be practicing and learning while solving this problem?
- ♦ How much time will we have?
- ♦ What is the product we should have at the end of this time?

Educational research indicates that providing students this simple information before they start leads to better learning and higher achievement. An example of an opening move is shown in Figure 11.1.

Middle Game (~ 35 minutes). This is the learning activity -- students work collaboratively to solve the problem. During this time, your role is one of coach. You circulate around the room, listening to what students in each group are saying and observing what the Checker/ Recorder is writing. You intervene when a group needs to be coached on an aspect of physics or is not functioning well. At the end of the allotted time, you have your groups draw and write on the board the parts of the solution that you specified in your opening moves.

Figure 11.1. Examples for beginning and ending a CPS session.

Example of Opening Moves

We have been studying the conservation principles in class -- the conservation of energy and the conservation of momentum. The problem you will solve today was selected to help you learn *when and how to apply these principles.*

You will have 35 minutes to work on the problem. At the end of that time, you will be asked to draw your diagrams and list the equations you used to solve the problem on the board.

Example End-game Questions

Look at the momentum vector diagrams on the board. How are they the same and how are they different?

Is there different physics represented in the diagrams, or the same physics?

Look at the diagrams for group #1 and #5. What is missing in these diagrams?

Does the order -- x direction first or y direction first -- make any difference to the final solution?

End Game (~ 10 minutes). The end game determines the mind-set students have when they leave the class -- do they think they learned something or do they think it was it a waste of their time. The purpose of the end game is to help students answer the following questions.

- ♦ What have I learned that I didn't know before?
- ♦ What did other students learn?
- ♦ What should I concentrate on learning next?

A good end game helps students consolidate their ideas and produces discrepancies that stimulate further thinking and learning. Typically the instructor gives students a few minutes to examine what each group produced, then leads a whole-class discussion of the results. Your role as the instructor is to *facilitate* the discussion, making sure students are actively engaged in consolidating their ideas.

There are several decisions to make and materials to prepare before teaching a CPS session, as shown outlined below.

1. Assign students to groups (if changing groups) and assign/rotate a role for each student. Make a copy of the group and role assignments (for preparing an overhead or for copying on the board before class begins).

2. Write/Adapt a context-rich problem that meets the criteria for a good group problem (see Chapter 15).

3. Write a problem solution following the problem-solving framework that you have decided to teach your students. Decide what part of the solution you want your groups to write on the board (or butcher paper, whiteboards) during class.

4. Make a computer projection or photocopy of the problem and information sheet for examples see pages // and //), and photocopies of a group Answer Sheet (optional) that contains the major cues for the problem-solving framework you are teaching (example on pages //-//).

Each of these preparations is discussed in the sections of this chapter.

Assign Students to Groups with Roles

How many groups will I have?

The optimal group size is three. For example, suppose you had 17 students in a discussion class. Then you would have 5 three-member groups and two students left over.

We have found that groups of four usually work better than pairs (see Chapter 8). So with seventeen students, you would normally assign students to five groups, three groups with three members and two four-member groups.

What criteria do I use to assign students to groups?

There are three criteria we use to assign students to groups.

1. Problem-solving Performance. The most important criterion for assigning students to groups is their problem solving performance based on past problem-solving tests. That is, a three-member group would ideally consist of a higher-performance, a medium-performance, and a lower-performance student. Four-member groups would ideally consist of a high performance, medium-high performance, medium-low performance, and a low-performance student. [See Chapter 9 for some research support for this criterion.] There are two other "rules of thumb" for assigning students to groups. These rules should be modified by your knowledge of the social interactions of your students.

> **Gender** We observed a group, consisting of a lower-performance man, a medium performance man and a high performance woman, having a vigorous discussion about the path of a projectile. The men insisted on a path following the hypotenuse of a triangle; while the woman argued for the correct parabolic trajectory.
> At one point, she threw a pen horizontally, commenting as it fell to the floor, "There see how it goes. It does ***not*** go in a straight line!" Even so, she could not convince the two men, who politely ignored her.

2. Gender. Our observations indicated that frequently groups with only one woman do not function well, especially at the beginning of class. To be on the safe side, avoid groups with only one woman. We found the difficulty is with the men, not the women (see example on previous page). Regardless of the strengths of the lone woman, many men in our groups tend to ignore her. On the other hand, it is not a good idea to assign all the students in a class to same-gender groups. The women notice and tend to suspect gender discrimination. Curiously, no one seems to notice when all mixed-gender groups have two women.

3. English as a Second Language (ESL). Students from other cultures often have difficulty adjusting to group work, especially in mixed-gender groups. Their difficulties are exacerbated if English is their second language (ESL). So to be on the safe side, whenever possible we assign ESL students to same-gender groups of three.

Prepare Problem & Information Sheet

The success of CPS depends on designing group practice problems that place "fences" or barriers on all paths that do not involve using a logical and organized problem-solving strategy. As described previously in Chapter 3, context-rich problems are specifically designed so that:

- ✓ It is difficult to manipulate a formula to get an answer.
- ✓ It is difficult to match an example solution pattern to get an answer.
- ✓ It is difficult to solve the problem without first analyzing the problem situation.
- ✓ Solution patterns are not cued by physics words such as "inclined plane," "starting from rest," or "inelastic collision".
- ✓ Logical analysis using fundamental physics concepts is reinforced.

In addition, to be effective a context-rich problem should have an appropriate *level of difficulty* for its intended use, in this case as a group practice problem (see Chapter 16).

Below are some suggestions for preparing an appropriate problem and information sheet.

Step ①. The first step is to decide on your goal(s) for the group practice problem. For example, suppose you have finished studying elastic collisions and introduced inelastic collisions in your lecture. Your students, however, have not had time to complete the more difficult homework problems on inelastic collisions. In this case the group practice problem should involve an inelastic collision, but should not be too difficult (i.e., involve vectors or both inelastic collisions *and* the transfer of momentum). You also know that your students are still having difficulty

Figure 11.2. The context-rich Skateboard Problem.

You are helping your friend prepare a skateboard exhibition. The idea is for your friend to take a running start and then jump onto a heavy duty 15-lb stationary skateboard. Your friend, on the skateboard, will glide in a straight line along a short, level section of track, then up a sloped concrete wall. The goal is to reach a height of at least 6 feet above the starting point before rolling back down the slope. The fastest your friend can run and safely jump on the skateboard is 20 feet/second. Can this program work as planned? Your friend weighs in at 125 lbs.

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distinguishing between conservation of momentum and conservation of energy. To confront this difficulty, you want a problem that requires both the conservation of momentum and the conservation of energy for a solution.

Step ②. The next step is to adapt a context-rich problem to meet your goal for the problem. Many textbooks have context-rich problems that can be adapted for group work. This book contains some context-rich problems (See Appendices B and C). Some problems are also available on our website (http://groups.physics.umn.edu/physed/Research/CRP/crintro.html). The skateboard problem, shown in Figure 11.2, is from Appendix B. If you cannot find and adapt a context-rich problem, you can adapt a traditional textbook problem, following a procedure described in Chapter 16.

Solve your problem and check that it is the right level of difficulty for a practice group problem (see also Chapter 14), using the Criteria shown in Figure 11.3. As this figure shows, the skateboard problem is suitable for a practice group problem, since it meets all the criteria.

If you know *how to arrive at a solution*, even if you don't know the answer, then the question is not a problem for you (see Chapter 4). Because you are an expert, the "problems" in an introductory physics course are not real problems for you, so you don't use a problem-solving framework to solve them. But they *are* problems for your students.

Step ③. The final step is to prepare an information sheet. The information sheet contains all constants and "equations" they need to solve the problem, as illustrated in Figure 11.4 (see also Chapter 3, pages //-//). The use of an information sheet is another example of applying the 3rd Law of Instruction -- *Make it easier for students to do what you want them to do and more difficult to do what you don't want.* In this case, we don't want students to spend time searching a textbook (or their memory) for appropriate equations or a matching example problem, so we supply them with the equations. Then groups spend their time discussing what the equations mean and how the concepts and principles should be applied to solve the problem. The information sheet "grows" with the course -- nothing is taken off, but new constants, fundamental principles, and concepts are added as more is learned.

Figure 11.3. Criteria for a good group problem**.**

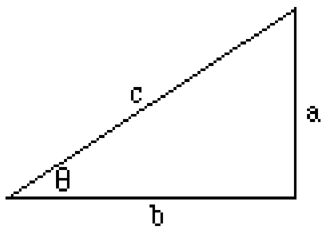
Criteria for Group Problem	Skateboard Problem
❶ A group problem must be designed so that:	
✓ There is something to discuss *initially* so that *everyone* (even the weakest member) can contribute to the discussion.	Students need to spend time initially drawing a picture of the situation.
✓ There are several decisions to make in solving the problem.	Students must decide what assumptions to make and what their target variable will be.
❷ A group problem must be *challenging* enough so that:	
✓ Even the best student in the group can not immediately see how to solve the problem, and all students feel good about their role in arriving at a solution.	The problem cannot be solved by substitution of known values into momentum or energy equations.
✓ Knowledge of basic physics concepts is necessary to interpret the problem.	Students must apply the conservation of momentum and the conservation of energy.
✓ Students' alternative conceptions about the physics naturally arise and must be discussed.	Students must understand the *difference* between conservation of energy and momentum, and when it is appropriate to use these principles.
❸ At the same time, the problem must be *simple* enough so that:	
✓ The mathematics is not excessive or complex.	The problem requires only simple algebra.
✓ The solution path, once arrived at, can be understood, appreciated, and easily explained to all members of the group.	Once students have decided when and how to apply the conservation of momentum and energy, the solution is straightforward.
✓ A majority of groups can reach a solution in the time allotted.	Figuring out *how* to solve the problem, which takes the most time, can be done in the time allotted (about 35 minutes).

Write the Problem Solution

In most CPS sessions (except for a group exam problem), students may not have time to complete the problem solution before you stop them to conduct the whole-class discussion. This makes some students anxious or uncomfortable, and it is very difficult to get groups to stop solving the problem. Because the class discussion cannot go over the entire problem solution, many students need reassurance that their group solution is correct (or at least on the right track). Anxiety is relieved when they know they will see a complete solution after the session (usually posted on you class web page). This makes it easier to stop the groups and have them participate in the whole-class discussion.

Figure 11.4. Example list of Useful Information: Calculus-based course

Useful Mathematical Relationships:



For a right triangle: $\sin\theta = \frac{a}{c}$, $\cos\theta = \frac{b}{c}$, $\tan\theta = \frac{a}{b}$,

$a^2 + b^2 = c^2$, $\sin^2\theta + \cos^2\theta = 1$

For a circle: $C = 2\pi R$, $A = \pi\square^2$

For a sphere: $A = 4\pi R^2$, $V = \frac{4}{3}\pi R^3$

If $Ax^2 + Bx + C = 0$, then $x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$; $\frac{d(z^n)}{dz} = nz^{n-1}$

Fundamental Concepts and Principles:

$$v_{x\ av} = \frac{\Delta x}{\Delta t} \qquad s_{ave} = \frac{dist}{\Delta t} \qquad a_{x\ av} = \frac{\Delta v_x}{\Delta t} \qquad E_f - E_i = E_{in} - E_{out} = \Delta E_{transfer} \qquad \vec{p}_f - \vec{p}_i = p_{in} - p_{out} = \Delta\vec{p}_{transfer}$$

$$v_x = \frac{dx}{dt} \qquad s = \frac{dr}{dt} \qquad a_x = \frac{dv_x}{dt} \qquad KE = \frac{1}{2}mv^2 \qquad \vec{p} = m\vec{v}$$

$$\frac{d\theta}{dt} = \frac{v}{r} \qquad \Sigma F_x = ma_x \qquad F_{12} = F_{21} \qquad E_{transfer} = \int F_x dx \qquad \vec{p}_{transfer} = \int \vec{F} dt$$

Under Certain Conditions:

$$x_f = \frac{1}{2}a_x(\Delta t)^2 + v_{ox}\Delta t + x_o, \quad a_r = \frac{v^2}{r}, \quad F = \mu_k F_N, \quad F \le \mu_s F_N, \quad F = k\Delta x, \quad PE = mgy,$$

$$PE = \frac{1}{2}kx^2, \quad p_{transfer} = F_{parallel}\Delta t$$

Useful constants: 1 mile = 5280 ft = 8/5 km, $g = 9.8\ m/s^2 = 32\ ft/s^2$

Step ①. The first step is to write the problem solution using the specific problem-solving framework that you have decided to teach your students (see Chapter 4 and Chapter 14). An example solution for the Skateboard Problem, following the *Competent Problem-solving Framework: Calculus Version,* is shown in Figure 11.5. In our experience writing problem solutions is difficult for instructors because the solution shows how you would solve the problem *if* you did not already know how to solve it. This is a difficult mindset for expert problem solvers to assume.

Remember

The specific framework you decide to teach your students in an introductory course should be a logical and organized guide through the solution of a problem. It gets them started, guides them to what to consider next, organizes their mathematics, and helps them determine if their answer is correct. *It does not always yield the most elegant solution but it is straightforward, easy to understand, and very general.*

Step ②. The next step is to decide what part of the problem solution you want each group to write on the board for the whole-class discussion. Typically, you select part(s) of the solution that you know are difficult for your students. For example, you know that students have difficulty distinguishing the conservation of energy from the conservation of momentum. So for the skateboard problem, you could have groups write on the board their physics diagrams/representations of the problem, in this case the momentum and energy descriptions of the problem.

Prepare An Answer Sheet (Optional)

If you are following the grading recommendations from Chapter 9, you may want to provide an Answer Sheet that contains specific cues for the problem-solving framework you are teaching your students. An example answer sheet for the calculus-based *Competent Problem-solving Framework* is shown in Chapter 5, page //. Notice that the skateboard problem example solution is written on this answer sheet.

Prepare Group Functioning Evaluation Sheets (as needed)

One of the elements that distinguish cooperative groups from traditional groups is the opportunity for students to discuss how well they are solving the problems together and how well they are maintaining effective working relationships among members. We found that our students needed to evaluate their group functioning fairly often at the beginning of a course. For this purpose, we used *the Group Functioning Evaluation* form shown in Chapter 9, page //. After students are comfortable working in groups and know how to co-construct group solutions, they typically need only an occasional opportunities to evaluate their group functioning.

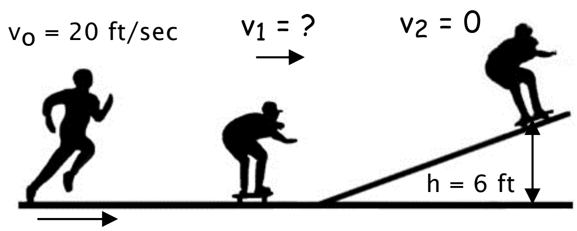
After the first 5 - 6 weeks of the introductory course, we use two rules of thumb to decide if students need to discuss their group functioning.

1. **Change to New Groups.** Typically groups are more effective when they evaluate their functioning the first time they work together as a *new* group.

Figure 11.5. Example Solution of Skateboard Problem Following the Competent Problem-solving Framework for Calculus-based Course

FOCUS on the PROBLEM

Picture and Given Information



W_r = 125 lbs (weight of runner, R)
W_s = 15 lbs (weight of skateboard, S)
v_o = 20 ft/sec (initial speed of runner)
v_1 = ? (speed of runner & skateboard just after runner jumps off skateboard)
h = 6 ft (desired height R&S would reach)

Question:

Will your friend reach the goal of gliding to a height of at least 6 ft?

Approach

Define the system as the runner and the skateboard. Use conservation of momentum to find v_1, then conservation of energy to find h.

Assume that when the runner jumps on the skateboard: (a); the vertical component of runner's momentum is so small it can be ignored, so there is no transfer of momentum and (b) the friction between ground and skateboard is so small it can be neglected, so there is no transfer of energy as the skateboard and runner move up the slope.

DESCRIBE the PHYSICS

Diagram(s) and Define Quantities

Momentum Diagrams			**Energy Diagrams**		
Momentum Before Collision	Momentum Transfer	Momentum After Collision	Energy before going up slope	Energy Transfer	Energy at top of slope
$\longrightarrow$ $\vec{p}_i = m_i\vec{v}_i$ $m_i = m_r = \frac{W_r}{g}$ $v_i = v_o$	assume NONE (neglect vertical component of initial momentum)	$\longrightarrow$ $\vec{p}_f = M\vec{v}_f$ $M = m_r + m_s = \frac{W_r + W_s}{g}$ $v_f = v_1 = ?$	$E_i = KE = \frac{1}{2}Mv_i^2$ $v_i = v_1 = ?$	assume NONE (neglect friction)	$E_f = PE_{grav.} = MgH$

Target Quantity: H = ?

Quantitative Relationships: $\vec{p}_f - \vec{p}_i = \Delta\vec{p}_{transfer} = 0$ and $E_f - E_i = \Delta E_{transfer} = 0$

PLAN the SOLUTION

Construct Specific Equations | Unknowns

Find H: Conservation of Energy — H

$$E_f - E_i = \Delta E_{transfer}$$

$$MgH - \frac{1}{2}Mv_1^2 = 0$$

① $H = \frac{v_1^2}{2g}$ — v_1

Find v_1: Conservation of Momentum

$$\vec{p}_f - \vec{p}_i = \vec{p}_{transfer}$$

$$Mv_1 - m_r v_o = 0$$

② $v_1 = \frac{m_r}{M} v_o$ — m_r/M

Find m_r/M: Use W=mg

③ $\frac{m_r}{M} = \frac{W_r/g}{(W_r + W_s)/g} = \frac{W_r}{W_r + W_s}$

So $H = \frac{\left(\frac{W_r}{W_r+W_s} v_o\right)^2}{2g}$

$$H = \left(\frac{W_r}{W_r+W_s}\right)^2 \frac{v_o^2}{2g}$$

Check for Units

$$\left(\frac{[\text{lbs}]}{[\text{lbs}]+[\text{lbs}]}\right)^2 \frac{[\text{ft}/\text{s}]^2}{[\text{ft}/\text{s}^2]} = [\text{ft}]$$

OK

EXECUTE the PLAN

Calculate Target Quantity(ies)

$$H = \left(\frac{125 \text{ lbs}}{125+15 \text{ lbs}}\right)^2 \frac{(20 \text{ ft/s})^2}{2 \bullet 32 \text{ ft/s}^2}$$

$$= 5.0 \text{ ft}$$

So H < 6 ft, so your friend will not reach the goal of 6 ft above the ground.

EVALUATE the ANSWER

Is Answer Properly Stated?

Yes. As expected, H has the units of feet.

Is Answer Unreasonable?

No. If the skateboard were massless, your friend could reach a height of only 6.3. So reaching a height of 5 feet with a 15 lb skateboard makes sense.

Is Answer Complete?

Yes. As required, we have shown that your friend can not reach the goal of gliding to a height of 6 feet.

2. **More than 20% Dysfunctional Groups.** If appropriate group problems, grading, group structures, and coaching are in place, then at any given time there should be no more than 20% dysfunctional groups (i.e., 1 in 5 groups). If you notice an increase in the number of dysfunctional groups in the past few weeks, then one possibility is personality conflicts within groups. In this case, groups need the opportunity to resolve these conflicts and decide how to function more effectively.

Every group evaluation should include the opportunity for students to think about and discuss two things:

- What is something each member did that was helpful for the group?
- What is something each member could do to make the group even better the next time they solve a problem together?

There are, of course, many ways to do this. If you are changing groups or if you do not have a clear idea why there is an increase in the number of dysfunctional groups, you may decide to use a general form similar to the one shown on page // of Chapter 9. If you have a hypothesis about the cause of the difficulty, you may decide to ask a few specific questions to test your hypothesis. An example form, shown Figure 10.6, could be adapted for this purpose. The instructor who used this form suspected that several groups contained students, with strong personalities, who were forcing their groups to follow the plug-and-chug approach of manipulating equations rather then the logical and organized approach the instructor was teaching.

Prepare Materials

The last step of advance preparation is to make the appropriate number of photocopies or prepare a projection of everything you need for your class. Below is a checklist.

- ❑ Problem & Useful Information *(one per students),* or make a projection of one or both.
- ❑ Problem Solution: post solution on your web site after class.
- ❑ Answer Sheet (optional) *(one per group)*
- ❑ Group Functioning Evaluation form (optional) *(one per group)*
- ❑ Extra copies of *Group Roles* sheets

Figure 11.6.5. An example of a Group-Functioning Evaluation Sheet

Date: ________________ **Group #:** ___

Complete the following questions as a team.

	Low				High
1. Did all the members of our group contribute ideas?	**1**	**2**	**3**	**4**	**5**
2. Did all the members of our group listen carefully to the ideas of other group members?	**1**	**2**	**3**	**4**	**5**
3. Did we *encourage* all members to contribute their ideas?	**1**	**2**	**3**	**4**	**5**

4. What are two **specific** actions we did today that helped us solve the problem?

5. How did each of us contribute to the group's success?

6. What is a specific action that would help us do even better next time?

Group Signatures: **Manager**: ________________________________.
Skeptic: ________________________________.
Recorder/Checker: ________________________________.
Summarizer: ________________________________.

Chapter 12
Teaching a CPS Session

In this chapter:

- ✓ An outline of instructor actions in teaching a typical CPS session.
- ✓ Description of each instructor action.

The previous chapter began with a description of the three parts of a typical CPS practice session: opening moves, the middle game, and the end game. The sections below contain a description of instructor actions for the opening moves, middle game, and end game. An outline of theses instructor actions is given in Figure 12.1.

Opening Moves

⓪ Be at the Classroom Early

The classroom will probably need some preparation, so it is best to go in and lock the door, leaving your early students outside. Check out the equipment you will need to use. Make sure the chairs are set up in appropriate groups. Write the group assignments and roles on the blackboard if they are new. Include a map of the room showing where each group sits. Write instructions for students to sit in their groups.

① State the Purpose of This CPS Session (~ 2 minutes)

(a) Communicate Goal of Session. First tell your students why the problem was selected and *what they should learn from solving the problem*. For example: "We have been studying the conservation of energy and the conservation of momentum. Today's problem illustrates when it is useful to apply each conservation law."

(b) Communicate Time Limits. Then tell the students how much time they will have and what their product should be. For example: "After about 30 minutes, I will

Figure 12.1. Outline for Teaching a CPS Session

Preparation Checklist

- ❑ Group/Role assignments (if necessary, projected or written on board)
- ❑ Projection or photocopies of Problem & Useful Information *(one per person)*
- ❑ Photocopies of Answer Sheet (optional) *(one per group)*
- ❑ Group Evaluation forms (optional one per group) and extra photocopies of Group Roles Sheet
- ❑ Problem solution for posting on website after class

	Instructor Actions	What the Students Do
Opening Moves ~3-5 min.	⓪ Be at the classroom early ① Introduce the problem by telling students: a) what they should learn from solving problem; b) the part of the solution you want groups to put on board ② Prepare students for group work by: a) showing group/role assignments and classroom seating map; b) passing out Problem & Useful Information and Answer Sheet.	· Students sitting and listening · Students move into their groups, and begin to read problem. · Checker/Recorder puts names on answer sheet.
Middle Game ~35 min.	③ Coach groups in problem solving by: a) Monitoring (diagnosing) progress of all groups b) Helping groups with the most need. ④ Prepare students for class discussion by: a) giving students a "five-minute warning" b) selecting one person from each group to put specified part of solution on the board. c) passing out Group Evaluation Sheet (optional)	· Solve the problem: - participate in group discussion, - work cooperatively, - check each other's work. · Finish work on problem · Write part of solution on board · Discuss their group effectiveness
End Game ~10 min.	⑤ Lead a class discussion focusing on what you wanted students to learn from solving the problem (your goals) a) Start by asking open-ended questions. b) Follow up with questions specific to your goal or observed common errors. ⑥ Discuss group functioning (optional) ⑦ Post the problem solution after class.	· Participate in class discussion

randomly select one person to write part of their group's solution on the board. Then we will compare them."

② Prepare Students for Group Work (~ 1 minute)

(a) Assign or Rotate Roles. If students are working in the same groups, remind them to rotate roles. If you have assigned new groups, remind students to briefly introduce themselves.

(b) Problem and Information Sheet. *Project of pass out the Problem/information Sheet (one per group) and an Answer Sheet* (one per group). As you do this, make sure all groups are seated according to your map -- facing each other, close together, but with enough space between groups for you to circulate.

Middle Game (~ 30 minutes)

There are two instructor actions during the middle game: coaching students in problem solving, and preparing students for the final class discussion. You will spend most of this time coaching groups.

③ Coach Groups in Problem Solving (~30 minutes)

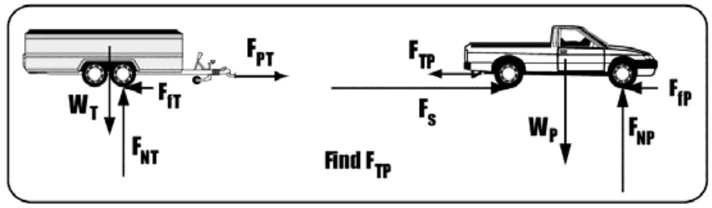
Below is a brief outline of coaching groups. For detailed suggestions of coaching and intervening techniques, see Chapter 13.

(a) Diagnose initial difficulties with the problem or group functioning. Once the groups have settled into their task, spend about five minutes circulating and ***observing*** all groups. Try not to begin coaching until you have observed all groups at least once. This will allow you to determine if a whole-class intervention is necessary to clarify the task (e.g., "I noticed that very few groups are drawing appropriate pictures. Be sure to draw the situation at all the important times.").

(b) Monitor groups and intervene to coach when necessary. Establish a circulation pattern around the room. Stop and observe each group to see how easily they are solving the problem and how well they are working together. Don't spend a long time with any one group. Keep well back from students' line of sight so they don't focus on you. Make a mental note about which group needs the most help.
Intervene and coach the group that needs the most help to get started. If you spend more than a few minutes with this group, then circulate around the room again, noting which group needs the most help. Keep repeating the cycle of (1) circulate and diagnose, (2) intervene and coach the group that needs

Figure 12.2. An example of the parts of a solution each group could be asked to write on a wall board.

Draw your group's free-body force diagrams



Write the equations for the physics principles and problem constrains that your group used to solve the problem.

$$\bar{a} = \frac{v_f - v_i}{\Delta t} \qquad F_{PT} = F_{TP} = P \qquad \Sigma F_y = m a_y = 0$$
$$W = mg \qquad \Sigma F_x = m a_x = ma$$
$$\bar{a}_P = \bar{a}_T = \bar{a} \qquad F_f = \mu F_N$$

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the most help. During this process decide on the part of the problem that would be most instructive to discuss.

(c) Five-minute Warning. About five minutes before you want students to stop, warn the class that they have only five minutes to wind up their solution. Then circulate around the class once more to determine the progress of the groups. Select one member of each group to bring part of their solution to the board. In the beginning of the course, select students who are obviously interested and articulate. Later in the course, it is sometimes effective to occasionally select a student who has not participated in their group as much as you would like. This reinforces the fact that *all* group members need to know and be able to explain what their group did.

④ Prepare Students for Class Discussion (~ 5 minutes)

(a) Posting Partial Group Solution. Announce which part of the problem is to be put on the board. An example is shown in Figure 12.2. Those not writing on the board should compare their solution with those of the other groups going on the board.

(b) Pass out Group Functioning Evaluation form (as needed). If you decided to have groups evaluate their effectiveness, pass out the forms (one per group) and have students complete the forms while the solution parts are going on the board. Give the people who you selected to write on the board time to participate in the group evaluation.

End Game (10 minutes)

The end-game discussion should focus on part of the problem that illustrates the primary goal. The purpose is to help students consolidate their ideas and produce discrepancies that stimulate further thinking and learning.

Give students a few minutes to compare the results from each group. Then lead the class discussion.

⑤ Lead a Class Discussion (~ 10 minutes)

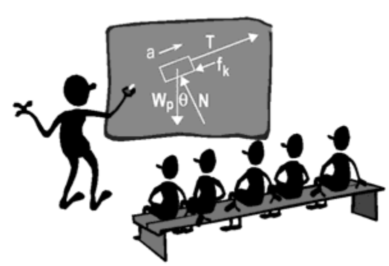
(a) Ask Open-ended Questions. The class discussion is always based around the groups, with individuals only acting as representatives of a group. This avoids putting one student "on the spot." Conduct a discussion about the problem solution without (a) telling the students the "right" answer or becoming the final "authority" for the right answers, and (b) without focusing on the "wrong" results of one group and making them feel stupid or resentful. To avoid these pitfalls, try starting with general, open-ended questions such as:

- How are the . . . (problem solution part) on the board similar?
- How do they different?

In the beginning of a course, students are usually reticent. They unconsciously play the “waiting game”. They know that by waiting long enough, the instructor will answer their own questions and they won’t have to think. We recommend counting silently up to at least 30 after you have asked a question. Usually students get so uncomfortable with the silence that somebody speaks out. If not, call on a group by number: “Group 3, what do you think?” Always encourage an individual to get help from other group members if he or she is "stuck."

(b) Ask Questions Related to Goals or Common Errors. After the general questions, you can become more specific. The questions you ask will depend on what you observed while groups were solving the problem and what your groups write on the board.

Encourage groups to talk to each other by redirecting the discussion back to the groups. For example, when a group reports their answer to a question, ask the rest of the class to comment: "What do the rest of you think about that?" This helps avoid the problem of you becoming the final "authority" for the right answer. Encourage students to go to the board, point out something a group wrote, and ask the groups questions.



⑥ Discuss group functioning (optional, ~ 5 minutes)

An occasional class discussion of group functioning is essential. Students need to *hear* the difficulties other groups are having, *discuss* different ways to solve these difficulties, and receive *feedback* from you (see Chapter 9, pages // and //). Randomly call on one member of from each group to report their group's answer to the following question on the Evaluation form:

- one difficulty they encountered working together, or
- one way they could interact better next time.

After each answer, ask the class for additional suggestions about ways to handle the difficulties. Then add your own feedback from observing your groups (e.g., "I noticed that many groups are coming to an agreement too quickly, without considering all the possibilities. What might you do in your groups to avoid this?")

⑦ Post the problem solution.

Posting the solution on your website is important to students. They need to see good examples of solutions to improve their own problem solving skills. Again, it is important to post them as the last thing you do – just after students have left the classroom. If you post the solution earlier, your students will ignore the class session.

Chapter 13
Coaching Students During Group Work

In this chapter:
- ✓ How to monitor groups and diagnose their difficulties.
- ✓ How to intervene and coach dysfunctional groups.
- ✓ How to intervene and coach groups having difficulty applying physics concepts and principles to solve a problem.

There are two important instructor actions involved in efficient and timely coaching of groups while they are working to solve a problem:

- ♦ monitoring all groups and diagnosing their difficulties; and
- ♦ intervening and coaching the groups that need the most help.

Coaching groups that are solving problems is similar to triage in a medical emergency room. When there are more patients than available doctors, doctors first *diagnose* what is wrong with each patient to decide which patients need immediate care and which can wait a short time. The doctors then treat the patient with the most need first, then the second patient, and so on. Similarly, with CPS the instructor needs to first diagnose the "state of health" of each group by observing and listening to each group (without interacting with the groups). With CPS, you diagnose:

- ♦ what physics concepts and problem-solving procedures each group does and does not understand; and
- ♦ what difficulties group members are having working together cooperatively.

As with medical triage, your next step is to intervene with the group that is having the most difficulty with the physics or with group functioning.

This chapter contains recommendations of how to monitor and coach groups.

Monitor and Diagnose

The following steps are helpful to monitor and diagnose the progress of *all* groups:

`Step 1.` Establish a circulation pattern around the room. Stop and observe each group to see how easily they are solving the problem and how well they are working together. Don't spend a long time observing any one group. Keep well back from students' line of sight so they don't focus on you.

`Step 2.` Make mental notes about each group's difficulty, if any, with group functioning or with applying physics principles to the problem solution, so you know which group to return to first.

`Step 3.` If several groups are having the **same** difficulty, you may want to stop the whole class and clarify the task or make short, additional comments that will help the students get back on track. For example, there is a tendency for students to immediately try to plug numbers into equations each time new physics principles are introduced. If about half of your groups are doing this, stop the whole class. Remind your students that the first step in problem solving is a thorough analysis of the problem *before* the generation of mathematical equations.

> If you begin intervening too soon (without first diagnosing all groups), it is not fair to the last groups. By the time you recognize that all groups may have the same difficulty, the last groups will have wasted considerable time.

Intervene and Coach

From your observations, decide if any group is struggling and needs attention urgently. Return to that group and watch for a few minutes to diagnose the exact nature of the problem, then ***join the group at eye level.*** You could kneel down or sit on a chair, but try not to loom over the students.

If you spend more than a few minutes with this group, circulate around the room again, noting which other groups need your help. Keep repeating the cycle of a) circulate and diagnose, (b) intervene with the group that needs the most help.

In *well functioning groups*, members share the roles of manager, checker, explainer, and skeptic, and role assumption usually fluctuates over the time students are solving a problem. Students in these groups do not need to be reminded to "stick to their roles." For dysfunctional groups, however, assigned group roles are an important part of intervention strategies. For example, one way to intervene with a dysfunctional group (e.g., a dominant student, one person working alone) is to ask: "Who is the manager (or skeptic/summarizer, or recorder/checker, depending on the dysfunctional group behavior)? What should you be doing to help resolve this problem?" If the student does not have any suggestions, then model several possibilities.

Coaching Dysfunctional Groups

First Few CPS Sessions.

In the first CPS sessions, students with no prior experience with cooperative learning often do not understand their role in a group co-construction of a problem solution. Students do not organize their own thoughts about a problem before launching into an algorithmic process. Naturally they are more comfortable comparing answers or equations than sharing their thought process about the physics. Without coaching, there is a tendency to solve problems individually, and compare progress to determine the "best" solution. The three examples below illustrate some common difficulties and possible interventions for the first CPS sessions.

Example 1: Individual Problem Solving.

You observe a group in which the members are not talking to each other, but solving the group problem individually.

Say something like: "I notice that you are solving the problem individually, not as a group. Who is the Recorder/Checker? You should be the only person writing the solution. Manager and Skeptic/Summarizer, put your pencils away and work with the Recorder/Checker to solve the problem."

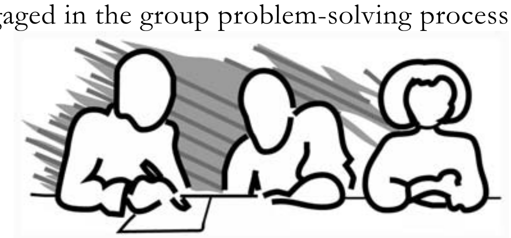
If necessary, make the students rearrange their chairs so they can all see what the Recorder/Checker is writing. If the students persist in solving the problem individually and only then return to the group to compare answers, explain again that they should be solving the problem together. Take the pencils from the Manager and Skeptic (return them at the end of class), and have the group read the Group Role sheet again. Do not leave until they have started solving the problem together.

Example 2: A Lone Problem Solver. You observe a group in which two members, including the Recorder/Checker, are working together, but one member, the "loner", is working alone to solve the problem. First, try to determine why the loner is solving the problem alone. Say something like: "I notice that while two of you are working together, you (loner) appear to be solving the problem by yourself. What are each of your group roles? Why are you (loner), as the group Manager (or Skeptic/Summarizer) solving the problem by yourself?"

Frequently, the loner will sheepishly mumble something about not being used to working in a group. This individual may need only a gentle reminder to give group work a try. Ask the Recorder/Checker to explain to the loner what they have done so far to solve the problem. If necessary, make the students rearrange their chairs so they can all see what the Recorder/Checker is writing.

Occasionally, a loner is more adamant about needing to solve the problem alone before talking with the group. Maintain a sympathetic attitude, but explain to the loner that the research shows that all students learn much more about physics and problem-solving procedures when they construct problem solutions together, which is why you are having them do it. Although it may seem difficult at first, insist that the loner try it. Tell the individual to put their pencil away and ask the Recorder/Checker to explain to the loner what they have done so far to solve the problem.

Example 3: A Non-participant. You observe a group in which one member does not appear to be engaged in the group problem-solving process.



Try to determine why the student appears to be disengaged. For example, if the students are sitting in a row and not facing each other, have the students to get up and rearrange the chairs so they sit facing each other. Ask the student to explain what the group is doing and why. [This emphasizes the fact that **all** group members need to be able to explain each step in solving a problem.] If the student can describe what the group is doing and why, then they may be a quiet student who pays attention, but does not speak as often as the others. You do not need to intervene further.

If the student does not have a clear idea of what the other group members are doing, they may be what is called a "free-rider" -- a person who leaves it to others to solve the problem. Ask the free rider: What is your group role? What should you be doing to help your group solve this problem?" [If necessary, have the free rider read the role description from the Group Role sheet.] If the free-rider is not the Manager, ask the Manager what could be done to make sure everyone, including the free-rider, participates in solving the problem.

Later CPS Sessions.

With appropriate structure (see Chapters 7 through 9) and coaching, most students learn to function in groups relatively well. Occasionally, however, a group may exhibit one of the following dysfunctional behaviors[1]:

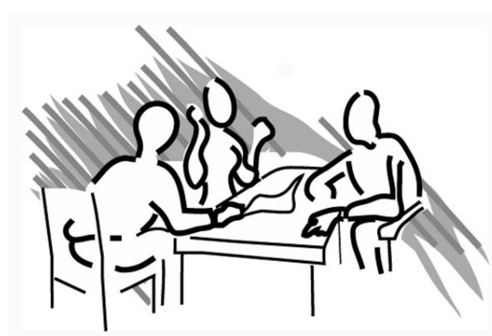
- Lower-achievement members sometimes "leave it to John" to solve the group problem, creating a free-rider effect. At the same time, higher achieving group members may expend decreasing amounts of effort because they feel exploited by the others, the sucker effect. This sucker effect is unusual when group problems are graded occasionally.
- Higher-performance group members may be deferred to and take over leadership roles in ways that benefit them at the expense of the other group members (the dominant student or the rich-get-richer effect).
- Groups with no natural leaders may avoid conflict by "voting" or not making any decision rather than discussing an issue (conflict avoidance effect).
- Group members argue vehemently for their point of view and are unable to listen to each other or come to a group consensus (destructive conflict effect).

The last section included an example of how to intervene in a group with a "free-rider." The two examples below suggest how to coach groups with a dominant students or a conflict.

Example 1: Dominant Student. You observe a group in which one member is doing almost all of the talking, while the other members appear somewhat disengaged and lethargic.

In this case, all members are failing in their roles. First tell the group: “I notice that one person appears to be doing all the talking in this group.“ Then ask: Manager, what could you be doing to make sure that all members of your group contribute their ideas?” If the manager has no ideas, then either have the group read their *Group Role* sheet (early in course) or make a suggestion, such as: “For each step in your problem solving process, ask each member of your group what they think.” Point to a specific part of the group’s solution and model some specific questions the Manager could ask.

Repeat this procedure with each group member. Ask: “Checker/Recorder, what could you be doing to make sure that all members understand and can explain everything that is written down?” [Periodically ask each member if they understand and agree with everything written down. Point to part of the group’s solution and model some specific questions.] Ask: “Skeptic, what could you be doing to make sure that alternative ideas are being considered by the group?” [Be sure to ask for a justification for an idea, and suggest alternative ideas. Point to specific parts of the group’s solution and model specific questions the skeptic could ask.]



Example 2: Conflict Avoidance or Destructive Conflict. You observe a group that cannot seem to reach a decision, but does not appear to have any strategy that leads to convergence (conflict avoidance) or a group that is arguing loudly, but does not appear to be resolving their conflict (destructive conflict). Ask the group: "Who is the Skeptic/Summarizer (or Summarizer in a four-member group)? I noticed that you are having difficulty deciding . . . . . Summarizer, what could you be doing to help the group come to a decision that is agreeable to all of you? If the Summarizer has no idea, then either have the group read the *Group Role* sheet again (early in course) or give some suggestions, such as: “Stop and summarize your different ideas. Then discuss the merits of each idea. For example, you could . . . ." The specific suggestions you give will depend on the exact nature of the decision.

Coaching Groups with Physics Difficulties

As the number of dysfunctional groups decreases, you will spend more of your time coaching groups that are having difficulty applying physics concepts and principles to solve the problem. The general approach to coaching is to give a group just enough help to get them back on track, then leave. That is, spend as little time as possible with a group, then go to the next group that needs help, and so on. Below are some general guidelines for coaching groups with physics difficulties.

Step 1. Before you intervene, listen to the discussion in a group for a few minutes and look at what the checker/recorder is drawing and writing. Diagnose the group's specific difficulty. The checklist for grading feedback (Chapter 9, page //) can be useful for this purpose.

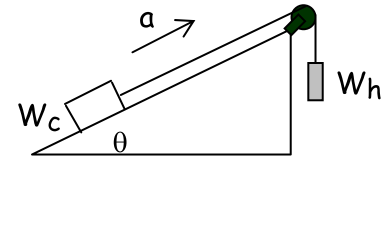
Step 2. Based on the nature of the group's difficulty, decide *how* to begin your coaching of the group. There are two general coaching approaches, depending on whether you can point to the difficulty on the group's answer sheet.

- **Use Group Roles.** Point to something on the answer sheet and state the general nature of the difficulty or error. Then ask: "Then ask: "Who is the manager (or skeptic/summarizer, or recorder/checker)? What could you be doing to help resolve this difficulty?" If the student/group does not have any suggestions, then model several possibilities.
- **General Questions.** If you can not point to something specific written on the group's answer sheet, begin by asking the group some general questions to find out what they are thinking, such as: (a) What are you doing? (b) Why are you doing it? and (c) How will that help you?

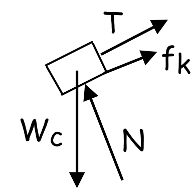
Step 3. Based on the answers you get to your initial question(s), ask additional questions until you get the group thinking about *how* to correct their difficulty. That is, try to give a group just enough help to get them back on track, then leave. Check back with the group later to see if your coaching was sufficient for the group to discuss the difficulty and get back on track.

Examples of Using Group Roles in Coaching

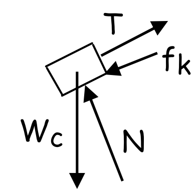
Suppose your students are solving a modified Atwood machine problem, as shown in the diagram at right. As part of the solution, students must find the tension of the rope. Below are some examples of a coaching technique that uses group roles.



Example 1: Misunderstanding of Physics Concept. You observe that a group has drawn the frictional force in the wrong direction on their diagram. Point to the diagram: "There is something wrong with one of the forces in this diagram. Skeptic, what questions could you ask about each of these forces?" When the group has responded (i.e., Does each interaction result in a push or a pull on the carton? In what direction?), then leave the group.



Example 2: Improper Construction of a Specific Equation. You observe that a group has drawn a correct force diagram, but there is an incorrect sign for the frictional force in their 2nd Law component equation:



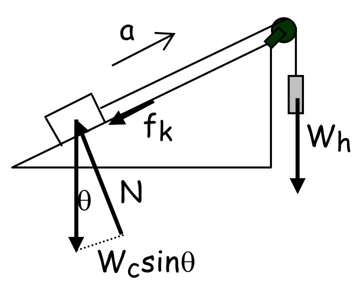
$$\sum F_x = ma_x$$

$$T - W_c \sin\theta + \mu W_c \cos\theta = \frac{W_c}{g} a$$

Point to the force diagram *and* the equation: "I think you made a mistake in translating from your diagram to this equation. Skeptic, what questions could you ask about each translation?" When the group has responded, then leave the group.

Example 3: Diagram Missing. You observe that a group has not drawn a separate force diagram. Their 2nd Law equation is correct except for the wrong sign for the frictional force. Point to the equation: "I think there is a mistake in this equation. Manager, what is an important part of analyzing a problem that could prevent a mistake in this equation?" When the group has responded with "a force diagram," leave the group. It they don't, you should suggest they draw one.

Example 4: Major Misconception. You observe a group that has not drawn separate force diagrams for the carton and the hanging weight. Instead, they sketched some forces on the picture, as shown at right. In addition, they did not start their equations with Newton's Second Law in its general form, $\Sigma F_x = ma_x$. Instead, the first equation is:



$$\begin{aligned} T &= W_h - f_k - W_c \sin\theta \\ &= W_h - \mu W_c \cos\theta - W_c \sin\theta \end{aligned}$$

Equations of this type often indicate a misconception about Newton's 2nd Law. We have found that about 20% of students in the calculus-based course solve Newton's Law problems by setting the unknown force (tension in this problem) equal to the sum of the known forces, in this case all the other forces acting on the carton and the hanging weight (see Chapter 14, pages //-//). [In addition, about 20% of students in a traditional class solve Newton's Law problems by setting the unknown force (e.g., tension) equal to "ma," or by setting the sum of the forces equal to zero even when there is an acceleration.]

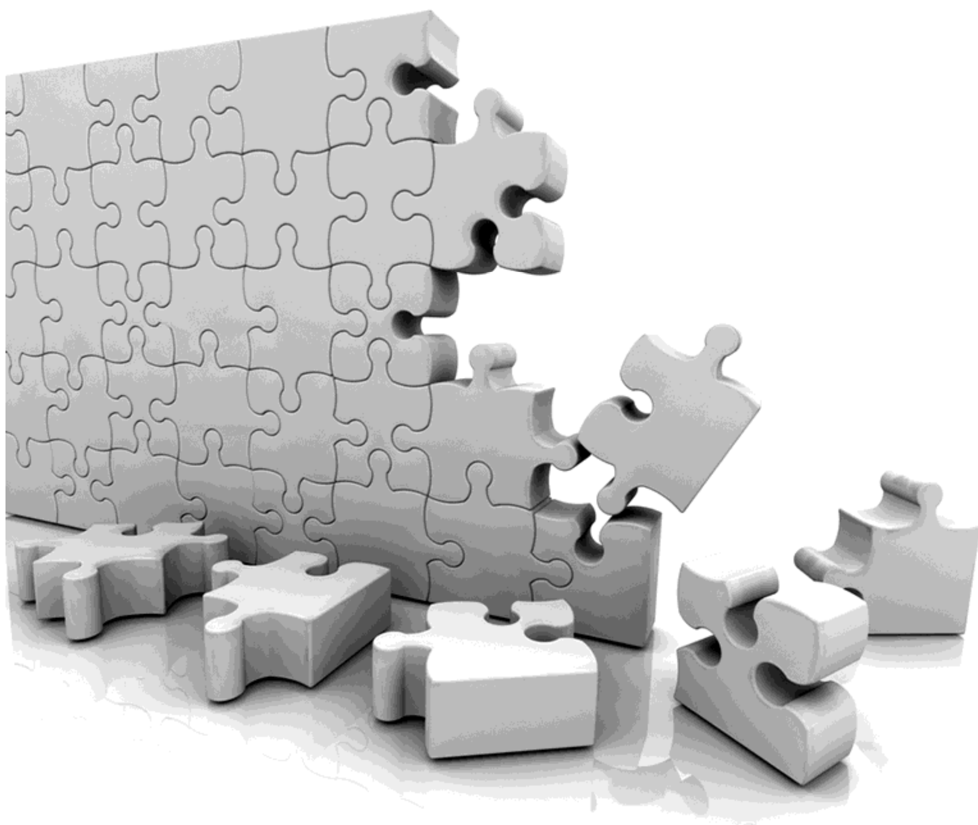
Point to the equation: "I don't understand this equation. Checker/recorder, could you describe how your group arrived at this equation?" Specific follow-up questions will depend on the response of the group. If you have Newton's Second Law ($\Sigma F_x = ma_x$) on the Problem & Information sheet, then you could point to this equation and ask the group what this equation means. Finally, you may need to coach the group through drawing free-body force diagrams for each object (carton and hanging weight).

General-Questions Coaching Technique

Sometimes, it is impossible to identify a specific error even though the solution path is obviously incorrect. By the time you get to a group, they may have several interrelated difficulties. Your intervention with this group will take longer. You can start coaching by asking the group: (a) What are you doing? (b) Why are you doing it? and (c) How will that help you? This often provides you with enough information to diagnose the problems and deal with them one at a time. Always try to ask questions, rather than give answers.

Part 4

Personalize a Problem-solving Framework and Problems



In this part . . .

This part of the book presents some advanced techniques in Cooperative Problem Solving (CPS). It assumes that minimally you have read or know the content of Chapters 2, 3, 4, 5, 7, and 11, and have tried some cooperative problem solving with your classes, using context-rich problems form Appendices B and C or from our on-line archive of context-rich problems.

Chapter 14 describes how to personalize a problem-solving framework to match your students and your goals and approach to your introductory course. It provides examples of different research-based, problem-solving frameworks that you can modify to fit your own needs.

Chapter 15 describes a procedure for constructing your own context-rich problems from textbook exercises or end-of-chapter problems. A review of the features of context-rich problems is followed by an explanation of the procedure, and examples are provided for your practice in writing your own context-rich problems.

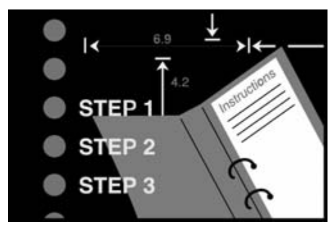
Chapter 16 provides guidance on how to judge the suitability of a context-rich problem for use by individuals or groups in either a practice or exam situation. Twenty-one traits are described that make a more difficult to solve, and a checklist is provided for determining the difficulty level of a problem and it's suitability for an intended use.

Chapter 14
Personalizing a Problem-solving Framework

In this chapter:

- ✓ Five steps for personalizing a problem-solving framework to fit your students and your approach to your introductory course.
- ✓ Examples of research-based, problem-solving frameworks.
- ✓ Examples of problem-solving frameworks in textbooks.

In Chapters 3 and 4 we described the problem-solving framework that we use in our introductory physics courses. A problem-solving framework is a logical and organized *guide* to help students arrive at a solution to real problems. It gets students started, guides them to what to consider, organizes their mathematics, and helps them determine if their answer is reasonable.



The problem-solving framework presented in Chapter 4 and expanded in Chapter 5 is one several that have been used for introductory physics. All research-based frameworks are specific adaptations of the general framework used by experts in *all* fields (see Chapter 4, page //). Although these frameworks are very similar, they were developed and tested for different populations of students. They divide important problem-solving actions into a different number of steps and sub-steps, describe the same actions in different ways and emphasize different heuristics depending on the backgrounds and needs of population of students for whom they were developed.

The details of the framework you decide to teach should be tailored to the needs and backgrounds of *your* students and to *your* approach to the course you are teaching. In this chapter we describe guidelines to help you personalize a framework for your students. There are five major steps in this procedure, outlined below.

A problem-solving *framework* is only a guide for students -- actions for students to consider as they solve problems. It does not present a set of linear steps to be followed. Problem solving usually involves looping back and forth both within steps and between steps.

Figure 14.1. Checklist of Symptoms of Students' Difficulties Solving Problems

- ❑ **1. Difficulty visualizing a physical situation**. Symptoms include physically impossible results. No pictures or diagrams are drawn.
- ❑ **2. Disconnect between physics and reality**. Symptoms include physically impossible results; excessive emphasis on the exact meaning of words (the "lawyer" approach to a problem); difficulty in applying knowledge to slightly different situations; and difficulty applying knowledge consistently within a single situation or across similar situations.
- ❑ **3. No recognition of common physics concepts**. Major symptom is difficulty in applying knowledge to slightly different situations.
- ❑ **4. Difficulty applying of fundamental concepts consistently**. Symptoms include difficulty in applying knowledge to slightly different situations; and incorrect alternative conceptions remain.
- ❑ **5. Lack of a coherent conceptual framework**. Symptom is that many misconceptions remain. Students have difficulty applying their knowledge to slightly different situations and difficulty applying knowledge consistently, even within a single situation.
- ❑ **6. Lack of a logical analysis**. Symptoms include random equations and/or the inability to begin a solution.
- ❑ **7. Over-reliance on pattern matching**. Symptom is students solving the "wrong" problem either through over-simplification or misreading of the problem.
- ❑ **8. Lack of mathematical rigor**. Symptoms include frequent "algebraic mistakes" and mathematical "magic" in solutions (e.g., $m_1 = m_2 = 1$).

Step ①. Clarify the situation. Analyze your students' solutions from homework and/or exams to determine your students' problem solving difficulties.

Step ②. Gather Additional Information. Determine how your students' difficulties have been addressed in some existing research-based, problem-solving frameworks.

Step ③. Construct Your Own Framework. Build a problem-solving framework for your students by adapting research-based frameworks.

Step ④. Reconcile Your Framework with Your Textbook Strategy. Figure out how you can use your framework in conjunction with the problem-solving strategies in your textbook.

Step ⑤. Evaluate Your Framework. Use your problem-solving framework with a class. Based on the results, modify your framework as necessary by repeating the above steps.

Each section of this chapter describes the specific actions in each of these steps, and provides some examples.

Step ①. Clarify the Situation

The first step in personalizing a problem-solving framework is to analyze your students' written solutions to practice and exam problems to determine the problem-solving difficulties of your students. Your experience in working with students during your office hours or other discussions will also help. The *Checklist of Symptoms of Student Difficulties* in Figure 14.1 and the table in Chapter 3, page //, might be helpful. Note that a symptom may have more than one cause.

It is tempting, at first glance, to ascribe many student difficulties to mathematical weaknesses because students' solutions tend to be mostly mathematics. Considerable research indicates, however, that mathematics is only a minor element of students' difficulties solving real problems.[i] For example, Figures 14.2 – 14.4 give the results of our analysis of student solutions to three final examination problems for a calculus-based physics course (two problems from the first semester and one from the second semester). These results were used to develop our problem-solving framework for that course.

The problems were selected because they required students to apply their knowledge to slightly different situations than they had encountered in their course and textbook. They were also standard problems for a calculus-based course. We began by analyzing the students' solutions to the *Modified Atwood-machine Problem* (Figure 14.2). At first, the major student errors seemed to in the mathematics. However, we adopted the following procedure:

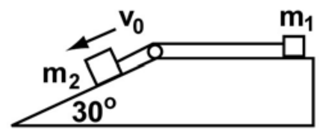
- Look first at the student's diagrams (if any).
- Then look at the first equation written down (or the first equation in each sub-part). Does the equation match the diagram? Is the equation an application of a fundamental concept or principle, or something else?

Based on this procedure, we classified over 250 student solutions to each problem for *major* errors (i.e., the error that prevented students from arriving at a correct solution).[ii] Approximately 40% of the students solved the three final examination problems correctly (or with minor errors). As expected, only about 10% of our students had major difficulties that could be directly ascribed to mathematics, as shown in Figure 14.2 and Figure 14.4.

In contrast, approximately 50% of our students failed to solve each problem because of incorrect approaches to the problems. These students either drew no pictures or diagrams or drew incomplete diagrams. This is a symptom of the problem-solving difficulty of *visualizing a physical situation* (Difficulty 1 in Figure 14.1). Approximately one-half of our students could not apply their knowledge consistently (primarily Difficulties 3 and 4) . They did not appear to have integrated their knowledge into a coherent conceptual framework (Difficulty 5 in Figure 14.1).

Figure 14.2. Major errors in students' solutions to Modified Atwood-machine Problem

Modified Atwood-machine Problem. In the diagram shown at right, block 1 of mass 1.5 kg and block 2 of mass 4 kg are connected by a light taut rope that passes over a frictionless pulley. Block 2 is just over the edge of the ramp inclined at an angel of 30°, and the blocks have a coefficient of sliding friction of 0.21 with the surface. At time t = 0, the system is given an initial speed of 11 m/s that starts block 2 down the ramp. Find the tension in the rope.



Major Type of Error in Students' Solutions	% (N=272)
1. Correct or minor errors Total:	**29**
2. Careless, many omissions, no sense of order	**9**
3. Incorrect Physics Approaches	**52**
a. **$F_{unknown}$ = ma.** The unknown force, in this case the tension in the rope, is mass times acceleration (ma). Usually no force diagram (or very minimal diagram) is drawn. These students appear to have no idea what the "tension" force is. $T = F = ma$, where the acceleration could be g, $g\sin\theta$, or v_o	6
b. **$F_{unknown} = \Sigma F_{known}$.** The unknown force (tension) is the sum of all the known forces acting on the two blocks. Usually only a minimal force diagram is drawn, often without the tension force. $T = F_1 + F_2 - F_3 \ldots$	22
c. **Tension = Friction.** The unknown force (tension) is the frictional force on m_1 or the sum of the frictional forces on m_1 and m_2. Usually minimal force diagrams are drawn, often without the tension force shown. (They may be setting $\Sigma F = 0$ in their heads). $T = \mu m_1 g$ or $T = f_1 + f_2 = \mu m_1 g + \mu m_2 g\cos\theta$	11
d. **Incomplete, can't tell.**	13
4. Mathematical Difficulties	**9**
a. Can't solve simultaneous equations	6
b. Trigonometry or algebra errors	3

Figure 14.3. Incorrect physics approaches in students' solutions to the Enterprise Problem

Enterprise Problem. The "Enterprise, a ride at the Valley Fair Amusement Park, consists of a vertical wheel if radius 9 meters rotating about a fixed horizontal axis with seats for the occupants around its outer edge. The wheel rotates so that the occupants are moving at 11 m/s. The seats pivot so the occupants' heads are towards the center of the wheel. When a 56-kg woman is upside down at the top of the wheel, what is the force she exerts on the seat?

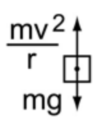
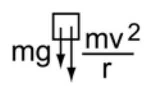
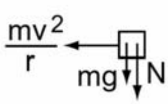
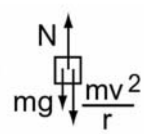
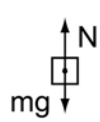
	% (N=289)
Incorrect Physics Approaches in Students' Solutions	**46**
a. **$F_{unknown}$ = ma.** The unknown force (in this case the force of the woman on the seat, usually written as "F") is mass times acceleration (ma). Usually no force diagram is drawn. $\Sigma F = ma = \frac{mv^2}{r}$	9
b. **$F_{unknown} = \Sigma F_{known}$.** The unknown force (force of the woman on the seat) is the sum of all the known forces in the problem. The force of the seat on the woman is not drawn (or given a symbol). F_C or mv^2/r is drawn as a force vector.  $F = \frac{mv^2}{r} - mg$ or  $F = \frac{mv^2}{r} + mg$	18
c. **$\Sigma F = 0$.** The sum of the forces acting on the object (woman) is zero. In this case, the unknown force (force of the seat on the woman) is shown in the force diagram. F_C or mv^2/r is drawn as a force vector.  $\Sigma F = \frac{mv^2}{r} - N - mg$, $N = \frac{mv^2}{r} - mg$ or  $\Sigma F = N - \frac{mv^2}{r} - mg$, $N = \frac{mv^2}{r} + mg$	11
d. **N in wrong direction.** The normal force is drawn in wrong direction, but the application of Newton's second law is correct.  $\Sigma F = ma$, $N - mg = \frac{mv^2}{r}$	8

Figure 14.4. Major errors in students' solutions to the Wave Problem

Wave Problem. A violin String 55 cm in length and placed near a loudspeaker is observed to respond strongly when the speaker is driven at a frequency of 1320 Hz exhibiting two nodes between endpoints. What is the tension in the spring if it has a mass of 0.5 grams?

Major Type of Error in Students' Solutions	% (N=217)
1. Correct ($T = v^2\mu$, $v = \lambda f$)	**44**
a. No mistakes	35
b. Math Mistake (units, calculator, forgot to square v)	9
2. Incorrect Approaches	**49**
Incorrect Approach for Determining String Density ($\mu = m$ or $\mu = mg$)	4
Incorrect Approach for Determining Wave Velocity (v). (No useful diagram drawn for determining λ)	45
a. Used $v = 2Lf_n/n$, with n = # of nodes.	13
b. Used formula for constructive interference $\Delta L = n\lambda$	12
c. Used velocity of sound	15
d. Used other relationships (pendulum, harmonic oscillator)	1
e. No idea how to calculate v, so quit.	4
3. Incorrect Approach for Determining T	**5**
4. Nothing or little written	**3**

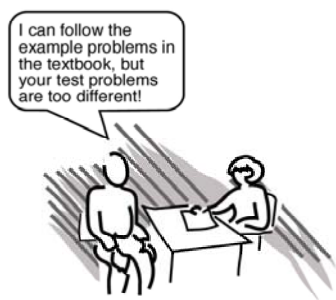
Student solutions to both mechanics problems (the *Modified Atwood-machine Problem* and *Enterprise Problem*) indicated similar incorrect approaches:

- $F_{unknown} = ma$. The unknown force is always the mass times the acceleration (ma).
- $F_{unknown} = \Sigma F_{known}$. The unknown force is always the sum of all the known forces in the problem.
- $\Sigma F = 0$. The sum of the forces acting on an object is always zero.

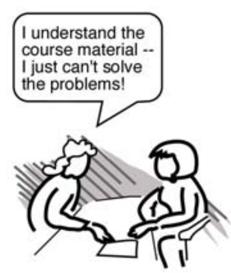
These three approaches could indicate students may have misconceptions about acceleration, the nature of forces, or about the meaning of Newton's second law (Difficulties 4 and 5 in Figure 14.1). This view is supported by our results for written conceptual questions (see Chapter 10, pages //-//). At the end of the quarter on mechanics, 57% of students still confused acceleration and velocity, 39% did not understand the nature of forces (e.g., included the force acceleration or the force of momentum), and 75% did not understand that acceleration is caused by the ***sum*** of the forces acting on a object.

The three incorrect approaches could also be due to the pattern-matching strategy for solving quantitative problems – memorizing the series of equations needed to solve different types of problems (Difficulties 5 and 7). That is, students use a different network of knowledge to answer conceptual questions than to answer quantitative questions. This view is supported in a study by Mel Sabello and Edward Redish.[iii] When administering a problem with several parts, they found that 17% of students set the sum of the forces equal to zero when solving the quantitative part of a problem, despite the fact that they drew a non-zero acceleration vector in an earlier qualitative part of the problem. This indicates a clear disconnect between their qualitative knowledge and their quantitative knowledge. The reality is probably somewhere between these two extremes, with students having misconceptions *and* exhibiting an over reliance on pattern matching.

Additional analysis of similar problems and open-ended conceptual problems indicated that the primary difference between the students in the algebra-based and calculus-based courses was their mathematical dexterity. About 90% of our calculus-based students had sufficient mathematical dexterity that, together with their over-reliance on pattern matching, they could arrive at a numerical answer. In our algebra-based course, students made more mathematical mistakes, some students would give up half-way through the algebra, and more students used the novice plug-and-chug strategy – randomly plugging numbers into memorized formulas until they arrive at a numerical answer (Chapter 2, pages // - //).



Pattern-matching Novice Strategy



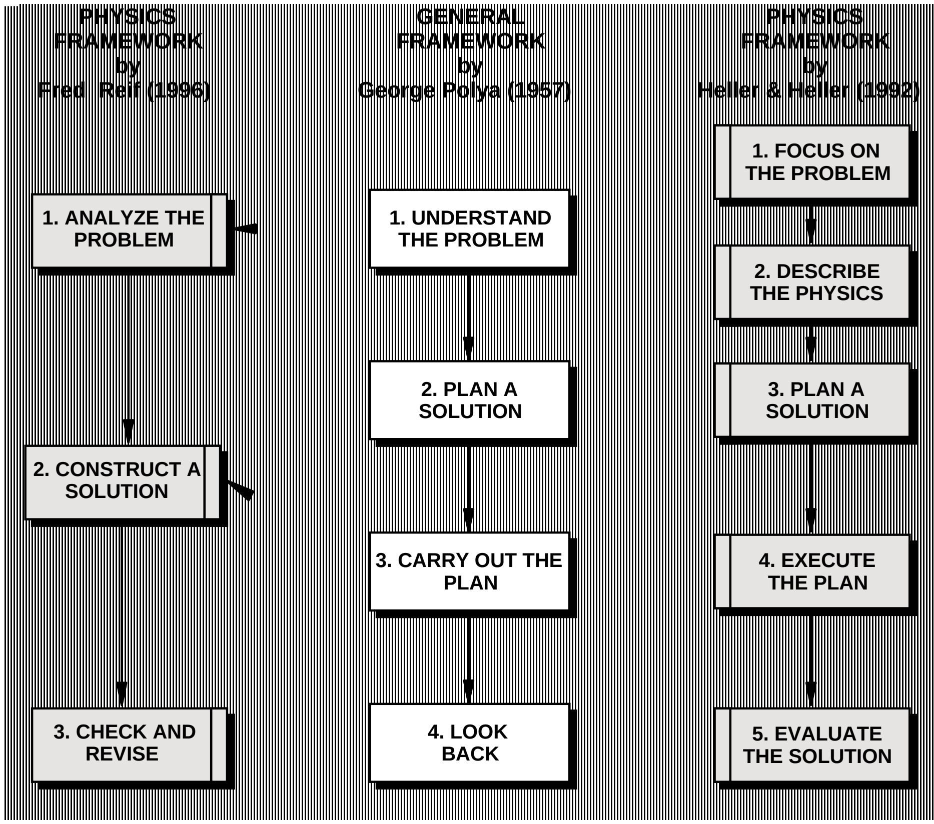
Plug-and-chug Novice Strategy

Our initial problem-solving framework was based on the observation that the majority of our students in both the algebra-based or the calculus-based physics course exhibited the problem-solving difficulties in Figure 14.1 on the final examination. Subsequent analysis of students' problem solutions led to the refinement and elaboration of slightly different frameworks for each course, as discussed in Chapter 5.

Step ②. Gather Additional Information

Once you have determined your students' problem-solving difficulties, the next step is to determine these difficulties have been addressed in some existing research-based, problem-solving frameworks. There are several research-based frameworks

Figure 14.5. Three examples of research-based problem-solving frameworks.



for physics that have been developed and successfully used,[iv,v,vi] all based on the general framework developed by George Polya,[vii] Two examples are outlined in Figure 14.5. Each framework divides the important actions into a different number of steps and sub-steps, describes the same actions in different ways, and emphasizes different heuristics depending on the backgrounds and needs of population of students for whom they were developed.

For each framework consider:

- the specific actions in each step;
- how these actions are carried out in solving a physics problem;
- the specific problem-solving difficulties each step and/or action is designed to help students overcome;
- the match with the problem-solving difficulties your students exhibit.

Step ③. Construct Your Problem-solving Framework

Begin with a Research-based Framework. Choose the research-based, problem-solving framework that best addresses the major difficulties of the majority of *your* students.

Figure 14.6. Outline of the Competent Problem-solving Framework: Calculus Version

1. **Focus the Problem.** Establish a clear mental image of the problem.
   - A. Visualize the situation and events by sketching a useful picture.
     - Show how the objects are related spatially, Show the time sequence of events, especially the initial and final states of the objects. Add times to the drawing when an object experiences an abrupt change in interaction.
     - Write down the known information, giving each quantity a symbolic name and adding that information to the picture.
   - B. Precisely state the question to be answered in terms you can calculate.
   - C. Identify physics approach(es) that might be useful to reach a solution.
     - Which fundamental principle(s) of physics (e.g., kinematics, Newton's Laws, conservation of energy) might be useful this problem situation.
     - List any approximations (e.g., assume kinetic friction is negligible) or problem constraints (e.g., constant acceleration, $T_1 = T_2$, uniform electric field) that are reasonable in this situation.
2. **Describe the Physics**
   - A. Draw any necessary diagrams (e.g., motion diagram, force diagram, momentum diagram, energy table) with coordinate systems that are consistent with your approach(es).
     - Define consistent and unique symbols for any quantities that are relevant to the situation.
     - Identify which of these quantities is known and which is unknown.
   - B. Identify the target quantity(s) that will provide the answer to the question.
   - C. Assemble the appropriate equations that mathematically give the physics principles, approximations, and problem constraints identified in your approach.
3. **Plan a Solution**
   - A. Construct a logical chain of equations from those identified in the previous step, leading from the target quantity to quantities that are known.
     - Choose an equation that contains the target quantity and write it down. Identify other unknowns in that equation.
     - Choose a new equation for one of these unknowns. Write down this equation and note the unknown quantity this equation was chosen to determine.
     - Continue this process for each unknown.
   - B. Determine if this chain of equations is sufficient to solve for the target quantity by comparing the number of unknown quantities to the number of equations.
4. **Execute the Plan**
   - A. Follow the outline from in the previous step.
     - Arrive at an *algebraic* equation for your target quantity by following your chain of equations in reverse from the order you constructed your plan.
     - Check the units of your algebraic equation before putting in numbers.
     - Use numerical values to calculate the target quantity.
5. **Evaluate the Answer**
   - A. Does the mathematical result answer the question asked with appropriate units?
   - B. Is the result unreasonable?

Figure 14.7. The problem-solving framework by Fred Reif for a calculus-based introductory course in mechanics.

1. **Analyze the Problem:** Bring the problem into a form facilitating its subsequent solution.
   A. Basic Description—clearly specify the problem by
      - describing the *situation*, summarizing by drawing diagram(s) accompanied by some words, and by introducing useful symbols; and
      - specifying compactly the *goal(s)* of the problem (wanted unknowns, symbolically or numerically)

   B. Refined Description—analyze the problem further by
      - specifying the *time-sequence* of events (e.g., by visualizing the motion of objects as they might be observed in successive movie frames, and identifying the *time intervals* where the description of the situation is distinctly different (e.g., where acceleration of object is different); and
      - describing the situation in terms of important *physics concepts* (e.g., by specifying information about velocity, acceleration, forces, etc.).

2. **Construct a Solution:** Solve simpler sub-problems repeatedly until the original problem has been solved.
   A. Choose sub-problems by:
      - examining the *status* of the problem at any stage by identifying the available known and unknown information, and the obstacles hindering a solution;
      - identifying available *options* for sub-problems that can help overcome the obstacles;
      - selecting a useful sub-problem among these options.

   B. If the obstacle is lack of useful information, then apply a *basic relation* (from general physics knowledge, such as $\mathbf{F}_{net} = m\mathbf{a}$, $\mathbf{f_k} = \mu\mathbf{N}$, $x = 1/2\ a_x t^2$) to some *object or system* at some *time* (or between some times) along some *direction*.

   C. When an available useful relation contains an unwanted unknown, eliminate the unwanted quantity by combining two (or more) relations containing this quantity.

   Note: Keep track of wanted unknowns (underlined twice) and unwanted unknowns (underlined once).

3. **Check and Revise:** A solution is rarely free of errors and should be regarded as provisional until checked and appropriately revised.
   A. Goals Attained? Has all wanted information been found?
   B. Well-specified? Are answers expressed in terms of known quantities? Are units specified? Are both magnitudes and directions of vectors specified?
   C. Self-consistent? Are units in equations consistent? Are signs (or directions) on both sides of an equation consistent?
   D. Consistent with other known information? Are values sensible (e.g., consistent with known magnitudes)? Are answers consistent with special cases (e.g., with extreme or specially simple cases)? Are answers consistent with known dependence (e.g., with knowledge of how quantities increase or decrease)?
   E. Optimal? Are answers and solution as clear and simple as possible? Is answer a general algebraic expression rather than a mere number?

Language. Next consider how your students would interpret language calling for the specific actions in the framework. Since the framework was developed as a specific implementation of Polya's general framework used by experts in *all* fields, it will probably need to be modified for your particular population of students.

1. Retain the elements of each research-based framework that:
   - seem directed at *your* students' most common problem-solving difficulties; and
   - are consistent with your course goals and approach.

2. Reduce or eliminate steps in the framework that address difficulties not exhibited by your students.
3. Make sure that the framework describes a complete, logical problem-solving procedure from your point of view. Fill in steps if necessary to assure completeness.
4. Choose language to describe the steps and actions of your modified problem-solving framework that is most meaningful to you and your students (see also Step ④).

Compare with Research-Based Frameworks. Make sure that all of the features of a research-based framework are incorporated (see Chapter 4, pages //-//).

Check Your Framework. Check that the framework will be useful in all parts of your course by solving problems using its procedure for different physics topics.

Remember, a problem-solving framework is most useful at the beginning of the course. When students become more familiar with the framework and comfortable using the framework, the different steps begin to merge. For example, towards the end of the first semester, our students in the calculus-based course merge Steps 1 and 2 into one description, and merge Steps 3 and 4 into one procedure. This is an example of our 2nd Law of Instruction: *Don't change course in midstream; structure early then gradually reduce the structure*

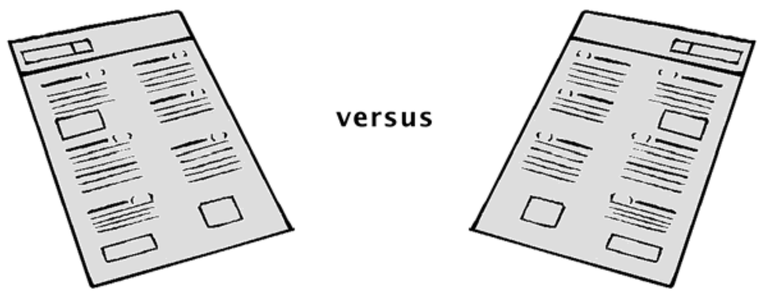
Example 1. Compare Algebra versus Calculus Versions

Figure 14.6 outlines our research-based *Competent Problem-solving Framework* for students in our calculus-based course.[viii] Compare this framework with the outline of our framework for students in our algebra-based course (Chapter 4, page //).[ix] Steps 1, 2, 4, and 5 in these frameworks are identical because we found that students in both our algebra-based and calculus-based courses had similar conceptual difficulties. These difficulties, addressed in steps 1, 2, and 5, were in visualizing a physical situation, connecting physics to reality, recognizing a common physics theme, applying fundamental concepts, and integrating knowledge into a coherent conceptual framework.

In addition almost all students had difficulty solving problems in a logical, organized progression. These difficulties were addressed in steps 3 and 4 of the framework. The actions in step 3 are slightly different because the calculus-based students have more dexterity in mathematics than the algebra-based students. Most of the calculus-based students (but not all) could follow their plan without needing to write an outline of their actions. The algebra-based students, on the other hand, needed this organizational tool to keep from becoming consumed by superfluous mathematical manipulation.

Example 2. Compare Two Calculus-based Versions

Figure 14.7 contains an outline of another research-based framework developed by Fred Reif and his group at Carnegie Mellon University[3] for use in the first semester (mechanics) calculus-based course. In contrast, our *Competent* framework (Figure 14.6) is generalized for use in both semesters of an introductory course. Reif's framework breaks the generalized procedure into three major steps, whereas our *Competent* framework has five major steps (see Figure 14.5).



Fred Reif's Framework   Our Competent Framework

One major difference between the two frameworks is in the heuristic emphasized to plan and construct a solution. A heuristic is a rule of thumb – a procedure that is both powerful and general, but not absolutely guaranteed to work. (see Chapter 4, pages // - //). Reif's framework emphasizes the heuristic of breaking a problem into sub-problems that you can solve (Step 2). Our *Competent* framework emphasizes the heuristic of working backwards[x] from the goal to the solution (Step 3). Working backwards is a powerful heuristic for students who have do not know where to start their mathematics solutions when faced with a problem that is slightly different than the example problems and solutions they encounter in class. Apparently the freshman students at Carnegie Mellon University do not use the pattern-matching novice strategy as much as our freshmen.

Another difference in the two frameworks is how they distinguish physics principles that apply in many topics (e.g., kinematics, Newton's second Law, conservation of energy) versus relationships that apply only in specific problem situations (e.g., constant force, potential energy when the gravitational interaction is near the Earth's surface). In Reif's framework, this difference is not emphasized, whereas in our *Competent* framework the difference is emphasized. Consequently, there are two sub-steps in the *Competent* framework (illustrated in Steps 1C and 2C) and only one sub-step in Reif's framework (illustrated in Steps 1B and 2B). There are two reasons for this difference.

1. Reif's framework is limited to one semester of mechanics.
2. The prominence of principles is part of our overarching goal of teaching physics through problem solving. Many students enter our courses believing that physics is a collection of disconnected concepts and equations. One of our goals is to help dispel this belief by approaching all topics with the same fundamental physics principles. [Here we go again. Lets see how the conservation of energy applies to electric circuits.]

Despite the differences in the two frameworks, the contents of the frameworks are remarkably similar. The two frameworks were developed independently at about the same time, as was the framework by Alan Van Heuvelen.[1] These problem-solving frameworks are based on the same problem-solving research, consequently they are similar in content, if not in specific heuristics and language.

Step ④. Reconcile Your Framework with Your Textbook Strategy (if necessary)

Physics textbooks often include problem-solving strategies that they use in all example problem solutions. *If your textbook has a problem-solving strategy*, then your next step is to figure out how you can use your problem-solving framework in conjunction with the textbook strategy.

Compare the Textbook Strategy with Your Framework. Examine the problem-solving strategy in your textbook and compare it with your framework. How is strategy similar to your framework? How is it different?

Identify Advantages and Disadvantages of Textbook Strategy. The next step is to identify the features of the textbook strategy that are advantages and those that conflict with your problem-solving goals

Reconcile. Build on the advantages of the textbook strategy and modify your framework to at least make use some of the same language as the textbook strategy.

An Example

Compare the Textbook Strategy with Your Framework. One physics textbook uses a Picture, Solve, and Check problem-solving format.[xi] An examination of this textbook strategy resulted in the following similarities and differences with Fred Reif's research-based framework (Figure 14.7).

1. The authors of this textbook do not provide general guidelines for what students should think about or do within each step. In some chapters the authors

Figure 14.8. Example of a specific problem-solving strategy in a textbook

Applying Newton's Second Law

PICTURE Make sure you identify all of the forces acting on a particle. Then determine the direction of the acceleration vector of the particle, if possible. Knowing the direction of the acceleration vector will help you choose the best coordinate axes for solving the problem.

SOLVE
1. Draw a neat diagram that includes the important features of the problem.
2. Isolate the object (particle) of interest, and identify each force that acts on it.
3. Draw a free-body diagram showing each of these forces.
4. Choose a suitable coordinate system. If the direction of the acceleration vector is known, choose a coordinate axis parallel to that direction. For objects sliding along a surface, choose one coordinate axis parallel to the surface and the other perpendicular to it.
5. Apply Newton's second law, $\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}}$, usually in component form.
6. Solve the resulting equations for the unknowns.

CHECK Make sure your results have the correct units and seem plausible. Substituting extreme values into your symbolic solution is a good way to check your work for errors.

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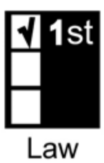
include a problem-solving strategy, but each is limited to a particular topic and/or problem constraint. For example, the strategy shown in Figure 14.8 is limited to contact forces on one solid object that can be considered a particle. Other specific strategies found within three chapters were: Applying Newton's Second Law to Problems with Two or More Objects; Solving Problems Involving Friction; and Solving Problems Involving Work and Kinetic Energy. In contrast, research-based frameworks emphasize deciding the approach to take, starting with fundamental principles, and adding the problem constraints that apply to the problem situation.

2. The three major steps in the textbook strategy (Picture, Solve, Check) are similar to, but not the same as, the major steps in the framework by Fred Reif: Analyze the Problem, Construct a Solution, and Check and Revise.
   - The Picture step in the textbook strategies include hints about what to be sure to remember in the limited problem constraint. It assumes that students already know what concepts and principles to apply (Reif's Step 1: Basic Description and Refined Description.
   - The Solve step in the textbook strategy overlaps Step 1 in Reif's framework. Moreover, the information in the Solve step does not include any heuristics on planning a solution and executing the plan.
   - The Check step in the same as the Check and Revise step in Reif's framework.

Identify Disadvantages of Textbook Strategy. The next step is to identify the features of the textbook strategy that are contrary to your specific problem-solving goals. The major disadvantage is that students cannot generalize the textbook strategies across topics.[xii] In fact, the strategies reinforce students pattern-matching novice strategies.

Reconcile. Build on any advantages with the textbook strategy. For example, you could take your framework and divide into the three major of Picture, Solve, and Check (i.e., use the same names as in the textbook). You could introduce your framework as an expansion or generalization of the specific strategies in the text.

Step ⑤. Evaluate Your Framework



Remember the Zeroth Law of Instruction (*If you don't grade for it, students' won't do it.*) and the 1st Law of Instruction (*Doing something once is not enough.*). Introduce your class to your problem-solving framework and require that your students use it. Model the entire framework whenever you solve problems for the class. Use context-rich problems on tests that require the use of your organized and logical problem-solving framework (see Chapter 3 and Chapter 9, pages //-//). Students also need to be able to practice solving context-rich problems before the tests, both individually and in cooperative groups (see Chapter 9, pages //-//.

Examine samples of students' problem solutions.

1. Focus on solutions in which students follow a logical and correct path up to a point but cannot correctly solve the problem.
2. Determine if there are common places in the framework where a significant number of students either make a jump to some incorrect or illogical solution path or simply stop. These are candidates for additional sub-steps in your problem-solving framework.
3. Examine existing research-based frameworks to see if they address this issue. If so, add that part of the framework to your own. If not, invent a sub-step that you believe would allow the student to continue following their logical path to a correct solution. These sub-steps should not be topic specific. If there is a physics difficulty, address that directly in your class using other techniques.

Endnotes

[i] See, for example, Maloney, D.P. (1994), Research on problem solving in physics. In D.L. Gabel (Ed.*), Handbook of research in science teaching and learning*, (pp. 327-354), NY, Macmillan; Leonard, W.J., Dufresne, R.J., & Mestre, J.P. (1996), Using

qualitative problem-solving strategies to highlight the role of conceptual knowledge in solving problems, *American Journal of Physics*, **64**(12): 1495-1503: and Pretz, J. E., Naples, A. J., & Sternberg, R. J. (2003), Recognizing, defining, and representing problems. In J. E. Davidson & R. J. Sternberg (Eds.), *The psychology of problem solving* (pp. 3-30). Cambridge, UK: Cambridge University Press.

[ii] We randomly selected 100 student solutions from each of three large sections of the calculus-based course, using the proportion of students in each section who received the grades of A, B, C, and D.

[iii] Sabella, M. and Redish, E.F. (2007). Knowledge organization and activation in physics problem polving, *American Journal of Physics*, **75**, 1017-1029.

[iv] Reif, F. (1995). *Understanding basic mechanics*: Text, NY: Wiley.

[v] Van Heuvelen, A. (1991), Overview, case study physics, *American Journal of Physics*, **59**(10), 898-907.

[vi] Leonard, W.J., Dufresne, R.J., Gerace, W.J. & Mestre, J.P. (2002). *Minds•On Physics*, NJ: Kendall Hunt.

[vii] Polya, G. (1957). *How to solve it: A new aspect of mathematical method.* Princeton, NJ: Princeton University Press.

[viii] Heller, K. & Heller, P. (2000). *The competent problem solver for introductory physics: Calculus*, McGraw-Hill Higher Education.

[ix] Heller, K. & Heller, P. (2000). *The competent problem solver for introductory physics: Algebra*, McGraw-Hill Higher Education.

[x] The working backwards heuristic starts with the ultimate goal and then deciding what would constitute a reasonable step just prior to reaching that goal. Then ask yourself what the step would be just prior to that, and so on until you reach the initial conditions of the problem.

[xi] Tipler, P.A., and Mosca, G. (2008). *Physics for scientists and engineers: 6th edition*, NY: WH Freeman.

[xii] Research over the last three decades indicates that students cannot learn complex skills, such as reading comprehension and problem solving by practicing specific sub-skills. It is a case of the complex skill being more than the sum of its sub-skills. This was the motivation behind the cognitive apprenticeship theory of instruction, which emerged after research studies of reading comprehension and problem solving in mathematics. In cognitive apprenticeship an expert models (demonstrates) the entire complex skill so students can form an initial, conceptual idea of the skill. Scaffolding and coaching as students practice the complex skill, sometimes concentrating on a sub-skill, accompany the modeling. Finally, the scaffolding and coaching is gradually faded. See Chapter 6 for additional explanations and references.

Chapter 15
Building Context-rich Problems from Textbook Problems

In this chapter:

- ✓ What are the properties of context-rich problems?
- ✓ How you can build your own context-rich problems based on textbook problems.
- ✓ Practice constructing context-rich problems.

In Chapter 2 we introduced the idea of *real* problem solving -- the process of arriving at a solution when you don't initially know what to do. Real problem solving involves making decisions in analyzing the problem situation, deciding what fundamental principles to apply, and deciding the information is needed to solve the problem. Most novice students tend to memorize rather that engage in logical decision making. They tend to plug numbers into memorized formulas and manipulate the formulas mathematically until they get an answer. Or they may memorize patterns of equations to solve different classes of problems (e.g., free fall problems, inclined plane problems). Traditional end-of-chapter problems do not promote the development of skills necessary to solve real problems.

In Chapter 3 we discussed how context-rich problems help student engage in real problem solving in order to improve their problem solving skills. In Chapter 8 we described the function of context-rich problems in cooperative group learning. This chapter suggests a procedure and provides some practice examples for you to construct context-rich problems that fit the needs of your students.

Review of the Properties of Context-rich Problems

Every context-rich problem has common properties that reinforce the development of students' problem-solving skills. These properties, described in more detail in Chapter 3 (pages // - //), are outlined below.

Short Story. The problem is a short story, in everyday language, in which the major character is the student. That is, each problem statement uses the personal pronoun "you."

Motivation. The problem statement includes a plausible motivation or reason for "you" to calculate something.

Real Objects. The objects in the problems are real (or can be imagined) -- the idealization process occurs explicitly.

No Diagrams. No pictures or diagrams are given with the problems. Students must visualize the situation by using their own experiences and knowledge of physics.

Two or More Steps. The problem solution requires more than one step of logical and mathematical reasoning. There is no single equation that solves the problem.

Fundamental Principles. The problem can be solved by the straightforward application of a fundamental principle of physics. (e.g. Newton's laws, conservation of energy).

In addition the **problem difficulty** can be adjusted to make the problem suitable for individual or group work. We have identified twenty-one traits that make problems more difficult to solve. These traits are explained in Chapter 16. A few common difficulty traits are listed below.

No explicit target variable. The unknown quantity is not explicitly specified in the problem statement (e.g., *Will this plan (design) work?* or *Should you fight this traffic ticket in court?*).

Excess Data. More information is given in the problem statement than is required to solve the problems (e.g., the inclusion of both the static and kinetic coefficients of friction).

Unusual Ignore/Neglect Assumption(s). The problem solution requires student to ignore a small effect (e.g., rotational energy of a pulley), or neglect a small, obvious effect (e.g., mass of an object other than the mass of a string).

Two Fundamental Principles. The problem requires more than one fundamental principle for a solution (e.g., kinematics and the conservation of energy).

Figure 15.1. Important problem features and properties of context-rich problems that support these features.

Problem Features That Promote the Development of Problem-solving Skills	Properties of Context-rich Problems that Support the Features
Feature 1. It is difficult to use novice strategies of plugging numbers into formulas or matching a solution pattern to get an answer.	**All Problems** • The problem is a short story, in everyday language, in which the major character is the student ("you"). • The objects in the problems are real (or can be imagined) -- the idealization process occurs explicitly. • The problem can be solved by the straightforward application of a fundamental principle of physics. **More Difficult Problems** • More information is given than is needed to solve the problem. • Problem solution requires student to neglect a small, obvious effect or ignore a small effect (assumptions).
Feature 2. It is difficult to solve the problem without first analyzing the problem situation.	**All Problems** • The problem is a short story, in everyday language, in which the major character is the student ("you"). • The problem statement includes a plausible motivation or reason for "you" to calculate something. • No pictures or diagrams are given with the problems. **More Difficult Problems** • The unknown quantity is not explicitly specified in the problem statement (e.g., "Will this plan work?") • The problem requires more than one fundamental principle for a solution (e.g., conservation of energy and kinematics).
Feature 3. Physics cues, such as "inclined plane", "starting from rest", or "projectile motion", are avoided in order to help students practice connecting physics knowledge to other things they know.	**All Problems** • The objects in the problems are real (or can be imagined) -- the idealization process occurs explicitly. **More Difficult Problems** • The problem solution requires using the geometry of the physical situation to generate a necessary mathematical expression.
Feature 4. The problem reinforces a logical analysis using fundamental principles.	**All Problems** • The problem solution requires more than one step of logical and mathematical reasoning. There is no single equation that solves the problem. • The problem can be solved by the straightforward application of a fundamental principle of physics. **More Difficult Problems** • The problem requires more than one fundamental principle for a solution (e.g., conservation of energy and kinematics).

Necessary Relationships from Diagram: The problem solution requires using the geometry of the physical situation to generate a necessary mathematical expression.

Figure 15.1 shows how some of the common properties and difficulty traits of context-rich problems that are related to the feature of problems that promote the development of problem solving skills.

The first thing people notice about context-rich problems is the use of the personal pronoun "you" and the inclusion of a plausible motivation or reason for "you" to calculate something. One reason to do this is because it makes the problem situation easier to visualize and more personal.
Using the personal pronoun "you" and providing motivations reinforces the value of physics in students' lives. Most students, especially students who are not physics majors, are not intrinsically motivated to study physics and see no value in being told to calculate an acceleration or electric field -- "Who cares?" We have found four categories of contexts/motivations that reinforce the value of physics in students' lives.

Curiosity

Solving context-rich problems should help students realize that the application of fundamental principles can satisfy their curiosity about objects and situations they did not think they could understand. Some motivations in this category include:

- You are . . . . (in some everyday situation) and need/want to figure out . . . .
- You are watching . . . . (an everyday situation) and wonder . . . .
- You are on vacation and observe/notice . . . . and wonder . . . .
- You are watching TV (or reading an article) about . . . and wonder . . .

Helpfulness

At the same time, context-rich problems can help students realize that physics can be useful both their student lives and the adult life they can envision. Some motivations in this category include:

- Because of your knowledge of physics, your friend asks you to help . . .;
- You are helping to design the opening ceremony for the next winter Olympics. One of the choreographers envisions . . .. You have been assigned the task of determining the minimum speed that the skater must have to . . ..
- Because of your interest in the environment and your knowledge of physics, you are a member of a Citizen's Committee (or Concern Group) investigating . . …
- You are volunteering with an ecology group investigating the feeding habits of eagles. During this research, you observe an eagle . . .. You wonder . . ..

Job Related

Many college students are motivated by the prospect of getting a summer or part time job, and can envision getting a job when they graduate.

- You have a summer job with a company that . . .. Because of your knowledge of physics, your boss asks you to . . ..
- You have been hired as a technical advisor for a TV (or movie) production to make sure the science is correct. In the script . . . ., but is this correct?
- You are a member of a team designing a new device to help trucks go down steep mountain roads at a safe speed even if their brakes fail. . .. The device is . . .. To help determine these forces, you decide to calculate . . ..

Application to unusual or unfamiliar objects or situations

Context-rich problems can also help students realize that a small number of physics principles apply to many situations, both everyday and unfamiliar, and at many different scales (atomic to the solar system).

- You have been hired by a college research group that is investigating (e.g., cancer prevention, the possibility of producing energy from fusion) . . . Your job is to determine. . . ."
- You are reading a magazine article about a satellite in orbit around the Earth that detects X-rays coming from outer space. The article states that the X-ray signal detected from one source, Cygnus X-3 . . . You realize that if this is correct, you can determine how much more massive the Cygnus X-3 neutron star is than our Sun. . ..
- You are working for a chemical company with a group trying to produce new polymers. You have been asked to help determine the structure of part of a polymer chain. . . Your boss wants to know . . . You boss asks you to calculate . . ..

Any context-rich problem will not motivate all students. You should strive for motivating at least half of the students at least half of the time.

Figure 15.2. Steps for building a context-rich problem from a textbook problem.

Tip

Step ❶ Decide on the goals of the problem.

☑ Choose the fundamental physics principle or principles to be featured.

☑ Decide on a difficulty level (e.g., individual or group practice or exam problem, just before or after students have studied principle or concept).

☑ Do you want students to confront and resolve a specific misconception?

☑ Do you want students to practice a specific technique (e.g. calculate the components of vectors, determine a functional dependence, compare a number with their experience)?

Step ❷ Construct the problem on the foundation of one that already exists.

☑ Find a textbook problem that could satisfy your goals.

☑ If the problem does not have one, invent a context (real objects with real motions or interactions) that seems *natural* for that problem.

☑ Decide on a motivation -- Why would the student want to calculate something in this context?

☑ Determine if you need to change the quantity to be calculated or the input quantities to:
- make the solution involve more one logical step using more than one equation; and
- correspond more naturally with the context or motivation.

Step ❸ Write the problem as if it were a short story that connects to your students' reality.

☑ Put the student into the problem.

☑ Make sure the context is as gender and ethnic neutral as possible.

☑ Use the motivation and context to reinforce the value of physics to the student.
- Job related (e.g., "You have a summer job . . .")
- Curiosity (e.g., "You wonder why . . .")
- Helpfulness (e.g., "You are helping your friend . . . ; ")
- Application to unusual or unfamiliar objects or situations (e.g., molecules instead of cars)

☑ Give the student the opportunity to make decisions.

Some suggestions for creating a problem with more decisions are (see Chapter 16 for more difficulty traits):
- Add extra information *that someone in that situation would be likely to have.*
- Leave out necessary but common-knowledge information (e.g. the boiling temperature of water).
- Write the problem so the target quantity is not explicitly stated but must be calculated to make a decision.
- Expand the problem so that two different major physics principles are needed.
- Change to unfamiliar objects or situations (e.g. change cars to electrons).
- Make the problem as short as possible by eliminating unnecessary words or descriptions. Make sure the problem is not obscured by a fog of unnecessary words or information.

Step ❹ Check to make sure that:

☑ the solution requires more than one logical step and one equation;

☑ the solution proceeds directly from fundamental principles and requires no subtle insights or subtile mathematics.

Step ❺ Check evaluation possibilities

Make sure the student has a reasonably straightforward way to check the correctness of the answer by:

☑ Checking the units of the answer.

☑ Comparing to similar quantities that the student should know; and/or

☑ Taking the function to limits where the student knows the behavior (calculus-based course).

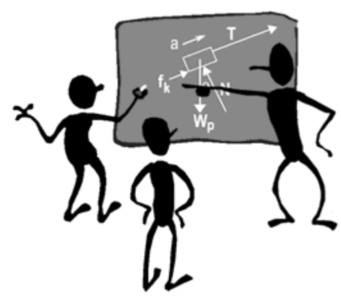
Constructing a Context-rich Problem from a Textbook Problem

You can always construct a context-rich problem from scratch, but the easiest way is to start from a textbook exercise or problem. Figure 15.2 contains a checklist for five steps to guide the construction of a context-rich problem from a textbook exercise or problem. The steps are explained below, using the textbook problem shown in Figure 15.3a.

Step ❶ Decide on the Goals of the Problem.

☑ Choose the fundamental physics principle or principles to be featured.

Imagine that you want students to practice using Newton's 2nd law as applied to uniform circular motion.



☑ Decide on the difficulty level (e.g., individual or group practice or exam problem, just before or after principles and concepts have been studied)

Group problems should be more difficult than individual problems. In addition, the problem must be easier for use just after a concept is introduced than for use towards the end of instruction on the concept.

Suppose you intend to use a context-rich problem as a group practice problem *just after* you introduced the connection between a constant force towards the center of the circle and centripetal acceleration. Since you have not yet modeled solving problems with more than one force, you need a problem situation with only one force. To make the problem more difficult for a group, however, you decide to use a problem involving the vector components of a force.

☑ Do you want students to confront and resolve a specific misconception?

Solving context-rich problems, especially in collaborative groups, can help students confront and overcome their alternative physics conceptions, called misconceptions for simplicity. In this case however, no specific misconceptions are targeted.

☑ Do you want students to practice a specific technique (e.g. calculate the components of vectors, determine a functional dependence, compare a number with their experience, solve simultaneous equations)?

Imagine that you have noticed that your students need more practice visualizing a problem situation and calculating vector components.

Figure 15.3. A textbook and context-rich problem of the textbook problem.

Figure 15.3a. A textbook problem

If an aircraft is banked during a turn in level flight at constant speed, the force F_a exerted by the air on the aircraft is directed perpendicular to a plane that contains the aircraft's wings and fuselage. Draw a free-body diagram for such an aircraft. (*Hint:* Note the similarity to the conical pendulum example in the Chapter 6.) An aircraft traveling at a speed v = 75 m/s makes a turn at a banking angle of 28°. What is the radius of curvature of the turn?

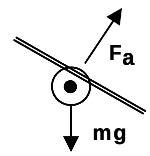


Figure 15.3b. The context-rich *Airplane Problem* based on the textbook problem

You are flying to a job interview when the pilot announces that there are airport delays and the plane will have to circle the airport. The announcement also says that the plane will maintain a speed of 400 mph at an altitude of 20,000 feet. To pass the time, you decide to figure out how far you are from the airport.

You notice that to circle, the pilot "banks" the plane so that the wings are oriented at about 10° from the horizontal. An article in your in-flight magazine explains that an airplane can fly because the air exerts a force called "lift" on the wings. The lift is always perpendicular to the wing surface. The magazine article gives the weight your type of plane as 100,000 pounds and the length of each wing as 150 feet. It gives no information on the thrust from the engines or the drag on the airframe.

Step ❷ Construct the Problem on the Foundation of One That Already Exists.

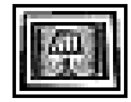
☑ Find a textbook exercise or problem that could satisfy your goals.
Imagine that you found the textbook problem shown in Figure 15.3a. This is the beginning of a good problem. It requires the application of Newton's 2nd law using the components of a single force and the connection of those forces to circular kinematics.

☑ If the problem does not have one, invent a context (real objects with real motions or interactions) that seems natural for that problem.

In this case, the textbook problem in Figure 15.3a has a reasonable object and interaction -- the force of the air (lift) on a banked airplane in uniform circular motion. Some suggestions for interactions and objects for other problems include:

- Physical work (pushing, pulling, lifting objects vertically, horizontally, or up and down hills or ramps)
- Suspending objects, falling objects
- Sports situations (falling, jumping, running, throwing, etc. while diving, bowling, playing golf, tennis, football, baseball, etc.)

- Situations involving the motion of bicycles, cars, sail boats, trucks, airplanes, windmills, etc.
- Astronomical situations (motion of satellites, planets)
- Rotating objects (wheel, doors on hinges, flywheel, sewer pipe rolling down ramp, large spools in a factory, etc.)



- Circuits in battery operated devices and home circuits (e.g., for lamps, kitchen appliances)
- More complicated circuits in devices (for application of conservation or charge and energy)
- Charged particles (e.g., pollen in ionization filter, electrostatic scale, special devices)
- Electric and Magnetic Fields (investigating CRT screen in lab, conducting bar sliding on two parallel conducting rails in safety device, Helmholz coils in various devices, etc.)

☑ Decide on a motivation -- Why would the student want to calculate something in this context?

Make the student a passenger of the airplane. To complete the motivation, answer the following questions: Some example answers are included.

- *Why is the student in an airplane?* Make the student a passenger.
- *Why does the airplane circle?* There is an airport delay and the airplane must circle the airport before it can land.
- *Why should students calculate the radius of the circle?* The student wonders how far the airplane is from the airport.
- *Where does the student get the necessary information?* The student can estimate the banking angle. The only information available to a passenger is an announcement from the pilot and, perhaps, the in-flight magazine.

☑ Determine if you need to change the quantity to be calculated or the input quantities to:

- make the solution more one logical step using more than one equation;
- correspond more naturally with the context or motivation.

In this case, there is no need to change the quantity to be calculated.

Step ❸ Write the Problem as If It Were a Short Story That Connects to Your Students' Reality.

In writing a story, eliminate the physics words and symbols so that the student gets practice supplying them. For example, in the context-rich airplane problem (Figure 15.3b), we eliminated the words *constant speed* and *banking angle*, and the symbol F_a. In addition, the units of quantities in a textbook problem may need to be changed to be consistent with the context. This information should be what would be naturally available in that context. For example, the velocity of the airplane was changed from meters per second to miles per hour. The weight of the airplane is in the common units of pounds, and the distance and length are in the common units of feet.

☑ Put the student into the problem.

Personalize and motivate the problem for the students by making the student the major character in the story (use the pronoun "you"). "You" also contributes to making problems gender neutral.

☑ Make sure the context is as gender and ethnic neutral as possible.

While flying in a commercial airplane is gender and ethnic neutral, not all students have this experience. However, most students have experience watching movies or TV shows involving flying.

☑ Use the motivation and context to reinforce the value of physics to the student.

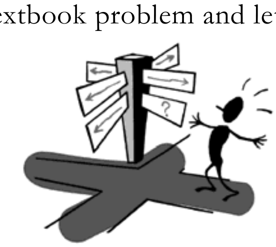
Another way to motive students is to name local landmarks in your problem, such as buildings, lakes, parks, or shopping malls. You can also include characters in popular TV shows or movies, and use movie names that are humorous take offs of popular current movies.

We also found that successful motivations are different for different introductory courses. For example, our algebra-based class consists of students majoring in architecture (40%), environmental science, forestry, veterinary medicine, and health-related fields such as food science and nutrition, nursing and pharmacy. On the average, these students are older than students in the calculus-based course for scientist and engineers, and about one-half of the students are women. They tend to like the curiosity and helpful motivations, and some practical job-related motivations.

The freshman in the calculus-based course are mostly engineering majors and tend to like the curiosity and job-related motivations, with more applications with unusual objects and situations. In our course for biology majors, we constructed many problems with contexts and motivations relevant to biology.

☑ Give the student the opportunity to make decisions.

Eliminate the physics words and symbols so that the student gets practice supplying them. Eliminate the diagram from the textbook problem and let the student decide on the best way to picture the airplane. Eliminate the hint of making a free-body diagram and the hint about the conical pendulum.



To make the problem more suitable for cooperative problem solving, add some characteristics that require groups to make more decisions. Notice that the statement of the context-rich airplane problem forces a connection with reality because the group must decide what "how far you are from the airport" means. The problem also gives more information than necessary to force the group to use physics principles rather than churn all the numbers that are available.

Beware: There is a tendency to make group problems too difficult. A good group problem does not have all of the characteristics that make a problem more difficult, but usually only a few of these characteristics (see Chapter 16).

☑ Make the problem as short as possible by eliminating unnecessary words or descriptions. Make sure the problem is not obscured by a fog of unnecessary words or information.

This is one of the most difficult characteristics to achieve. The elimination of a picture or diagram requires a clear and concise description of the problem situation. This increases the length of context-rich problems compared to most textbook exercises or problems. The motivation of the problem also needs to be as short as possible. For example, in the context-rich airplane problem (Figure 15.3b), the motivation and some of the necessary information is given in the first three sentences.

Step ❹ Check the Solution to Make Sure That:

☑ the solution requires more than one logical step and one equation;

☑ the solution proceeds directly from fundamental principles and requires no subtle insights or special mathematics.

The problem can be solved with a straightforward application of Newton's 2nd law in the vertical and horizontal directions. The most difficult part of the problem is drawing the free-body diagram and relating the banking angle to the components of the lift force.

Step ❺ Check Evaluation Possibilities

Make sure the student has a reasonably straightforward way to check the correctness of the answer by:

☑ checking for correct units

☑ comparing to similar quantities that the student should know;

☑ taking the function to limits where the student knows the behavior.

The function for radius indicates that the radius decreases as the banking angle increases, and increases as the velocity increases. Both of these relationships are consistent with experiences in the circular motion lab.

Practice Building Context-rich Problems

Practice Example 1: Electric Field and Gauss' Law

As a change of pace, practice constructing a context-rich problem for the second semester of a calculus-based introductory physics course -- a more sophisticated problem involving electric fields and Gauss's Law. This example assumes that the students have already had one semester of a physics course based on real problem solving.

There are two ways you could use this example. You could simply read through the example to learn more about the procedure for building a context-rich problem from a textbook problem. Or you could read Step ❶ (Decide the goals of the problem) below, accept or modify these goals, then use the guidelines in Figure 15.2 to practice constructing your own context-rich problem based on the textbook problem in Figure 15.4.

Step ❶ Decide on the Goals of the Problem.

☑ Choose the fundamental physics principle or principles to be featured. The major goal is for students to practice using Gauss' Law for electric fields to calculate something interesting about the behavior of real charged objects by using it in conjunction with mechanics.

☑ Decide on a difficulty level. This will be a difficult problem, suitable for students working in groups.

☑ Do you want students to confront and resolve a specific misconception? No known misconceptions will be addressed, but the concept of Gauss's Law is very difficult for some students to grasp.

☑ Determine if you need to change the quantity to be calculated or the input quantities. Students practice:

- choosing a simple Gaussian surface and determining the charge inside that surface from a charge density;
- using concepts from the first semester of the course, in this case conservation of energy;
- doing appropriate integration. Practice using numbers to get used to appropriate magnitudes for realistic charged objects.

Figure 15.4. A textbook exercise and context-rich problem about electric fields

Figure 15.4a. A textbook exercise about electric fields and Gauss's Law

An infinitely long cylinder of radius R carries a uniform (volume) charge density r. Calculate the field everywhere inside the cylinder.

Figure 15.4b. The context-rich *Fusion Problem* based on the textbook exercise

You have a job in a research laboratory investigating the possibility of producing power from fusion. The device being tested confines a hot gas of positively charged ions in a very long cylinder with a radius of 2.0 cm. The charge density in the cylinder is 6.0×10^{-5} C/m^3. Positively charged Tritium ions are to be injected into the cylinder perpendicular to its axis in a direction toward its. Your job is to determine the speed that a Tritium ion should have when it enters the cylinder, so that it just reaches the axis of the cylinder.

Tritium is an isotope of Hydrogen with one proton and two neutrons. You look up the charge of a proton and mass of the tritium in your trusty Physics textbook and find them to be 1.6×10^{-19} C and 5.0×10^{-27} Kg.

Step ❷ Construct the Problem on the Foundation of One That Already Exists.

☑ **Find a textbook problem that could satisfy your goals.** The textbook exercise in Figure 15.4a could be modified to meet the goals.

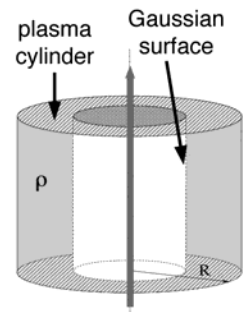
☑ **If the problem does not have one, invent a context.** One natural context is a cylindrical volume containing a charge density could be part of a containment vessel.

☑ **Decide on a motivation.** A possible motivation is that "you" want to know the functional form of the field so that you can inject a charged particle into the existing ions. "You" might want to inject the charged objects (call it Tritium) to investigate fusion as a source of power.

☑ **Determine if you need to change the quantity to be calculated or the input quantities.** Change the calculated quantity to the speed that a charged object must have to penetrate to the center of the cylinder. Specify the radius of the cylinder and the charge density as in the original exercise. Add the charge and mass of the object to be shot into the cylinder.

Step ❸ Write the Problem as If It Were a Short Story That Connects to Your Students' Reality.

☑ **Put the student into the problem Use the motivation and context to reinforce the value of physics.** The context and motivation for the context-rich problem in Figure 15.4a is job related: You have a job in a research laboratory investigating the possibility of producing power from fusion.



The device being tested . . . . Your job is to determine the speed that a Tritium ion should have when it enters the cylinder, so that it just reaches the axis of the cylinder. The context is gender and ethic neutral for students in a calculus-based course for scientists and engineers.

☑ Give the student the opportunity to make decisions. The students must decide to use conservation of energy and then decide to use Gauss' Law to find the force on the Tritium at every point in its flight. The student must also decide to assume that the charge density of the existing ions is uniform.

Step ❹ Check the Problem Solution to Make Sure That:

☑ The solution requires more than one logical step and one equation. The solution requires equations for Gauss' Law, conservation of energy, and the charge density. It is not a one-step problem.

☑ The solution proceeds directly from fundamental principles and requires no subtle insights or subtile mathematics. A simple path of the Tritium is specified so that the application of the physics principles is straightforward. Only a simple integral is required. This is a sophisticated problem but the application of Gauss' Law, conservation of energy, and the mathematics is not subtle.

Step ❺ Check Evaluation Possibilities

Make sure the student has a reasonably straightforward way to check the correctness of the answer by:

☑ Checking the units of answer. The units of the answer can be determined as a check.

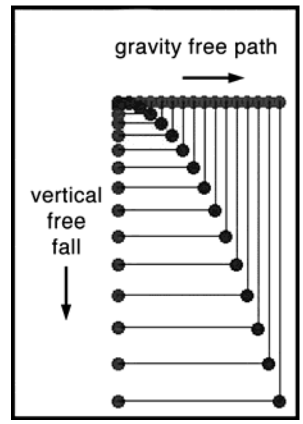
☑ Comparing to similar quantities that the student should know. The magnitude of the answer can be compared with the speed of light on the fast side.

☑ Taking the function to limits where the student knows the behavior. The algebraic solution behavior can be checked by increasing the ion charge density (increasing repulsion so the injection speed should increase) and increasing cylinder radius (going a longer distance should require an increased injection speed).

Practice Example 2.

At the other end of the difficulty scale, construct a context-rich problem suitable for kinematics at the beginning of an introductory physics course. This problem assumes that students are just beginning problem solving.

Step ❶ Decide on the Goals of the Problem.



- ☑ Choose the fundamental physics principle or principles to be featured. Practice using simple two-dimensional kinematics emphasizing the independence of orthogonal components of motion.
- ☑ Decide on a difficulty level. This will be an easy problem suitable for students working alone.
- ☑ Do you want students to confront a specific misconception? Several misconceptions will be addressed.
  - The velocity does not depend on the mass of the object falling.
  - The time an object takes to fall is independent of its horizontal velocity.
- ☑ Do you want students to practice a specific technique? Students should practice using a coordinate system and choosing its origin. Students practice getting a solution without using numbers.

Step ❷ Construct the Problem on the Foundation of One That Already Exists.

- ☑ Find a textbook problem that could satisfy your goals. Start with the textbook problem in Figure 15.5a.
- ☑ If the problem does not have one, invent a context. Use the rifle in the context of hunting.
- ☑ Decide on a motivation. The motivation is a crime scene investigation. There is a possible hunting accident. "You" need to determine the relationship between where the bullet hits the ground and its muzzle velocity to set up special effects for a movie.
- ☑ Determine if you need to change the quantity to be calculated or the input quantities. Have students practice solving a problem without numbers.

Figure 15.5. A textbook and context-rich problem for two-dimensional motion

Figure 15.5a. A textbook problem about two-dimensional motion

A rifle is aimed horizontally at a target 100 ft away. The bullet hits the target 0.75 in. below the aiming point. (a) What is the bullet's time of flight? (b) What is its muzzle velocity?

Figure 15.5b. The context-rich problem based on the textbook problem

You have a job working on the special effects team for a murder mystery movie. The movie opens with a body discovered in a field during the hunting season. A hunter was seen shooting a rifle horizontally in the same field. The hunter claimed to be shooting at a deer, missed, and saw the bullet kick up dirt from the ground. The detective later finds a bullet in the ground. In order to satisfy the nitpickers who demand that movies be realistic, the director has assigned you to calculate the distance from the hunter that this bullet should hit the ground as a function of the bullet's muzzle velocity and the rifle's height above the ground.

Step ❸ Write the Problem as If It Were a Short Story That Connects to Your Students' Reality.

☑ **Use the motivation and context to reinforce the value of physics.** The context and motivation for the context-rich problem in Figure 15.5a is again job related: "You have a summer job working on the special effects team for a murder mystery movie . . . . In order to satisfy the nitpickers who demand that movies be realistic, the director has assigned you to calculate the distance from the hunter that this bullet should hit the ground as a function of the bullet's muzzle velocity and the rifle's height above the ground."

☑ **Make sure the context is as gender and ethnic neutral as possible.** The context and motivation are gender and ethnic neutral because "you" and the hunter are not identified by name. Some cultures object to hunting. Similarly, we rarely make "you" the participant in a sport because many students may have participated or want to participate in the sport.

☑ **Give the student the opportunity to make decisions.** Eliminate breaking the problem into parts to give the student freedom to decide on their approach. The student must decide that the time for the horizontal motion is the same as the time for the vertical motion. The student must also decide on the origin of the coordinate system and to neglect air resistance.

Step ❹ Check the Solution to Make Sure That:

☑ The solution requires more than one logical step and one equation. The solution requires equations for both the horizontal motion of the bullet and its vertical motion. It is not a one-step problem.

☑ The solution proceeds directly from fundamental principles and requires no subtle insights or special mathematics. This is a straightforward application of the definitions of velocity and acceleration as applied to the vertical constant acceleration motion and the horizontal constant velocity motion. The mathematics is neither subtle nor difficult.

Step ❺ Check Evaluation Possibilities

☑ Checking the units of the answer. The units of the answer can be determined as a check.

☑ Taking the function to limits where the student knows the behavior. The behavior of the algebraic solution can be checked for increasing the muzzle velocity (increasing muzzle velocity gives greater distance) and increasing the height (increasing height gives greater distance).

Chapter 16
Judging Problem Suitability for Individual or Group Work

In this chapter:

- ✓ What are the traits of a problems that make them more difficult to solve
- ✓ What are the steps for deciding on the suitability of a problem for different purposes: as an individual or group problem to use just after or towards the end of students' study of a topic.
- ✓ Examples of Judging Difficulty Level

For class time in blocks of 50 minute, instructors of CPS typically create three context-rich problems for an exam: one group problem and two individual problems. The group problem is given in one 50-minute class period. Two individual problems, often accompanied by some multiple choice conceptual questions, are given in another 50 minute class period. [See Chapter 9, page //.]

The group problem should be more difficult to solve than either of the two individual problems. The group exam problem solving session serves as an intense "study group" for the individual exam. One of the individual problems is usually relatively easy to solve while the other is of medium difficulty. In addition instructors create one group practice problem each week (except exam weeks). Because these problems occur when material is first being introduced, they should be less difficult than an exam problem.

As you can probably imagine, the consistent design of context-rich problems with the appropriate difficulty level requires some attention. You may find it helpful to use the judging criteria described in this section. The criteria involve identifying and counting the difficulty traits of a problem, then using a set of "rules of thumb" and your own specialized knowledge of your class to judge whether the problem's difficulty level is appropriate for its intended use.

Since difficult mathematics is best practiced by individuals, the increased difficulty for group problems should be primarily conceptual, not mathematical. An exception might be if your students have a conceptual barrier to carrying out the mathematics. For example, at the beginning of an introductory physics course many students have not developed an organized technique for doing problems involving many steps of algebra. Such a problem would make an appropriate group practice problem at that stage of the course. Generally, problems that involve more

mathematics than physics or problems that require the use of a shortcut or "trick" do not make good group problems. The best group problems involve the straightforward application of the fundamental principles (e.g., the definition of velocity and acceleration, the independence of motion in the vertical and horizontal directions) rather than the repeated use of derived formulas. **Beware** -- it is tempting to make group problems too complex and difficult for groups to solve.

The first section of this chapter describes 21 problem difficulty traits and the research base supporting these traits. The second section describes the four criteria for judging the suitability of a problem for its intended use: individual practice, individual exam, group practice, or group exam. Finally, the third section provides several examples of judging the suitability of problems for their intended use.

Problem Difficulty Traits

In our research we have identified twenty-one traits of a problem that affect students' ability to solve it.[i] Each of these traits causes difficulty for an introductory level student because it represents a qualitative difference between an expert and novice approach to a problem solution. Moving a student toward more expert-like problem solving requires them to perform at these difficulty levels. Some of these traits can be found in any problem, while others are more likely to turn up in specific topic areas. A good group problem should have between 2 to 5 of these difficulty traits, depending on the purpose of the problem and the background of the students.

The difficulty traits have been classified into three major categories, each with two or three subcategories. While these categories are not mutually exclusive, they are helpful in deciding the total number of difficulty traits for a problem. The three categories are Approach to the Problem, Analysis of the Problem, and Mathematical Solution. The traits in each category are explained in more detail below.

Approach to the Problem

The traits in the Approach category affect how a student decides which concepts, principles and laws apply to a problem. In traditional problems this is often given to the students either by a direct statement, such as "the carts have an inelastic collision" or merely by placing the problem at the end of the chapter under a subheading such as "Inelastic Collisions." Without such cues, the following 7 problem traits can make it more difficult for students to decide how to approach a problem.

1. **Problem statement lacks standard cues.** Novice problem solvers often decide on an approach from concrete "cues" in a problem statement. The two difficulty traits in this subcategory thwart this tendency.

   A. **No explicit target variable.** The target quantity is not explicitly stated. Problems with this difficulty trait typically include statements

such as: *Will this plan (design) work?* or *Should you fight this traffic ticket in court?* (see traffic-ticket problem, Chapter 8, page //). Novice problem solvers often use the explicit statement of the desired quantity (e.g., find a) as a cue to the concepts and principles they should apply to the problem.[ii] Problems without this type of cue are more difficult for students to solve.[iii]

B. **Unfamiliar context.** The context of the problem is very unfamiliar to the students. If the students have no experience with the objects in a context, such as molecules or galaxies, they have difficulty creating the mental connections from the problem statement to their understanding. This translation is critical for any successful problem solution.[iv] The Fusion problem (Chapter 15, page //) is an example of a problem that is more difficult to solve because students' lack familiarity with the objects in that context.

2. **Solution requires multiple connections among principles.** As novices in physics, most students are not initially adept at connecting recently learned fundamental principles to other principles they know. This is especially true if the principles are several steps removed from their direct experience. The next three difficulty traits are examples of how problem statements can force students into become fluent with principles.

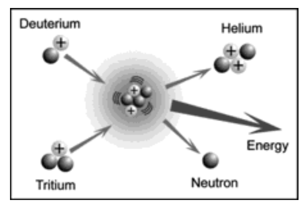
A. **Very abstract concept.** A central concept required to solve the problem is an abstraction of another abstract concept. Most physics concepts are abstract (e.g., forces, energy, field), while college freshmen tend to be concrete thinkers.[v] Some concepts, which we are calling *very* abstract, are themselves an abstraction of an abstract concept. For example, electric potential is an abstraction of the abstract concept of electric potential energy. Most *very abstract concepts*, such as magnetic flux and Gauss's law, are encountered in electromagnetism. The fusion problem, Chapter 15, page //) has this difficulty trait.

B. **Choice of useful principles.** The problem has more than one possible set of useful fundamental principles that *could* be applied to reach a correct solution. For example, consider a problem with a box sliding down a ramp. Typically either Newton's Laws or the conservation of energy will lead to a solution, but the act of deciding which to use in a particular problem is difficult for students. Novice students tend to try to memorize solution patterns for problems based on the objects or actions in a problem. For example they try to solve *all* "object-sliding-down-an-inclined-plane" problems with the same solution pattern, rather than decide which fundamental principles would be most useful to solve that particular problem.[vi] Problems that require students to make these decisions are more difficult for students. For example, traffic-ticket problem, Chapter 8, page //, has this difficulty trait if given on a final exam. Students must decide whether to use forces and kinematics or conservation of energy

C. **Two Fundamental Principles.** The solution requires students to use two or more fundamental physical principles. Examples include pairings such as Newton's Laws and kinematics (see traffic-ticket problem, Chapter 8, page //), conservation of energy and conservation of momentum, conservation of energy and kinematics, linear kinematics and torque, or Gauss's Law and conservation of energy (see fusion Problem, Chapter 15, page //). Combining what the students learned several weeks ago with a current principle is difficult for the beginning student, who perceives physics as a set of incoherent topics.[vii]

3. **Unfamiliar Application of Concepts and Principles.** Students typically begin learning new concepts or principles by practicing in situations that require only a simple, straightforward application. For example, students learn Coulomb's Law by solving problems that require the determination of the total force on a charge located at known distances from other charges. The two difficulty traits in this subcategory require students to generalize their problem-solving knowledge to situations beyond those already practiced.

A. **Atypical situation.** The setting, constraints, or complexity is unusual compared to practice problems and examples. That is, the problem combines objects or interactions in a manner unfamiliar to the student. The fusion problem (Chapter 15, page //) is an example of an atypical situation. This trait challenges the students' novice pattern-matching problem-solving technique.[viii]



B. **Unusual target quantity.** That is, the problem requires students to solve for a quantity that is usually *supplied* in their homework problems or examples. For example, when students first encounter the concept of work, the external force is usually supplied and students are asked to calculate the work. A problem that requires students to determine the external force would have this difficulty trait. This trait also challenges the students' novice pattern-matching, problem-solving strategy.

Analysis of the Problem

The difficulty traits in the Analysis category tax the novice plug-and-chug and pattern-matching problem solving strategy plunging into equations without taking sufficient time and care to analyze the problem. Problem analysis is the translation of the written problem statement into a complete physics description of the problem. It includes a determination of which physics concepts apply to which objects or time intervals, specification of coordinate axes, physics diagrams (e.g., a vector momentum diagram), assigning symbols to important quantities (including subscripts), and the determination of special conditions, constraints, and boundary conditions. The next 9 traits are all examples of how problems that require a careful and complete qualitative analysis are more difficult for students to solve.

4. Excess or Missing Information. Typical practice problems tend to give exactly the information necessary. Students believe that manipulating these values will solve the problem. Excess or missing information in a problem thwarts this novice strategy and requires students to analyze the problem situation, using physics concepts, to decide how to proceed.

   A. Excess data. The problem statement includes more data than is needed to solve the problem. For example, the inclusion of both the static and kinetic coefficients of friction in the traffic-ticket problem (Chapter 8, page //) require students to decide which frictional force is applicable to the situation. The airplane problem (Chapter 15, page //) also has excess information (length of airplane wings). Students often show the symptom of novice problem solving by forcing *all* the given information into the solution.

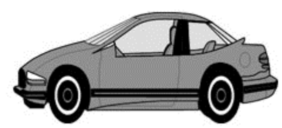
   B. Missing Numbers. The problem requires students to either use a common number, such as the boiling temperature of water, or to estimate a number, such as the height of a person. This case is more difficult than having too much information because the student must first decide that data is missing and then generate that data by connecting physics to the rest of their knowledge base.

   C. Unusual Ignore/Neglect Assumption(s). The problem requires students to ignore or neglect a small effect to solve the problem. *All* problems with a context require students to use their common sense knowledge of how the world works (e.g., boats move through water and not through the air.). If assumptions, such as frictionless surfaces or massless strings, are always made for students in every example or practice problem, deciding to make this type of simplifying assumption in a new context will be difficult. For example, most students quickly learn to ignore air resistance so that assumption does not usually add to the difficulty level of a problem. On the other hand, ignoring the frictional force on an ice hockey puck would. There are two classes of unusual simplifying assumptions: neglect and ignore. The first class is instances where the students must *neglect* a quantity that obviously, to them, makes no difference (e.g. neglecting the mass of a flea when compared to the mass of a dog). The next class of assumptions involves *ignoring* small but noticeable effects that cannot be easily expressed mathematically (e.g. the modified Atwood's machine problem involving

a string moving downward (see Chapter 5, page //). Although the string changes the downward force on the system, students must assume a constant downward force to solve the problem.

5. **Seemingly missing information.** The problem requires students to generate a mathematical expression from the specific situation of the problem. There are three traits in this subcategory.

   A. **Verbal math statement.** The problem includes a verbal statement that can be expressed as an equation. For example, if the problem states "A is proportional to B," then the students must translate the written statement into a mathematical expression and decide how to use it. Examples of verbal math statements are: "the counter-weight is always twice the mass of the package on the ramp" and the statement in the illustration at right. The fusion problem (Chapter 15, page //) has this trait because the statement, "Tritium is an isotope of hydrogen consisting of one proton and two neutrons" must be translated into information about charge and mass.



The car has a weight of 1400 pounds, and 75% of that weight is carried by the front tires.

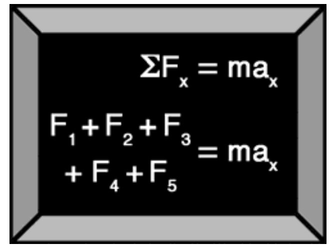
   B. **Special Conditions or Constraints.** The problem requires students to generate a mathematical expression from the special conditions or constraints of the problem. An example is the generation of the relationship $a_1 = a_2$ for two masses connected by a taut string.

   C. **Necessary Relationships from Diagram**: The problem solution requires using the geometry of the physical situation to generate a necessary mathematical expression. This characteristic adds difficulty to a problem because it emphasizes the necessity of a diagram, a skill many novice problem-solvers lack.[ix]

6. **Additional Complexity.** The more "pieces" students have to keep track of, the more difficult the problem.

   A. **More than two subparts.** Some problems require students decompose the problem into more than two sub-parts. More than two sub-parts can arise because there are more than two interacting objects or more than two important time intervals. Changing systems of interest can be hard for students. In addition, novice problem-solvers often lose sight of the problem goal through numerous subparts. Examples of this trait include such classic problems as the ballistic pendulum (which requires using conservation of energy both before and after the impact, but using conservation of momentum during the impact), and the massive-pulley Atwood machine (which requires analyzing the torques on the pulley and the forces on both suspended weights).

B. **More than 4 terms per equation.** The problem involves five or more non-zero terms in an equation. After a required principle has been identified by a student (e.g., Newton's 2nd Law), the principle must be translated into an equation. Problems that have five or more terms in that equation push the limits of student's short-term memory.[x] Students must have a procedural knowledge base that includes organizing principles and logical connections to solve this type of problem. Typical examples include problems in which 5 or more forces are acting on a single object along one axis, and problems in which there are 5 or more energy terms in the conservation of energy equation. Problem statements with this trait require special care in specifying quantity names and determining the sign for each term.



C. **Vector components.** The problem requires students to apply the same principle (e.g., forces or conservation of momentum) in orthogonal directions. This requires the definition of a coordinate system, the decomposition of the physics quantities, and the careful labeling of those quantities. For example, deciding that it is necessary to determine electric field vector components is one of the stumbling blocks in integrating the field of continuous charge distributions.

Mathematical Solution

Mathematical difficulty is last category of traits. Some of these are included in the last five traits.

7. **Algebraic solution.** A strictly algebraic solution is challenging for many novice problem-solvers. There are three problem types that can require algebraic solutions.

A. **No numbers.** The problem statement does not use any numbers. Expert problem-solvers routinely solve problems without substituting in numbers until the very end. Beginning students, however, do not. Many students use numbers as placeholders to help them remember which quantities are known and which are unknown.[xi] The mystery movie problem (Chapter 15, page //) has this difficulty trait.

Figure 16.1. The context-rich Airplane Problem

You are flying to a job interview when the pilot announces that there are airport delays and the plane will have to circle the airport. The announcement also says that the plane will maintain a speed of 400 mph at an altitude of 20,000 feet. To pass the time, you decide to figure out how far you are from the airport.

You notice that to circle, the pilot "banks" the plane so that the wings are oriented at about 10° to the horizontal. An article in your in-flight magazine explains that an airplane can fly because the air exerts a force called "lift" on the wings. The lift is always perpendicular to the wing surface. The magazine article gives the weight your type of plane as 100 thousand pounds and the length of each wing as 150 feet. It gives no information on the thrust from the engines or the drag on the airframe.

B. **Unknown cancels.** This category includes problems in which an unknown quantity, such as a mass, ultimately factors out of the final solution. Students must not only decide how to solve the problem without all the information they expect, but also define symbols for quantities they neither know nor can determine.

C. **Simultaneous equations.** This category includes problems in which each unknown quantity cannot be independently determined, as shown in the equations at right. A typical circuit-analysis problem best illustrates this trait. Simultaneous equations are difficult for students because of their tendency to solve each equation for a definite numerical answer before moving to another equation. Solving simultaneous equations also requires a logical, organized strategy that most beginning students do not initially possess.

Find a:

$$m_2 g - T = m_2 a$$

$$T - m_1 g\sin\theta - \mu m_1 g\cos\theta = m_1 a$$

8. **Unfamiliar Mathematics.** The problems in this subcategory require students to use mathematics that is new and still unfamiliar.

A. **Calculus or vector algebra.** The problem requires calculus, or vector cross products for a correct solution. Most students are still learning these skills in their math courses and resist using them in the unfamiliar situation of a physics problem. This is Difficulty Trait 3 applied to mathematics.

B. **Lengthy or detailed algebra.** A successful solution to the problem is not possible without working through a series of algebraic calculations. While these calculations may not be difficult, they require a careful labeling of quantities and an organized execution.

Decision Strategy for Judging the Suitability of Problems for Their Intended Use

Imagine that you have just introduced the mechanics of circular motion -- the relationship between the sum of the forces towards the center of a circle and the centripetal acceleration. Using the criteria for designing context-rich problems, you may have created the airplane problem (Chapter 15, page //) to use as the first practice group problem on this topic. The airplane problem is also shown in Figure 16.1. You designed the problem to help students confront and resolve their conceptual difficulty of associating centripetal acceleration with a single force in the direction of that acceleration. You also wanted to give students further practice in using Newton's 2ndLaw.

Figure 16.2. Steps for judging a problem's suitability for an intended use.

❶ *Read* the problem statement. *Draw* the diagrams and *determine* the equations needed.

❷ *Reject* if the problem:

- can be solved in one step;
- involves long, tedious mathematics, but little physics; or
- can only be solved easily by using a "trick" or shortcut.

❸ *Identify* and *count* the difficulty traits of the problem.

Problem Approach	Analysis of Problem	Mathematical Solution
1. Cues Lacking ☐ A. No target quantity ☐ B. Unfamiliar context **2. Multiple Connections** ☐ A. Choice of principle ☐ B. Two principles ☐ C. Very abstract concept **3. Non-Standard** Application ☐ A. Atypical situation ☐ B. Unusual target quantity	**4. Excess or Missing** Information ☐ A. Excess data ☐ B. Missing numbers ☐ C. Unusual Assumption (ignore/neglect) **5. Seemingly Missing** Information ☐ A. Verbal Math ☐ B. Special constraint ☐ C. Relationship from diagram **6. Additional Complexity** ☐ A. >2 subparts ☐ B. ≥ 5 terms/equation ☐ C. Vector Components	**7. Algebraic solution** ☐ A. No numbers ☐ B. Unknown cancels ☐ C. Simultaneous equations. **8. Unfamiliar Math** ☐ A. Calc/vector algebra ☐ B. Detailed algebra TOTAL = __ traits

❹ Use the rules-of-thumb below to *decide* the suitability of the problem for its intended use.

Type of Problem	Timing - - Students have	Difficulty
Group Practice Problems should be shorter and mathematically less complex	Just been introduced to concept(s). Just finished study of concept(s).	2 - 3 3 - 4
Group Exam Problems can be longer with more complex math.	Just been introduced to concept(s). Just finished study of concept(s).	3 - 4 4 - 5
Individual Problems: Easy	Just been introduced to concept(s). Just finished study of concept(s).	0 - 1 1 - 2
Medium-difficult	Just been introduced to concept(s). Just finished study of concept(s)	1 - 2 2 - 3
Difficult	Just been introduced to concept(s). Just finished study of concept(s).	2 - 3 3 - 4

There are four steps involved in determining whether a problem is suitable to use as an individual or group practice problem at different stages of instruction on a topic. These steps are outlined in Figure 16.2 and described in more detail below.

Step ❶ If you have not done so, complete at least a partial solution of the problem -- draw diagrams and determine the equations needed.

Step ❷ Decide if the problem should be rejected. There are three reasons to reject a problem. First, problems that can be solved with only one equation will not discourage students from using their novice plug-and-chug or pattern-matching techniques. Second, problems that involve more mathematics than physics make poor problems for group discussion. Finally problems that require a special shortcut or "trick," while intriguing, do not emphasize the use of fundamental principles. The Airplane Problem passes the three rejection tests.

Step ❸ Identify and count the number of problem difficulty traits. This can be accomplished in three stages: (1) difficulty traits associated with deciding on a problem approach, (2) traits associated with the analysis of the problem, and (3) traits associated with the mathematical solution.

- Problem Approach. Determine if the problem has traits that make it difficult for students to decide on how to start. First compare the airplane problem to your textbook's problems for the topic and to problems that you have solved in class.
- Analysis of the Problem. Determine if the problem has traits that make it difficult for students to solve without a careful and complete analysis of the situation.
- Mathematical Solution. Determine if the problem has traits that make it difficult for students to reach a mathematical solution.

A detailed analysis of the difficulty traits for the Airplane Problem is given in Figure 16.3. The airplane problem has a total difficulty rating of 3 - 4 traits.

Step ❹ The last step is to decide if a difficulty rating of 3 - 4 is appropriate for its intended use, in this case as a group practice problem. There are three factors to consider: have about 35 minutes to solve a group practice problem, so it should be less difficult than a Group Exam Problem, where students have about 50 minutes.

- How much time do the students have to solve the problem? Students typically have 20 minutes to solve an individual exam problem, 35 minutes to solve a group practice problem, and 50 minutes to solve a group exam

Figure 16.3a. Difficulty traits of the Airplane Problem related to the problem approach.

1. **Cues Lacking.** This problem does not have an explicit target quantity. Although the problem does specify a distance, there are two choices for distance -- the radius of the horizontal circle or the actual distance from the plane to the ground. The context includes familiar objects (airplane).
2. **Multiple Connections.** The only fundamental principle needed to solve the problem is dynamics (Newton's Laws).
3. **Non-standard Application.** The situation is similar to problems found in many textbooks. The problem does not have an unusual target quantity.

Problem Approach

1. **Cues Lacking**
   - ☑ A. No target variable
   - ☐ B. Unfamiliar context
2. **Multiple Connections**
   - ☐ A. Choice of principle
   - ☐ B. Two principles
   - ☐ C. Very abstract concept
3. **Non-Standard** Application
   - ☐ A. Atypical situation
   - ☐ B. Unusual target

Figure 16.3b. Difficulty traits of the Airplane Problem related to the analysis of the problem.

4. **Excess or Missing Information.** The airplane problem statement does include excess data -- the length of the plane wings. There is no missing data, nor is there any unusual ignore/neglect assumption that must be made.
5. **Seemingly missing information.** The problem does not contain a verbal math statement, and there is no special constraint. If students decide to solve for the distance from the plane to the ground, then they do need a geometric relationship from the diagram.
6. **Additional Complexity.** There is one additional complexity trait -- students must use vector components to solve the problem.

Analysis of Problem

4. **Excess or Missing** Information
   - ☑ A. Excess data
   - ☐ B. Missing numbers
   - ☐ C. Unusual Assumption (ignore/neglect)
5. **Seemingly Missing** Information
   - ☐ A. Verbal Math
   - ☐ B. Special constraint
   - ? C. Relationship from diagram
6. **Additional Complexity**
   - ☐ A. >2 subparts
   - ☐ B. ≥ 5 terms/equation
   - ☑ C. Vector Components

Figure 16.3b. Difficulty traits of the Airplane Problem related to the mathematical solution

7. **Algebraic Solution.** The airplane problem does not require an algebraic solution.
8. **Unfamiliar Math.** The problem does not require use of unfamiliar mathematics.

Mathematical Solution
7. Algebraic Solution
☐ A. No numbers
☐ B. Unknown cancels
☐ C. Simultaneous equations.
8. Unfamiliar Math
☐ A. Calc/vector algebra
☐ B. Detailed algebra

problem. The problem difficulty should by easy-to medium for the individual problems, more difficult for the group practice problems, and the most difficult for the group exam problems.

- Is this a new topic? Problems featuring a topic that has just been introduced should have fewer difficulty traits than a problem featuring a topic about which the students have seen example problem solutions.
- What is the students' familiarity with both a logical, organized problem-solving strategy and with context-rich problems? Problems used early in the course should have fewer difficulty traits than those later in the course.

Taking these three factors into account, some "rules of thumb" can be used to determine if your problem is about the right level of difficulty for its intended use. These rules of thumb are shown in Figure 16.2 for this step.

If the airplane problem (difficulty rating of 3-4 traits), is intended for use as a group practice problem before the students much experience solving uniform circular motion problems, it would be challenging for most groups to make significant progress toward a solution in 35 minutes. This problem could be modified by removing one difficulty trait to make it more appropriate, or could be used as a group practice problem several days later in the course.

There is considerable overlap in the rules of thumb. Many problems can be appropriate for several uses. For example, the Airplane Problem could be used as a medium-difficult individual problem on an exam. As with any procedure, your judgment of the appropriateness of a difficulty level should be modified by your knowledge of your students.

Practice Judging Context-rich Problems

Judging the difficulty level of a problem depends on the students and the instructor's view of the course. Even though instructors do not always agree on the specific difficulty traits of a problem, there is reasonable agreement on the total number of these traits. This is because these criteria are not mutually exclusive. For example, it is unusual to find verbal-mathematics statements in problems. In that case, some instructors check the Atypical Situation trait, and some check the Verbal Math trait. The only important consideration is not to count the *same* problem difficulty twice!

Practice Example 1:The Skateboard Problem.

Suppose you have just finished teaching about using the conservation of momentum and conservation of energy in collisions. You design the skateboard problem, shown in Figure 16.4, for a group exam problem. The problem is designed to test students' ability to recognize when both conservation of energy and conservation of momentum are needed. You may want to determine the suitability of the Skateboard problem for a group exam problem by following the steps in Figure 16.2 before reading the analysis below.

Figure 16.4. The Skateboard Problem

You are helping a friend prepare for a skateboard exhibition. The idea is for your friend to take a running start and then jump onto a heavy-duty 15-lb stationary skateboard. Your friend, on the skateboard, will glide in a straight line along a short, level section of track, then up a sloped concrete wall. The goal is to reach a height of at least 6 feet above the starting point before rolling back down the slope. The fastest your friend can run and safely jump on the skateboard is 20 feet/second. Can this plan work? Your friend weighs 125 lbs.

The Suitability of the Skateboard Problem

Step ❶ *Read* the problem statement. *Draw* the diagrams and *determine* the equations needed. [Note: One solution to this problem is shown in Chapter 11, pages // to //.]

Step ❷ Decide if the problem should be rejected.

The skateboard problem cannot be solved in one step, does not involve long, tedious mathematics, and requires only the straightforward application of the conservation of energy and momentum. It should not be rejected.

Step ❸ *Identify* and *count* the difficulty traits of the problem.

The analysis outlined in Figure 16.5 indicates that the skateboard problem has 3-4 difficulty traits: no specific target variable, two principles needed to solve the problem, unusual assumption required, and possibly the choice of which principles to use.

Step ❹ Use the rules-of-thumb to *decide* the suitability of the problem for its intended use.

The difficulty rating of 3-4 for the problem indicates it may be too easy for a 50-minute group exam problem for most students. The skateboard problem is probably more suitable for use as a group practice problem or, perhaps, a difficult final exam problem.

Practice Example 2: Gravitational vs Electric Forces

This practice example problem, shown in Figure 16.5, is from the second semester of an introductory course. Analyze the problem to determine the number of

Figure 16.5a. Difficulty traits of the Skateboard Problem related to the problem approach.

1. Cues Lacking. This problem does not have an explicit target quantity (Can this plan work?). The context includes familiar objects (people,

Problem Approach
1. Cues Lacking

skateboard).

2. **Multiple Connections.** Two fundamental principles are needed to solve this problem -- conservation of energy and conservation of momentum. Students might spend some time deciding whether to use the conservation of energy or kinematics to relate the velocity of the friend and skateboard after the inelastic collision with the height above the starting point. However, at this point in the course, it should not be a big issue.

☑ A. No target quantity
☐ B. Unfamiliar context

2. Multiple Connections
? A. Choice of principle
☑ B. Two principles
☐ C. Very abstract concept

3. Non-Standard Application
☐ A. Atypical situation
☐ B. Unusual target

3. **Non-standard Application.** The situation is not atypical of energy and momentum problems, nor is the target quantity unusual. Students can solve for the height to determine if it is less than 6 feet, or the initial velocity needed to reach a height of 6 feet and compare it to 20 feet/sec.

Figure 16.5b. Difficulty traits of the Skateboard Problem related to the problem analysis.

4. **Excess or Missing Information.** The skateboard problem does not include excess data, nor are any numbers missing. There is, however, an *unusual* ignore-or-neglect assumption (besides the usual assumption of ignoring friction). Students must ignore the vertical component of the runner's momentum during the inelastic collision with the skateboard. (That is, there is a small transfer of momentum out of the system.)

5. **Seemingly missing information.** There is no seemingly missing information in this problem.

6. **Additional Complexity.** There is no additional complexity for the skateboard problem. There are only 2 subparts (conservation of momentum during inelastic collision and conservation of energy after the collision), 2 terms/equation, and no vectors components.

Analysis of Problem

4. Excess or Missing Information
☐ A. Excess data
☐ B. Missing numbers
☑ C. Unusual Assumptions (ignore/neglect)

5. Seemingly Missing Information
☐ A. Verbal Math
☐ B. Special constraint
☐ C. Relationships from diagram

6. Additional Complexity
☐ A. >2 subparts
☐ B. ≥ 5 terms/equation
☐ C. Vector Components

Figure 16.5c. Difficulty traits of the Skateboard Problem related to the mathematical solution.

7. **Algebraic Solution.** The problem does not require a strictly algebraic solution -- a number is expected. There are no unknowns that cancel and simultaneous equations are not required for a solution

8. **Targets unfamiliar math.** The math of the problem is simple and straightforward.

Mathematical Solution

7. Algebraic Solution
☐ A. No numbers
☐ B. Unknown cancels
☐ C. Simultaneous equations.

8. Targets Unfamiliar Math
☐ A. Calc/vector algebra
☐ B. Detailed algebra

Figure 16.6. Gravitational versus Electric Forces Problem

You and a friend are reading a newspaper article about nuclear fusion energy generation in stars. The article describes the helium nucleus, made up of two protons and two neutrons, as very stable so it doesn't decay. You immediately realize that you don't understand why the helium nucleus is stable. You know that the proton has the same charge as the electron except that the proton charge is positive. Neutrons you know are neutral. Why, you ask your friend, don't the protons simply repel each other causing the helium nucleus to fly apart? Your friend tells you that the gravitational force keeps the nucleus together.

Your friend's model is that the two neutrons sit in the center of the nucleus and gravitationally attract the two protons. Since the protons have the same charge, they are always as far apart as possible with the neutrons directly between them. It sounds good but you are not sure you believe it. To check, you decide to determine if the mass the neutron must have for this model of the helium nucleus to work.

Looking in your physics book, you find that the mass of a neutron is about the same as the mass of a proton and that the diameter of a helium nucleus is 3.0×10^{-13} cm.*

* Constants, such as the charge of the electron, the universal gravitational constant and the electric constant, are typically given on an information sheet, but are not included here.

difficulty traits. Then determine the possible uses of this problem in an introductory course.

> This problem has a difficulty rating of 4 - 5: an unfamiliar context, requires two principles, an unusual target quantity, and either verbal math or relationship from a diagram. The difficulty is primarily in the visualization of the problem – determining that the distance between the two neutrons is one-half the diameter of the nucleus. It is also possible that the students have not used the gravitational force lately, which makes the problem more difficult.
>
> This problem is most suitable for individual practice or a group exam problem, If this were a group problem, you could expect groups to spend most of their time discussing the problem situation and the physics to apply. Once that is resolved, the mathematical solution follows easily.

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Appendix A
Department Survey

1. In your opinion, what is the primary reason your department requires students to take this physics course?

2. How many quarters of physics do you think should be required for your students?0 1 2 3 4 5 6

3. Many different goals could be addressed through this course. Would you please rate each of the following possible goals in relation to its importance for your students on a scale of 1 to 5?

1 = unimportant	2 = slightly important	3 = somewhat important	4 = important	5 = very important

Know the basic principles behind all physics (e.g. forces, conservation of energy, ...)	1	2	3	4	5
Know the range of applicability of the principles of physics (e.g. conservation of energy applied to fluid flow, heat transfer, plasmas, ...)	1	2	3	4	5
Be familiar with a wide range of physics topics (e.g. specific heat, AC circuits, rotational motion, geometrical optics,...)	1	2	3	4	5
Solve problems using general quantitative problem solving skills within the context of physics	1	2	3	4	5
Solve problems using general qualitative logical reasoning within the context of physics	1	2	3	4	5
Formulate and carry out experiments	1	2	3	4	5
Analyze data from physical measurements	1	2	3	4	5
Use modern measurement tools for physical measurements (e.g.. oscilloscopes, computer data acquisition, timing techniques,...)	1	2	3	4	5
Program computers to solve problems within the context of physics.	1	2	3	4	5
Overcome misconceptions about the behavior of the physical world	1	2	3	4	5
Understand and appreciate 'modern physics' (e.g. solid state, quantum optics, cosmology, quantum mechanics, nuclei, particles,...)	1	2	3	4	5
Understand and appreciate the historical development and intellectual organization of physics.	1	2	3	4	5
Express, verbally and in writing, logical, qualitative thought in the context of physics.	1	2	3	4	5
Use with confidence the physics topics covered.	1	2	3	4	5
Apply the physics topics covered to new situations not explicitly taught by the course.	1	2	3	4	5
Other goal. Please specify here	1	2	3	4	5

- **Please place a star (*) next to the TWO goals listed above that you consider to be the MOST IMPORTANT for your students.**

4. In three quarters it is impossible to cover every topic in physics, so some topics need to be left out. The purpose of this question is to inform us of your priorities of the topics we might cover in the course. Below

are the chapter headings from a typical textbook at this level. Please place the number of weeks you would like to see spent on each chapter. Each week consists of three lectures, one recitation, and a two-hour laboratory. The total number of weeks should equal 24 since there is a week of introduction and organization at the beginning of the quarter and a week of review at the end of the quarter. Please do not use half-week units.

______ Units, dimensions and vectors
______ Linear motion
______ Two dimensional motion
______ Forces and Newton's Laws
______ Applications of Newton's laws
______ Kinetic energy and work
______ Potential energy and conservation of energy
______ Momentum and collisions
______ Rotations and torque
______ Angular momentum
______ Statics
______ Gravitation
______ Simple harmonic motion
______ Waves (e.g. standing waves, sound, Doppler effect)
______ Superposition and interference of waves
______ Properties of fluids (e.g. pressure, continuity, Bernoulli's equation)
______ Temperature and ideal gas
______ Heat flow and the first law of thermodynamics
______ Molecules and gases (e.g. probability distributions of velocity, equipartition theory)
______ The second law of thermodynamics
______ Properties of solids (e.g. stress, strain, thermal expansion)
______ Electric charge (e.g. Coulomb's law, charge conservation)
______ Electric field
______ Gauss' law
______ Electric potential
______ Capacitors and dielectrics
______ Currents in materials (e.g. resistance, insulator, semiconductors)
______ DC circuits
______ Effects of magnetic fields (e.g. magnets, magnetic force, Hall effect)
______ Properties of magnetic fields (e.g. Ampere's law, Biot-Savart law)
______ Faraday's law
______ Magnetism and matter (e.g. ferromagnetism, diamagnetism)
______ Magnetic Inductance
______ AC circuits
______ Maxwell's equations and electromagnetic waves
______ Light (e.g. reflection and refraction)
______ Mirrors and lenses
______ Interference
______ Diffraction
______ Other. Please specify.
__24__ Total number of weeks

Please place a star (*) next to the FOUR chapters listed above that you consider to be the MOST IMPORTANT for your students.

5. The laboratory associated with this course is typically taught by graduate teaching assistants and could be structured in several ways. Please place an 'X' by that structure that you feel would be most appropriate for the students.

   ___ A lab with well defined directions which verifies a physical principle previously explained to the students using the given apparatus.

   ___ A lab where the students are given a specific question or problem for which they must conduct an experiment with minimal guidance using the given apparatus.

   ___ A lab where the students are given a general concept from which they must formulate an experimental question, then design and conduct an experiment from a choice of apparatus.

   ___ Other. Please describe.

6. The recitation sections associated with this course is typically taught by graduate teaching assistants and could be structured in several ways. Please place an 'X' by that structure that you feel would be most appropriate for the students.

   ___ Students ask the instructor to solve specific homework problems on the board.

   ___ Instructor asks students to solve specific homework problems on the board.

   ___ Instructor asks students to solve unfamiliar textbook problems, then discusses solution with class.

   ___ Students work in small collaborative groups to solve real-world problems with the guidance of the instructor.

   ___ Other. Please describe.

7. Would you please give examples of topics or subjects covered in your curriculum that assume some knowledge, skills or understanding which should be imparted by this physics course? Specific course numbers would also be helpful.

Thank you for completing this questionnaire. If you have any material which illustrates the topics or subjects covered in your curriculum which assume knowledge, skill, or understanding which should be imparted in Physics, we would appreciate receiving a copy.

In order for us to ask you more detailed questions and consult with you as the need arises, we ask that you complete the following information. Thank you.

Name: ______________________________

Department / program: ______________________________

Campus address: ______________________________

Campus phone: ______________________________

Appendix B
Context-rich Mechanics Problems

1. (average speed) You and your friend run every day no matter what the weather (well almost). One winter day the temperature is a brisk 0^o F. Your friend insists that it is OK to run. You agree to this madness as long as you both begin at your house and end the run at her nice warm house in a way that neither of you has to wait in the cold. You know that she runs at a very consistent pace with an average speed of 3.0 m/s, while your average speed is a consistent 4.0 m/s. Your friend starts first so that she will arrive at her house and unlock the door before you arrive. Five minutes later, you notice that she dropped her keys and start after her. How far from your house will she be when you catch up if you run at your usual pace?

2. (constant speed) You have been hired to assist in a research laboratory doing experiments investigating the mechanism by which chemicals such as aspirin relieve pain. Your task is to calibrate your detection equipment using the properties of a radioactive isotope that will later be used to track the chemical through the body. You have been told that your isotope decays by first emitting an electron and then, some time later, emitting a photon. You set up your equipment to determine the time between the electron emission and the photon emission. Your apparatus detects both electrons and photons. You determine that the electron and photon from a decay arrive at your detector at the same time if the detector is 2.0 feet from your radioactive sample. A previous experiment has shown that the electron from this decay travels at one half the speed of light. You know that the photon travels at the speed of light, 1.0 foot per nanosecond.

3. (average speed) You have joined the University team racing a solar powered car. The optimal average speed for the car depends on the amount of sun hitting its solar panels. Your job is to determine strategy by programming a computer to calculate the car's average speed for a day consisting of different race conditions. To do this you need to determine the equation for the day's average speed based on the car's average speed for each part of the trip. As practice you imagine that the day's race consists of some distance under bright sun, the same distance with partly cloudy conditions, and twice that distance under cloudy conditions.

4. (average velocity) It is a beautiful weekend day and you and your friends decide to spend it outdoors. Two of your friends just want to relax while the other two want some exercise. You need some quiet time to study. To satisfy everyone, the group decides to spend the day on the river. Two people will put a canoe in the river and just drift downstream with the 1.5 mile per hour current. The second pair will begin at the same time as the first but from 10 miles downstream. They will paddle upstream until the two canoes meet. Since you have been canoeing with these people before, you know that they will have an average velocity of 2.5 miles per hour relative to the shore when they go against this river current. When the two canoes meet, they will come to shore and you should be there to meet them with your van.

5. (Constant velocity) You have a job in a University laboratory investigating possible accident avoidance systems for oil tankers. Your group is concerned that an oil spill in the North Atlantic could be caused by a super tanker running into an iceberg. The group has been developing a new type of down-looking radar that can detect large icebergs. They are concerned about its rather short range of 2 miles. Super tankers are so huge that it takes a long time to turn them. Your job is to determine how much time would be available to turn the tanker to avoid a collision once the iceberg is detected. The radar signal travels at the speed of light, 186,000 miles per second, but once the

signal arrives back at the ship it takes the computer 5 minutes to process it. A typical sailing speed for super tankers during the winter on the North Atlantic is about 15 miles per hour. Assume that the tanker is heading directly at an iceberg that is drifting at 5 miles per hour in the same direction that the tanker is going.

6. (Constant velocity, group) You have a job in a research laboratory that has been investigating possible accident avoidance systems for automobiles. You have just begun a study of how bats avoid obstacles. In your study, a bat is fitted with a transceiver that broadcasts the bat's velocity to your instruments. Your research director has told you that the signal travels at the speed of light, 1.0 ft/nanosecond. You know that the bat detects obstacles by emitting a forward going sound pulse (sonar) that travels at 1100 ft/s through the air. The bat detects the obstacle when the sound pulse reflects from the obstacle and that reflected pulse is heard by the bat. You are told to determine the maximum amount of time that a bat has after it detects the existence of an obstacle to change its flight path to avoid the obstacle. In the experiment your instruments tell you that a bat is flying straight toward a wall at a constant velocity of 20.0 ft/s and emits a sound pulse when it is 10.0 ft from the wall.

7. (1-D Kinematics) You are part of a citizen's group evaluating the safety of a high school athletic program. To help judge the diving program you would like to know how fast a diver hits the water in the most complicated dive. The coach has the best diver perform for your group. The diver, jumps from the high board, and near the end of the dive, passes the lower diving board, 3.0 m above the water. Using a stopwatch, you determine that it took 0.20 seconds to enter the water from the time the diver passed the lower board.

8. (1-D Kinematics) You have a summer job working for a University research group investigating the causes of the ozone depletion in the atmosphere. The plan is to collect data on the chemical composition of the atmosphere as a function of the distance from the ground using a mass spectrometer located in the nose cone of a rocket fired vertically. To make sure the delicate instruments survive the launch, your task is to determine the acceleration of the rocket before it uses up its fuel. The rocket is launched straight up with a constant acceleration until the fuel is gone 30 seconds later. To collect enough data, the total flight time must be 5.0 minutes before the rocket crashes into the ground.

9. (1-D Kinematics) Just for the fun of it, you and a friend decide to enter the famous Tour de Minnesota bicycle race from Rochester to Duluth and then to St. Paul. You are riding along at a comfortable speed of 20 mph when you see in your mirror that your friend is going to pass you at what you estimate to be a constant 30 mph. You will, of course, take up the challenge and accelerate just as she passes you until you pass her. If you accelerate at a constant 0.25 miles per hour each second until you pass her, how long will she be ahead of you?

10. (1-D Kinematics, group) Because of your knowledge of physics, you have been assigned to investigate a train wreck between a fast moving passenger train and a slower moving freight train both going in the same direction. You have statements from the engineer of each train and the stationmaster as well as some measurements you make. To check the consistency the description of events leading up to the collision, you decide to calculate the distance from the station that the collision should have occurred if everyone’s statement were accurate. You will then compare that result to the actual position of the wreck that is 0.5 miles from the station. In this calculation you decide that you can ignore all reaction times. Here is what you know:

a. • The stationmaster claims that she noted that the freight train was behind schedule. As regulations require, she switched on a warning light just as the last car of the freight train passed her.
b. • The freight train engineer says he was going at a constant speed of 10 miles per hour.
c. • The passenger train engineer says she was going at the speed limit of 40 miles per hour when she approached the warning light. Just as she reached the warning light she saw it go on and immediately hit the brakes.
d. • The train reaches the warning light 2.0 miles before it gets to the station.
e. • The passenger train's brakes slow it down at a constant rate of 1.0 mile per hour for each minute they are applied.

11. (1-D Kinematics, group) Because parents are concerned that children are learning "wrong" science from TV, you have been asked to be a technical advisor for a new science fiction show. The show takes place on a space station at rest in deep space far away from any stars. In the plot, a vicious criminal escapes from the space station prison. She steals a small space ship and blasts off to meet her partners somewhere in deep space. Your job is to calculate how long her partners have to transport her off her ship before it is destroyed by a photon torpedo from the space station? In the story, the stolen ship accelerates in a straight line at its maximum possible acceleration of 30 m/sec^2. After 10 minutes all of the fuel is burned and the ship coasts at a constant velocity. Meanwhile, the hero learns of the escape and fires a photon torpedo. Once fired, a photon torpedo travels at a constant velocity of 20,000 m/s. By that time the escapee has a 30 minute head start on the photon torpedo.

12. (1 D Kinematics) You are an assistant technical advisor for a new gangster movie. In one scene the robbers try to evade capture by driving from one state to another. In the script, they are speeding down the highway, and pass a concealed police car that is stopped by the roadside a short distance from the state line. The instant they pass the police car, it pulls onto the highway and accelerates after them at its maximum rate. The writers want to know how far this chase has to start from the state line so that the robbers just make it across in dramatic style. The director wants you to give the answer in terms of the constant speed of the gangster's car and the maximum acceleration of the police car sot the car's can be chosen.

13. (1-D Kinematics, group) You have been hired as the assistant technical advisor for a new action movie. In this exciting scene, the hero pursues the villain up to the top of a bunge jumping apparatus. The villain appears trapped but to create a diversion drops a bottle filled with a deadly nerve gas on the crowd below. The script calls for the hero to quickly strap the bunge cord to his leg and dive straight down to grab the bottle while it is still in the air. Your job is to determine the length of the unstretched bunge cord needed to make the stunt work. You estimate that the hero can jump off the bunge tower with a maximum velocity of 10 ft/sec. straight down by pushing off and can react to the villain's dropping the bottle by strapping on the bunge cord and jumping in 2 seconds.

14. (1-D Kinematics, group) The University Skydiving Club has asked you to evaluate their plan a stunt for an air show. The plan is for two skydivers to step out of opposite sides of a stationary hot air balloon 5,000 feet above the ground. The second skydiver will leave the balloon 20 seconds after the first skydiver but both are to land on the ground at the same time. To get a rough idea of the situation, you decide to assume that a skydiver's air resistance will be negligible before the parachute opens. As soon as the parachute is opened, the skydiver falls with a constant velocity of 10 ft/sec. The first skydiver waits 3 seconds after stepping out of the balloon before opening the parachute. To

decide if this stunt will work, calculate how long the second skydiver waits after leaving the balloon before opening the parachute?

15. (1-D Kinematics) You have a summer job as the technical assistant to the director of an adventure movie. The script calls for a large package to be dropped onto the bed of a fast moving pick-up truck from a helicopter that is hovering above the road. The helicopter is 235 feet above the road, and the bed of the truck is 3 feet above the road. The truck is traveling down the road at 40 miles/hour. You must determine when to tell the helicopter to drop the package so it lands in the truck.

16. (1-D Kinematics, group) Because movie producers have come under pressure for teaching children incorrect science, you have been appointed to help a committee review a script for a new Superman movie. In the scene under consideration, Superman rushes to save Lois Lane who has been pushed from a window about 300 feet above a crowded street. Superman swoops down in the nick of time, arriving when Lois is half way to the street, and stopping her just at ground level. Lois changes her expression from one of horror at her impending doom to a smile of gratitude as she gently floats to the ground in Superman's arms. The committee wants to know if there is really enough time to express this range of emotions, even if there is a possible academy award on the line. The chairman asks you to calculate the time it takes for Superman to stop Lois's fall. To do the calculation, you assume that Superman applies a constant force to Lois and that she weighs 110 lbs. While thinking about this scene you also wonder if Lois could survive the force that Superman applies to her so you calculate that also.

17. (2-D Kinematics) While on a vacation you find an old coastal fort probably built in the 16th century. Large stone walls rise vertically from the shore to protect the fort from cannon fire from pirate ships. Walking around the ramparts, you find the fort's cannons mounted such that they fire horizontally out of holes near the top of the walls facing the ocean. Leaning out of one of these gun holes, you drop a rock that hits the ocean 3.0 seconds later. You wonder if a ship 500 meters away would be in range of the fort's cannon? To answer that question you need to calculate how fast the cannon ball would have to leave the cannon to hit the ship.

18. (2-D Kinematics) You read in the newspaper that rocks from Mars have been found on Earth. Your friend says that the rocks were shot off Mars by the large volcanoes there. You are skeptical so you decide to calculate the magnitude of the velocity that volcanoes eject rocks from the geological evidence. You know the gravitational acceleration of objects falling near the surface of Mars is only 40% that on the Earth. You assume that you can look up the height of Martian volcanoes and find some evidence of the distance rocks from the volcano hit the ground from pictures of the Martian surface. If you assume the rocks farthest from a volcano were ejected at an angle of 45 degrees, what is the magnitude of the rock's velocity as a function of its distance from the volcano and the height of the volcano for the rock furthest from the volcano?

19. (2-D Kinematics) You have a job on a team testing the software for an air traffic control center at the nation's busy airports. Part of the system is based on a single wide acceptance radar dish that can detect airplanes almost from horizon to horizon. The system determines an airplane's average velocity from measurements at two different times. As a test you will have the aircraft fly directly over the radar antenna on its downward going landing path. The system measures the distance of the airplane from the antenna and its angle from the horizontal at two times, before it passes over the antenna and after it has passed over the antenna. The distances, angles, and times are input into a computer and the software calculates both the direction and magnitude of the airplane's average velocity. You check the software by making the calculation.

20. (2-D Kinematics) You have a summer job with an insurance company and have been asked to help with the investigation of a tragic "accident." When you visit the scene, you see a road running straight down a hill that has a slope of 10 degrees to the horizontal. At the bottom of the hill, the road goes horizontally for a very short distance becoming a parking lot. The parking lot ends at the top of a high cliff. Looking down from the edge of the parking lot, you see the wreck of the car on the horizontal ground below the cliff. Police measurements give that the car is 30 feet from the base of the cliff. The only witness claims that the car was parked on the hill, he can't exactly remember where, and the car just began coasting down the road. He did not hear an engine so he thinks that the driver was drunk and passed out knocking off his emergency brake. He remembers that the car took about 3 seconds to get down the hill. Your boss drops a stone from the edge of the cliff and, from the sound of it hitting the ground below, determines that it takes 5.0 seconds to fall to the bottom. After looking pensive, she tells you to calculate the car's average acceleration coming down the hill based on the statement of the witness and the other facts in the case. She reminds you to be careful to write down all of your assumptions so she can evaluate the applicability of the calculation to this situation. Obviously, she suspects foul play.

21. (2-D Kinematics) You have a summer job with an insurance company and have been asked to help with the investigation of a tragic "accident." When you visit the scene, you see a road running straight down a hill that has a slope of 10 degrees to the horizontal. At the bottom of the hill, the road goes horizontally for a very short distance becoming a parking lot. The parking lot ends at the top of a high cliff. Looking down from the edge of the parking lot, you see the wreck of the car on the horizontal ground below the cliff. Police measurements give that the car is 30 feet from the base of the cliff. Was it possible that the driver fell asleep at the wheel and simply drove over the cliff? After looking pensive, your boss tells you to calculate the speed of the car as it left the top of the cliff.

22. (2-D Kinematics, group) Because of your physics background, you have been hired as a consultant for a new movie about Galileo. In one scene, he climbs up to the top of a tower and, in frustration over the people who ridicule his theories, throws a rock at a group of them standing on the ground. The rock leaves his hand at 30° from the horizontal. The script calls for the rock to land 15 m from the base of the tower near a group of his detractors. It is important for the script that the rock takes precisely 3.0 seconds to hit the ground so that there is time for a good expressive close-up. The set coordinator is concerned that the rock will hit the ground with too much speed causing cement chips from the plaza to injure one of the high priced actors. You are told to calculate that speed.

23. (2-D Kinematics) You are watching people practicing archery when you wonder how fast an arrow is shot from a bow. With a flash of insight you decide that you can easily determine what you want to know by a simple measurement. You ask one of the archers to pull back the bowstring as far as possible and shoot an arrow horizontally. The arrow strikes the ground at an angle of 86 degrees from the vertical at 100 feet from the archer's feet.

24. (2-D Kinematics) You have a summer job working on the special effects team for a new movie. A body is discovered in a field during the fall hunting season and the sheriff begins her investigation. One suspect is a hunter who was seen that morning shooting his rifle horizontally in the same field. He claims he was shooting at a deer and missed. You are to design the "flashback" scene that shows his version of firing the rifle with the bullet kicking up dirt where it hits the ground. The sheriff later finds a bullet in the ground. She tests the hunter's rifle and finds the velocity that it shoots a bullet (muzzle velocity). In order to satisfy the nitpickers who demand that movies be realistic, the director

has assigned you to calculate the distance from the hunter that this bullet should hit the ground as a function of the bullet's muzzle velocity and the rifle's height above the ground.

25. (2-D Kinematics) You have been hired as a member of a team investigating the ecosystem development around active volcanoes. One of your assignments is to write a computer program to help ensure the team's safety. You first decide to calculate the time a piece of volcanic material takes to reach an observing team. For this scenario, the team already knows the height of the volcano above them and their horizontal distance from the mouth of the volcano. They observe the angle that the volcanic material leaves the mouth of the volcano.

26. (2-D Kinematics) In your new job, you are helping to design stunts for a new movie. In one scene the writers want a car to jump across a chasm between two cliffs. The car is driving along a horizontal road when it goes over one cliff. Across the chasm, which is 1000 feet deep, is another road at a lower height. They want to know the minimum speed of the car so that it does not fall into the chasm. They have not yet selected the car so they want an expression for the speed of the car in terms of the car's mass, the width of the chasm, and the height of the upper road above the lower road. They will plug in the actual numbers after they have purchased a car for the stunt.

27. (2-D Kinematics) While you are watching a baseball game, the batter hits a ball that is barely off the ground. The ball flies through the air and looks as if it is going to go over the fence 200 ft away for a home run. In a spectacular play, the left fielder runs to the wall, leaps high, and catches the ball just as it clears the top of 10 ft high wall. You wonder how much time the fielder had to react to the hit and make the catch. You estimate that the ball left the bat at an angle of 30^{o}.

28. (2-D Kinematics) You are a member of a citizen's committee investigating safety in the school sports program. You are interested in knee damage to athletes participating in the long jump. The coach has the best long jumper demonstrate the event for you. He runs down the track and, at the take-off point, jumps into the air at an angle of 30 degrees from the horizontal. He comes down in a sandpit at the same level as the track 8.5 m away from the take-off point. With what velocity (both magnitude and direction) did he hit the ground?

29. (2-D Kinematics, group) Your friend has decided to make some money during the State Fair by inventing a game of skill for the Midway. In the game, the customer shoots a rifle at a 5.0 cm diameter target 7.0 meters above the ground. At the instant that the bullet leaves the rifle, the target is released from its holder and drops straight down. When shooting, the customer stands 20 meters from where the target would hit the ground if the bullet misses. Your friend asks you to calculate where to aim to hit the target in the center. The rifle's muzzle velocity is 350 m/sec according to the manual.

30. (2-D Kinematics, group) Your group has been selected to serve on a citizen's panel to evaluate a new proposal to search for life on Mars. On this unmanned mission, the lander will leave orbit around Mars falling through the atmosphere until it reaches 10,000 meters above the surface of the planet. At that time a parachute opens and takes the lander down to 500 meters. Because of the possibility of very strong winds near the surface, the parachute detaches from the lander at 500 meters and the lander falls through the thin Martian atmosphere with a constant acceleration of 0.40g for 1.0 second. Retrorockets then fire to bring the lander to a soft landing on the surface. A team of biologists has suggested that Martian life might be very fragile and decompose quickly in the heat from the lander. They suggest that any search for life should begin at least 9 meters from the base of the lander. This biology team has designed a probe that is shot from the lander by a spring mechanism in the lander 2.0 meters above the surface of Mars. To return the data, the probe cannot

be more than 11 meters from the bottom of the lander. For the probe to function properly it must impact the surface with a velocity of 8.0 m/s at an angle of 30 degrees from the vertical. Can this probe work as designed?

31. (Circular kinematics) You have been hired as a consultant for a new Star Trek TV series to make sure that any science on the show is correct. In this episode, the crew discovers an abandoned space station in deep space far from any stars. This station is a large wheel-like structure where people lived and worked in the rim. In order to create "artificial gravity," the space station rotates on its axis. The special effects department wants to know at what rate a space station 200 meters in diameter would have to rotate to create "gravity" equal to 0.7 that at the surface of Earth.

32. (Circular kinematics) While watching some TV you see a circus show in which a woman drives a motorcycle around the inside of a vertical ring. You determine that she goes around at a constant speed and that it takes her 4.0 seconds to get around when she is going her slowest. If she is going at the minimum speed for this stunt to work, the motorcycle is just barely touching the ring when she is upside down at the top. At that point she is in free fall. She just makes it around without falling off the ring but what if she made a mistake and her motorcycle fell off at the top? How high up is she?

33. (Forces, group) You have been asked to test a new device for precisely measuring the weight of small objects. The device consists of two light wires attached at one end to a support. An object is attached to the other end of each wire. The wires are far enough apart so the objects hanging on them don't touch. One of the objects has a very accurately known weight while the other object is the unknown. A power supply is slowly turned on to give each object an electric charge causing them to slowly move away from each other because of the electric force. When the power supply is kept at its operating value, the objects come to rest at the same horizontal level. At that point, each of the wires supporting them makes a different angle with the vertical and that angle is measured. To test the device, you calculate the weight of an unknown sphere from the measured angles and the weight of a known sphere. You will then check your calculation in the laboratory using a variety of different objects.

34. (Forces, friction) You are driving your car uphill along a straight road. Suddenly, you see a car run a red light and enter the intersection just ahead of you. You slam on your brakes and skid in a straight line to a stop, leaving skid marks 100 feet long. A policeman observes the whole incident and gives a ticket to the other car for running a red light. He also gives you a ticket for exceeding the speed limit of 30 mph (about 44 ft/s). When you get home, you read your physics book and estimate that the coefficient of kinetic friction between your tires and the road was 0.60, and the coefficient of static friction was 0.80. You estimate that the hill made an angle of about 10° with the horizontal. You look in your owner's manual and find that your car weighs 2,050 lbs. Will you fight the traffic ticket in court?

35. (Forces, friction, group) Because of your physics background, you have a job with a company devising stunts for an upcoming adventure movie. In the script, the hero has just jumped on the train as it passed over a lake so he is wearing his rubber wet suit. He climbs up to the top of the locomotive and begins fighting the villain while the train goes down a straight horizontal track at 5 mph. During the fight, the hero slips and hangs by his fingers on the top edge of the front of the locomotive. The locomotive has a smooth steel front face sloped at 20^{o} from the vertical so that the bottom of the front is more forward that the top. Now the villain stomps on the hero's fingers so he will be forced to let go, slip down the front of the locomotive, and be crushed under its wheels. Meanwhile, the hero's partner is at the controls of the locomotive trying to stop the train. To add to

the suspense, the brakes have been locked by the villain. It will take 7 seconds to open the lock. To save him, she immediately opens the throttle causing the train to accelerate forward such that the hero stays on the front face of the locomotive without slipping down. This gives her time to save his life. The movie company wants to know the minimum acceleration necessary to perform this stunt. The hero weighs 180 lbs in his wet suit. The locomotive weighs 100 tons. You look in a book giving the properties of materials and find that the coefficient of kinetic friction for rubber on steel is 0.50 and its coefficient of static friction is 0.60.

36. (Forces, friction) While working in a mechanical structures laboratory, your boss assigns you to test the strength of ropes under different conditions. Your test set-up consists of two ropes attached to a 30 kg block which slides on a 5.0 m long horizontal table top. Two low friction, light weight pulleys are mounted at opposite ends of the table. One rope is attached to each end of the 30 kg block. Each of these ropes runs horizontally over a different pulley. The other end of one of the ropes is attached to a 12 kg block which hangs straight down. The other end of the second rope is attached to a 20 kg block also hanging straight down. The coefficient of kinetic friction between the block on the table and the table's surface is 0.08. The 30 kg block is initially held in place by a mechanism that is released when the test begins so that the block is accelerating during the test. During this test, what is the force exerted on the rope supporting the 12 kg block?

37. (Forces, circular motion) After watching a TV show about Australia, you and some friends decide to make a communications device invented by the Australian Aborigines. It consists of a noise-maker swung in a vertical circle on the end of a string. You are worried about whether the string you have will be strong enough, so you decide to calculate the string tension when the device is swung with a constant speed at a constant radius. You and your friends can't agree whether the maximum string tension will occur when the noise maker is at the highest point in the circle, at the lowest point in the circle, or is always the same. To settle the argument you decide to compare the tension at the highest point to that at the lowest point.

38. (Forces, circular motion) One weekend, you decide to visit an amusement park and take your neighbor's children. They want to ride on the 20 ft diameter Ferris wheel. The Ferris wheel has seats on the rim of a circle and rotates at a constant speed making one complete revolution every 20 seconds. While you wait, you decide to calculate the total force (both magnitude and direction) on one of the children who weighs 45 lbs when they are at one quarter revolution past the highest point. Because the Ferris wheel can be run at different speeds, you also decide to make a graph that gives the magnitude of the force on the child at that point as a function of the period of the Ferris wheel.

39. (Forces, circular motion) You are reading a magazine article about the aesthetics of airplane design. One example in the article is a beautiful new design for commercial airliners. As a practical person, you want to know if the thin wing structure is strong enough to be safe. The article explains that an airplane can fly because the air exerts a force, called "lift," on the wings such that the lift is always perpendicular to the wing surface. For level flying, the wings are horizontal. To turn, the pilot "banks" the plane so that the wings are oriented at an angle to the horizontal. This causes the plane to go in a horizontal circle. The specifications of the 100×10^3 lb plane require that it be able to turn with a radius of 2.0 miles at a speed of 500 miles/hr. The article also states that tests show that the new wing structure will support a force 4 times the lift necessary for level flight.

40. (Forces, circular motion, group) You are flying to Chicago when the pilot tells you that the plane can not land immediately because of airport delays and will have to circle the airport. You are told that the plane will maintain a speed of 450 mph at an altitude of 25,000 feet while traveling in a horizontal circle around the airport. To pass the time you decide to figure out the radius of that circle. You notice that to circle, the pilot "banks" the plane so that the wings are oriented at 10^o to the horizontal. An article in your in-flight magazine explains that an airplane can fly because the air exerts a force, called "lift," on the wings. The lift is always perpendicular to the wing surface. The magazine article gives the weight of the type of plane you are on as 100×10^3 pounds and the length of each wing as 150 feet. It gives no information on the thrust of the engines or the drag of the airframe.

41. (Forces, circular motion, group) You are working with an ecology group investigating the feeding habits of eagles. During this research, you observe an eagle circling in the air at a height that you estimate to be 300 feet. You suspect that the eagle is circling around its prey on the ground below. You wonder how far the eagle is from its prey so you decide to calculate it. Looking up at the eagle, you determine that its wings are banked to make an angle of 15^o from the horizontal and that it takes 18 seconds to go around the circle.

42. (Forces, circular motion) Your team is designing a package moving system in the new, improved post office. The device consists of a large circular disc (i.e. a turntable) that rotates once every 3.0 seconds at a constant speed in the horizontal plane. Packages are put on the outer edge of the turntable on one side of the room and taken off on the opposite side. The coefficient of static friction between the disc surface and a package is 0.80 while the coefficient of kinetic friction is 0.60. If this system is to work, what is the maximum possible radius of the turntable?

43. (Forces, circular motion) You were hired you as technical advisor for a new "James Bond" film. The scene calls for Bond to chase a villain onto a merry-go-round. An accomplice starts the merry-go-round rotating in an effort to toss him off into an adjacent pool filled with hungry sharks. You must determine a safe rate of rotation such that the stunt man will not fly off the merry-go-round and into the pool. You measure the diameter of the merry-go-round to be 50 meters. You also determine that the coefficient of static friction between Bond's shoes and the merry-go-round surface is 0.7 and the coefficient of kinetic friction is 0.5.

44. (Forces, circular motion) You are driving with a friend who is sitting to your right on the passenger side of the front seat. The road you are on has some large turns in it. Without putting on a seatbelt your friend might slide into you causing you to swerve and have an accident. When you stop for lunch, you try to convince your friend that this is possible. To do this you decide to calculate the minimum radius you could turn on a level road for your friend not to slide, based on the coefficient of static friction between your friend and the seat of the car and the car's constant speed. You also determine the direction of the turn to make your friend slide into you and not away from you.

45. (Forces, circular motion) You have been hired as a member team reviewing the safety of northern freeways. The section of road you are studying has a curve that is 1/8 of a circle with a radius of 0.5 miles. The road has been designed with a curve banked at an angle of 4^o to the horizontal. To begin the study, you have been assigned to calculate the maximum speed for a standard passenger car (about 2000 lbs) to complete the turn while maintaining a horizontal path along the road if it is covered by a slick coating of ice. You then need to compare your results to the case of a dry, clear road with a coefficient of kinetic friction of 0.70 and coefficient of static friction of 0.80 between the tires and the road.

46. (Forces, circular motion) On a trip through Florida, you find yourself driving in your along a flat level road at 50 mph. The road makes a turn that you take without changing speed. The curve is approximately an arc of a circle with a radius of 0.05 miles. You notice that the road is flat and level with no sign of banking. There is no warning sign but you wonder if it would be safe to go 50 mph around the curve in the rain when the wet surface has a lower coefficient of friction. To satisfy your curiosity, you decide to determine the minimum coefficient of static friction between the road and your car's tires that will allow your car to make the turn. Your owner's manual says that your car weighs 3000-lbs.

47. (Conservation of energy) You are driving your car uphill along a straight road. Suddenly, you see a car run a red light and enter the intersection just ahead of you. You slam on your brakes and skid in a straight line to a stop, leaving skid marks 100 feet long. A policeman observes the whole incident and gives a ticket to the other car for running a red light. He also gives you a ticket for exceeding the speed limit of 30 mph. When you get home, you read your physics book and estimate that the coefficient of kinetic friction between your tires and the road was 0.60, and the coefficient of static friction was 0.80. You estimate that the hill made an angle of 10^{o} with the horizontal. In your owner's manual you find that your car weighs 2,050 lbs. Will you fight the traffic ticket in court?

48. (Conservation of energy) You are watching a National Geographic Special on television. One segment of the program is about the archer fish of Southeast Asia. This fish "shoots" water at insects to knock them into the water so it can eat them. The commentator states that the archer fish keeps its mouth at the surface of the stream and squirts a jet of water from its mouth at 13 feet/second. You watch an archer fish shoot a moth off a leaf into the water. You estimate that the leaf was about 2 feet above the stream. You wonder at what minimum angle from the horizontal the water can be ejected from the fish's mouth to hit the moth. Since you have time during the commercial, you quickly calculate this angle.

49. (Conservation of energy, force) At the train station, you notice a large horizontal spring at the end of the track where the train comes in. This is a safety device to stop the train so that it will not go plowing through the station if the engineer misjudges the stopping distance. While waiting, you wonder what would be the fastest train that the spring could stop by being fully compressed, 3.0 ft. To keep the passengers as safe as possible when the spring stops the train, you assume that the maximum stopping acceleration of the train, caused by the spring, is g/2. You make a guess that a train might have a mass of 0.2 million kilograms. For the purpose of getting your answer, you assume that all frictional forces are negligible.

50. (Conservation of energy) You are the technical advisor to a TV show about "death defying" stunts. Your task is to design a stunt in which a 5 ft 6 in, 120 pound actor jumps off a 100-foot tall tower with an elastic cord tied to one ankle with the other end of the cord tied to the top of the tower. This 75 ft cord is very light but very strong and stretches so that it can stop the actor without pulling a leg off. Such a cord exerts a force with the same mathematical form as a spring. To minimize the force that the cord exerts on the leg, you want it to stretch as far as possible. You must determine the elastic force constant that characterizes the cord so that you can purchase it. For maximum dramatic effect, the jump will be off a diving board at the top of the tower. From tests you have made, the maximum speed of a person coming off the diving board is 10 ft/sec.

51. (Conservation of energy) You are the technical advisor to a TV show about "death defying" stunts. Your task is to design a stunt in which an 80 kg actor is shot out of a cannon elevated 40^{o} from the

horizontal. The "cannon" is actually a 3-foot diameter tube that uses a stiff spring and a puff of smoke rather than an explosive to launch the human cannonball. The spring constant is 900 Newtons/meter. The spring is compressed by a motor until its free end is 1.0 meters above the ground. A small seat is attached to the free end of the spring for the actor to sit on. When the spring is released, it extends an additional 3 meters to the mouth of the tube. Neither the seat nor the chair touch the sides of the tube. At the place that the actor would hit the ground you will put an airbag that exerts an average retarding force of 500 Newtons. You need to determine the minimum airbag thickness to stop the actor.

52. (Conservation of energy, 2-D kinematics, group) You have a summer job with a company designing the ski jump for the next Winter Olympics. The ski jump ramp is coated with snow and a skier glides down it reaching a high speed. The bottom of the ramp is shaped so that the skier leaves it traveling horizontally. The winner is the person who jumps the farthest after leaving the end of the ramp. Your task is to write a computer program to determine the distance of the starting gate from the bottom of the ramp. For safety reasons, your design should be such that for a perfect run down the ramp, the skier's speed before leaving the ramp and sailing through the air should not exceed a safe speed that will be input later. For a fixed angle of the ski ramp with the horizontal and a maximum safe speed, your calculation should allow you to place the gate based on the skier's starting speed, the coefficient of friction between the snow and the ski, and the mass of the skier. Since the ski-jumpers bend over and wear very aerodynamic suits, you decide to neglect the air resistance.

53. (Conservation of momentum, kinematics) You have been hired as a technical consultant for an early-morning cartoon series for children to make sure that the science is correct. In the script, a wagon containing two boxes of gold (total mass of 150 kg) has been cut loose from the horses by an outlaw. The wagon starts from 50 meters up a hill with a 6^{o} slope. The outlaw plans to have the wagon roll down the hill and across the level ground and then crash into a canyon where the gang waits. Luckily, the Lone Ranger (mass 80 kg) and Tonto (mass 70 kg) are waiting in a tree 40 meters from the edge of the canyon. They drop vertically into the wagon as it passes beneath them. The script states that it takes the Lone Ranger and Tonto 5.0 seconds to grab the gold and jump out of the wagon, but do they have that much time?

54. (Conservation of momentum, kinematics) You have been asked to write part of the software for a new computer game. In one part of the game, the hero must get a magic fruit down from a tree by shooting it with an arrow. The hero aims the arrow so that when it is at the highest part of its path, it hits the fruit. The arrow sticks in the fruit and they both fall together to the ground. You need to calculate the distance from the tree that the fruit hits the ground in terms of the speed that the arrow leaves the bow, the angle the arrow leaves the bow, the height of the tree, the mass of the arrow, and the mass of the fruit.

55. (Conservation of momentum) As a concerned citizen, you have volunteered to serve on a committee investigating injuries to Junior High School students participating in sports programs. Currently the committee is looking into the high incidence of ankle injuries on the basketball team. You are watching the team practice, looking for activities that can result in large horizontal forces on the ankle. Observing the team practice jump shots gives you an idea, so you try a small calculation. A 40-kg student jumps 0.6 meters straight up and shoots the 0.80-kg basketball at his highest point. From the trajectory of the basketball, you deduce that the ball left his hand at 30^{o} from the horizontal at 10 m/s. To help estimate the force, you calculate his horizontal velocity when he hits the ground?

56. (Conservation of momentum) You are a volunteer at the Campus Museum of Natural History and have been asked to assist in the production of an animated film about the survival of hawks in the wilderness. In the script, a 1.5-kg hawk is hovering in the air so it is stationary with respect to the ground when it sees a goose flying below it. The hawk dives straight down at 60 km/hr. When it strikes the goose it digs its claws into the goose's body. The goose, which has a mass of 2.5 kg, was flying north at 30 km/hr just before it was struck by the hawk and killed instantly. The animators want to know the velocity (magnitude and direction) of the hawk and dead goose just after the strike.

57. (Conservation of momentum, 2-D kinematics, or center of mass, group) While waiting in a supermarket line you read in a tabloid newspaper that an alien spaceship exploded while hovering over the center of a remote town. The wreckage of the spaceship was found in three large pieces. One piece (mass = 300 kg) of the spaceship landed 6.0 km due north of the center of town. Another piece (mass = 1000 kg) landed 1.6 km to the southeast (36 degrees south of east) of the center of town. The last piece (mass = 400kg) landed 4.0 km to the southwest (65 degrees south of west) of the center of town. There were no more pieces of the spaceship. The paper reports that data from air traffic control radar show the spaceship was hovering motionless over the center of town before it exploded and that just after the explosion the pieces initially moved horizontally. The article speculates that the spaceship did not explode on its own accord but was hit by a missile. When you get home you decide to determine whether the fragments reported are consistent with the spaceship exploding spontaneously. If not, you want to know the direction from which the missile came.

58. (Conservation of momentum, 2-D kinematics) While visiting a friend's house, you a fascinated by the behavior of the cat. This 8 lb cat sits on the floor eyeing a stationary 20 lb chair that is 1.3 m away along the floor. The seat of the chair is 45 cm above the floor. The cat jumps up and lands on the seat of the chair just as she reaches the maximum height of her trajectory. She puts out her claws and hangs on. The chair sits on a part of the floor that has just been waxed and is very slippery. As the chair with the cat slides along the floor, you wonder what its speed was just after the cat lands on it?

59. (Conservation of momentum) You have been asked to serve on a citizen's commission investigating the safety of bridges across the Mississippi River. Because of increased barge traffic, you worry about bridge damage from being hit by runaway barges. A past accident serves as an example of what can go wrong. A fully loaded grain barge being pushed down river by tugboat rammed an empty barge being pushed directly across the river to a loading dock. At that time, the ropes tying each barge to its tugboat broke so that the barges were free. Records of the event give the speed of each barge and its mass before the accident. The record also gives the speed of the empty barge and its direction just after the accident. Unfortunately the speed and direction of the loaded barge just after the accident was not recorded so you decide to calculate them.

60. (Conservation of energy, conservation of momentum, group) Because of your physics background, you have been hired to help design stunt equipment for the movies. In this particular stunt, the actor is in a runaway car that skids in a complicated path down an icy hill. Your task is to design a method of stopping the car at the bottom of the hill without harming the actor. To do this you decide to have a steel plate at the bottom of the hill that the car can hit. The plate it attached to a stiff spring with the other end attached to a wall. The steel plate will be magnetized so that the car will stick to it. To choose the spring, you need to determine its spring constant based on the mass of the car, the mass of the steel plate, the length of the spring, and the height that the car starts above the stopping device.

61. (Conservation of energy, conservation of momentum) You are helping a friend prepare for a skate board exhibition. For the program, your friend plans to take a running start and then jump onto a heavy duty 15-lb stationary skateboard. The skateboard will glide in a straight line along a short, level section of track, then up a sloped concrete wall. The goal is to reach a height of at least 10 feet before turning to come back down the slope. You have measured your friend's maximum running speed to safely jump on the skateboard at 7 feet/second. Can this program be carried out as planned? Your friend weighs 120 lbs.

62. (Conservation of energy, conservation of momentum) You have been hired as a technical advisor for a new James Bond movie. In the script, Bond and his latest love interest, who is 2/3 his weight (including skis, boots, clothes, and various hidden weapons), are skiing in the Swiss Alps. She skis down a slope while he stays at the top to adjust his boot. When she has skied down a vertical distance of 100 ft, she stops to wait for him and is captured by the bad guys. Bond looks up and sees what is happening. He notices that she is standing with her skis pointed downhill while she rests on her poles. To make as little noise as possible, Bond starts from rest and glides down the slope heading right at her. Just before they collide, she sees him coming and lets go of her poles. He grabs her and they both continue downhill together. At the bottom of the hill, another slope goes uphill and they continue to glide up that slope until they reach the top of the hill and are safe. The writers want you to calculate the maximum possible height that the second hill can be relative to the position where he grabbed her. Both Bond and his girl friend are using new, top-secret frictionless stealth skis developed for the British Secret Service.

63. (Conservation of energy, conservation of momentum) You have been able to get a job with a medical physics group investigating ways to treat inoperable brain cancer. One form of cancer therapy being studied uses slow neutrons to deposit energy in the nucleus of the atoms that make up cancer cells. To test this idea, your research group decides to determine the change of energy of an iron nucleus after a neutron knocks out one of its neutrons. In the experiment, a neutron goes into the nucleus with a speed of 2.0×10^7 m/s and you detect two neutrons coming out at angles of 30^o and 15^o. To a good approximation, the heavy nucleus does not move during the interaction. The mass of a neutron is 1.7×10^{-27} kg.

64. (Conservation of energy, conservation of momentum) Your friend has just been in a traffic accident and is trying to negotiate with the insurance company of the other driver to pay for fixing the car. You are asked to prove that it was the other driver's fault. At the scene of the crash you determined what happened. Your friend was traveling North and entered an intersection with no stop sign. At the center of the intersection, this car was struck by the other car that was traveling East. The speed limit on both roads entering the intersection is 50 mph. From the visible skid marks, the cars skidded locked together for 56 feet at an angle of 30^o north of east before stopping. At the library you determine that the weight of your friend's car was 2600 lbs and that of the other car was 2200 lbs, where you included the driver's weight in each case. The coefficient of kinetic friction for a rubber tire skidding on dry pavement is 0.80. It is not enough to prove that the other driver was speeding, you must also show that your friend was under the speed limit.

65. (Conservation of energy, heat) You are helping a veterinarian friend to do some minor surgery on a cow. Your job is to sterilize a scalpel and a hemostat by boiling them for 30 minutes. You do as ordered and then quickly transfer the instruments to a well-insulated tray containing 200 grams of sterilized water at room temperature which is just enough to cover the instruments. After a few minutes the instruments and water will come to the same temperature, but can you hand to your friend without being burned? You know that both the 50 gram scalpel and the 70 gram hemostat are

made from stainless steel that has a specific heat of 450 J / (kg °C). They were boiled in 2.0 kg of water with a specific heat of 4200 J / (kg °C).

66. (Conservation of energy, heat) During the spring your friend will have an outdoor wedding. You volunteered to supply the perfect lemonade. Unfortunately, the ice used to cool the lemonade melts diluting it too much unless you have planned for this extra water. To help your planning, you look up the specific heat capacity of water (1.0 cal/(gm °C)), the specific heat capacity of ice (0.50 cal/(gm °C)), and the latent heat of fusion of water (80 cal/gm). You assume that the specific heat capacity of the lemonade is the same as water. Using that information, you calculate how much water you get from the ice melting when you add just enough to 6 quarts (5.6 kg) of lemonade at room temperature (23 °C) to bring it down to 10°C. The ice comes straight from the freezer at -5.0 °C.

67. (Conservation of energy, Conservation of momentum, Phase change) In a class demonstration, a 2.0-gram lead bullet was shot into a 2.0-kg block of wood hung from strings. The block of wood with the bullet stuck in it swung up to a height 0.50 cm above its initial position. Is it possible that the bullet melts when it comes to rest in the block? Assume that the bullet had a temperature of 50°C when it left the gun. The melting temperature of lead is 330°C. It has a specific heat capacity of 130 J/(kg °C) and a latent heat of fusion of 25 J/g.

68. (Conservation of energy, heat, pressure, group) While working for a mechanical engineering company, your boss asks you to determine the efficiency of a new type of pneumatic elevator for use in a two level storage facility. The elevator is supported in a cylindrical shaft by a column of air, which you assume to be an ideal gas with a specific heat of 12.5 J/mol °C. The air pressure in the column is 1.2×10^5 Pa when the elevator carries no load. The bottom of the cylindrical shaft is connected to a reservoir of air at room temperature (25° C). Seals around the elevator assure that no air escapes as the elevator moves up and down. The elevator has a cross-sectional area of 10 m^2. A cycle of elevator use begins with the unloaded elevator. The elevator is loaded with 20,000 kg causing the elevator to sink while the air temperature stays at 25° C. The air in the shaft is then heated to 75° C and the elevator rises. The elevator is then unloaded, while the air remains at 75° C. Finally, the air in the system is cooled to room temperature again, returning the elevator to its starting level. While the elevator is moving up and down, you assume that it moves at a constant velocity so that the pressure in the gas is constant.

69. (Center of mass, calculus, group) You have been hired as part of a research team consisting of biologists, computer scientists, engineers, mathematicians, and physicists investigating the virus that causes AIDS. This effort depends on the design of a new centrifuge to separate infected cells from healthy cells by spinning a container of these cells at very high speeds. Your design team has been assigned the task of specifying the mechanical structure of the centrifuge arm that holds the sample container. For aerodynamic stability, the arm must have uniform dimensions. Your team has decided the shape will be a long, thin rectangular strip. The properties of this strip will be optimized by a computer program. The arm must be stronger at one end than at the other so your team decided to use new composite materials to accomplish this. With these materials one can continuously change the strength by changing the density of the arm along its length while keeping its dimensions constant. The density will vary linearly along the length of the strip from a low value at its tip to a high value at its base. To calculate the strength of the brackets necessary to support the arm, you must determine the position of the center of mass of the arm as a function of the dimensions of the arm, its mass, the density at the tip, and the rate of change of the density along the arm.

70. (Kinematics, rotation) You are working in a group investigating more energy efficient city busses. One option is to store energy in the rotation of a flywheel when the bus stops and then use it to accelerate the bus. The flywheel under consideration is 1.5 m diameter disk of uniform construction except that it has a massive, thin rim on its edge. Half the mass of the flywheel is in the rim. When the bus stops, the flywheel rotates at 20 revolutions per second. When the bus is going at its normal speed of 30 miles per hour, the flywheel rotates at 2 revolutions per second. The material holding the rim to the rest of the flywheel has been tested to withstand an acceleration of up to 20g but you are worried that it might not be strong enough. To check, you consider the case of the bus initially going 30 miles per hour making an emergency stop in 0.50 seconds. You assume that during this time the flywheel has a constant angular acceleration. You know that the moment of inertia of a disk rotating about its center is half that of a ring with the same mass and radius rotating about the same axis.

71. (Conservation of energy, moment of inertia, rotation) You have applied for a summer job working with a special effects team at a movie studio. As part of your interview you have been asked to evaluate the design of a stunt in which a large spherical boulder rolls down an inclined track. At the bottom, the track curves up into a vertical circle so that the boulder can roll around on the inside of the circle and come back to ground level. It is important that the boulder not fall off the track and crush the actors standing below. You have been asked to determine the relationship between the height of the boulder's starting point on the ramp, as measured from the center of the boulder, and the maximum radius the circular part of the track. You also know the properties of the boulder. You are told that the moment of inertia of a sphere rotating about its center is 2/5 that of a ring with the same mass and radius rotating about the same axis. After some thought you decide that the boulder will stay moving in a vertical circle if its radial acceleration at the top is just that provided by gravity.

72. (Torque, moment of inertia) After watching a news story about a fire in a high rise apartment building, you decide to design an emergency escape device from the top of a building. Because of the possibility of power failure, you will make a gravitationally powered elevator. The design has a large, heavy horizontal disk that is free to rotate about its center on the roof with a cable wound around its edge. The free end of the cable goes horizontally to the edge of the building roof, passes over a heavy vertical pulley, and then hangs straight down. A strong box that can hold 5 people is then attached to the hanging end of the cable. When people enter the box and release the cable, it unrolls from the horizontal disk lowering the people safely to the ground. To see if this design is feasible you decide to calculate the acceleration of the fully loaded elevator to make sure it is much less than g. The device specifications are that the radius of the horizontal disk as 1.5 m and its mass is twice that of the fully loaded elevator box. The disk that serves as the vertical pulley has 1/4 the radius of the horizontal disk and 1/16 its mass. You know that the moment of inertia of a disk about its center is 1/2 that of a ring with the same mass and radius about the same axis.

73. (Torque, moment of inertia, kinematics) Because of your physics background, you have been asked to be a stunt consultant for a motion picture about a genetically synthesized prehistoric creature that escapes from captivity and terrorizes the city. The scene you are asked to review has three actors chased by the creature through an old warehouse. At the exit of the warehouse is a thick steel fire door 10 feet high and 6.0 feet wide weighing about 2,000 pounds. In the scene, the three actors flee from the building and close the fire door, thus sealing the creature inside. They have 3.0 seconds to shut the door and you need to know if they can do it. You estimate that each actor can each push on the door with a force of 50 pounds. When they push together, each actor needs a space of about 1.5 feet between them and the next actor. The door, with a moment of inertia around its hinges of 1/3 of what it would be if all of its mass were concentrated at its outside edge, needs to rotate 120 degrees to close.

74. (Conservation of energy or torque, moment of inertia, kinematics) While working on a paper about the technology of settlers crossing the Great Plains, you need to know the moment of inertia of a wooden wagon wheel. You decide make a measurement on a wagon wheel from the museum. This wheel has a mass of 70 kg and a diameter of 1.3 m. You mount the wheel vertically on a low-friction bearing then wrap a light cord around the outside of the rim to which you attach a 20-kg block. When the block is released, it falls 1.5 m in 0.33 s.

75. (Conservation of energy or torque, moment of inertia, kinematics) While you watch a TV program about life in the ancient world, you see that the people in one village used a solid sphere made out of clay as a kind of pulley to help get water from a well. A well-greased wooden axle was placed through the center of the sphere and fixed in a horizontal orientation above the well, allowing the sphere to rotate freely. To demonstrate the depth of the well, the host of the program completely wraps a thin rope around the sphere and ties the bucket to its end. When the bucket is released, it to falls to the bottom of the well unwinding the string from the rotating sphere as it goes. It takes 2.5 seconds. You wonder about the depth of the well so you decide to calculate it. You estimate that the sphere has twice the mass of the bucket. You look up the moment of inertia of a sphere about an axis through its center and find it is 2/5 that of a ring of the same mass and radius rotating about the same axis.

76. (Conservation of energy, torque, moment of inertia) You have been hired as a stunt advisor for a movie to be shot in Minnesota during the winter. The villain attempts to crush the hero by releasing a large sewer pipe from rest on a boat ramp. It rolls without slipping down the ramp and at the bottom of the ramp it encounters the horizontal, slick ice of a frozen stream. Having crossed the frozen stream, the pipe starts up a second ramp that is covered with slick ice. The hero is standing at the top of this ramp. The director wants the sewer pipe to almost reach her. Your assistant has measured the maximum height (above the frozen stream) that the center of the sewer pipe can reach. He has also measured the mass and radius of the pipe and the different angles that the two ramps make with the horizontal. You checked that frictional forces can be neglected on the slick ice. At what height do you tell the crew to place the center of the sewer pipe before releasing it on the first ramp?

77. (Conservation of energy, circular motion, moment of inertia) You are on a development team investigating a new design for computer magnetic disk drives. You have been asked to determine if the standard disk drive motor will be sufficient for the test version of the new disk. To do this you decide to calculate how much energy is needed to get the 6.4 cm diameter, 15 gram disk to its operating speed of 2000 revolutions per second. The test disk also has 4 different sensors attached to its surface. These small sensors are arranged at the corners of a square with sides of 1.2 cm. To assure stability, the center of mass of the sensor array is in the same position as the center of mass of the disk. The disk's axis of rotation also goes through the center of mass. You know that the sensors have masses of 1.0 grams, 1.5 grams, 2.0 grams, and 3.0 grams. The moment of inertia of your disk is one-half that of a ring of the same mass and radius rotating about the same axis.

78. (Force, torque, center of mass) The automatic flag raising system on a horizontal flagpole attached to the vertical outside wall of a tall building has become stuck. The management of the building wants to send a person crawling out along the flagpole to fix the problem. You have been asked whether or not this is possible. The flagpole is a 120 lb steel I-beam that is very strong and rigid. One end of the flagpole is attached to the wall of the building by a hinge so that it can rotate vertically. Nine feet away, the other end of the flagpole is attached to a strong, lightweight cable. The cable goes up from the flagpole at an angle of 30^{o} until it reaches the building where it is bolted

to the wall. The mechanic who will climb out on the flagpole weighs 150 lbs including equipment. From the specifications of the building construction, both the bolt attaching the cable to the building and the hinge have been tested to hold a force of 500 lbs. Your boss wants to know if the mechanic will be ok at the far end of the flagpole, nine feet from the building.

79. (Force, torque, moment of inertia) You have been asked to design a machine to move a large cable spool up a factory ramp at a constant speed. The spool is made of two 6.0 ft diameter disks of wood with iron rims connected together at their centers by a solid cylinder 3.0 ft wide and 5.0 ft long. Sometime later in the manufacturing process, cable will be wound around the cylinder. For now the cylinder is bare but the spool still weighs 200 lbs. Your plan is to attach a light weight string around the cylinder and pull the spool up the ramp with the string coming off the top of this cylinder. The spool will then roll without slipping up the ramp on its two outside disks. To finish the design you need to calculate how strong the string must be to pull the spool when it is moving up the ramp at a constant speed. The ramp has an angle of 27^{o} from the horizontal and the string will be parallel to the ramp.

80. (Force, torque, kinematics) You are working with an archeological team reconstructing the technique that an ancient civilization used to move heavy stone blocks along level ground on a high mountain plateau. The theory you are testing claims that the ancients pulled a block along a greased wooden road using a rope with one end attached to the block and the other end attached to a large rock. The large rock was then dropped off a nearby cliff. To better control the motion of the block, the rope passed over a large vertical wheel at the edge of the cliff supported by an axle through its center. The wheel was free to rotate vertically. To recreate the situation, you need to determine the force that the rope will exert on the block if it accelerates uniformly from rest and goes 150 m in 2.5 minutes. You will drop a 50 kg rock and use a wheel with a mass of 250 kg and a radius of 75 cm. The wheel is constructed of a heavy iron rim that contains essentially all of its mass supported by light wooden spokes.

81. (Force, torque, kinematics) While watching a mechanical clock mounted in a tower, you observe that the motion of the entire device is governed by a small lead ball hanging on one end of a string wound around a large pulley. As the ball descends, the pulley rotates and the string unwinds without slipping on the pulley. You estimate that the mass of the lead ball is 125 grams and the mass of the pulley is 2.5 kg. The pulley is shaped like a flat disc of radius 15 cm. You remember that the moment of inertia of a disk is half that of a ring of the same mass and radius when they rotate about an axis through their center. If the ball starts from rest, you wonder how far it moves in 2 s?

82. (Conservation of energy, rotation, moment of inertia, group) You are a member of a team designing a new device to help trucks go down steep mountain roads at a safe speed even if their brakes fail. The device is a flywheel, a large disk that rotates about its center and mounted in the truck. A system of gears attaches the flywheel to the truck's wheels so that when these gears are engaged, half of the truck's kinetic energy is in the flywheel. In order to design the flywheel, your team must know the forces on its structure. To help determine these forces, you decide to calculate the maximum radial and tangential components of the acceleration of any part of the flywheel when the truck starts from the top of a hill of a known height and goes down a straight road sloped at a known angle. The mass of the truck and radius of the flywheel have already been determined for this design. The mass of the flywheel is 5% that of the loaded truck. The moment of inertia of the flywheel is one half that of the same mass and radius ring rotating about its center. The flywheel gears can only be switched to the engaged position if both the truck and the flywheel are stopped.

83. (Conservation of angular momentum, moment of inertia, calculus) You have been asked to help evaluate a proposal to build a device to determine the speed of hockey pucks shot along the ice. The device consists of a rod that rests on the ice and is fastened to the ice at one end so that it is free to rotate horizontally. The free end of the rod has a small, light basket that will catch the hockey puck. The puck slides across the ice perpendicular to the rod and is caught in the basket. The rod then rotates. The designers claim that knowing the mass of the rod and puck, the length of the rod, and the rate of the rotation of the rod with the puck in the basket, you can calculate the speed of the puck as it moved across the ice before it hit the basket. To check their claim, you try to make the calculation?

84. (Conservation of energy, conservation of angular momentum, moment of inertia) You are helping to design the opening ceremony for the next winter Olympics. One of the choreographers envisions skaters racing out onto the ice and each one grabbing a very large ring (the symbol of the Olympics). Each ring is held horizontally at shoulder height by a vertical pole stuck into the ice. The pole is attached to the ring on its circumference so that the ring can rotate horizontally around the pole. The plan is to have a skater grab the ring at a point on the opposite side from where the pole is attached and, holding on, glide around the pole in a circle. You have been assigned the task of determining the minimum speed that the skater must have before grabbing the ring in terms of the radius of the ring, the mass of the ring, the mass of the skater, and the constant frictional force between the skates and the ice. The choreographer wants the skater and ring to go around the pole at least five times. The skater moves tangent to the ring just before grabbing it.

85. (Conservation of energy, conservation of angular momentum, moment of inertia) You have been asked to design a new stunt for the opening of an ice show. A small 50 kg skater glides down a ramp and along a short level stretch of ice. While gliding along the level stretch the skater bends to be as small as possible finally grabbing the bottom end of a large 180 kg vertical rod that is free to turn vertically about a axis through its center. The plan is to hold onto the 20 foot long rod while it swings the skater to the top. You have been asked to give the minimum height of the ramp. Doing a quick integral tells you that the moment of inertia of this rod about its center is 1/3 of what its moment of inertia would be if all of its mass were concentrated at one of its ends.

86. (Conservation of energy, conservation of angular momentum, moment of inertia, group) You are a member of a group designing an air filtration system for allergy suffers. To optimize its operation you need to measure the mass of the common pollen in the air where the filter will be used. You have an idea of how to do this using a micro-machine. You imagine that a pollen particle enters an opening in your device. Once inside the pollen is given a positive electric charge and accelerated by an electrostatic force to a speed of 1.4 m/s. The pollen then hits the end of a very small, bar that is hanging straight down from a pivot at its top. Since the bar has a negative charge at its tip, the pollen sticks to it as the bar swings up. Measuring the angle that the bar swings up would give the particle's mass. After the angle is measured, the charge of the bar is reversed, releasing that particle so that the device is ready for the next particle. Your friend insists it will never work and asks you to calculate the length of the bar needed. You assume you want an angle of about 2^o for a typical pollen particle of 4×10^{-9} grams. Your plan calls for a bar with a mass of 7×10^{-4} grams and a moment of inertial 1/3 as much as if all of its mass were concentrated at its end.

87. (Conservation of angular momentum, Conservation of energy, moment of inertia) In the physics lab your group did not get the result you expected when a metal ring was dropped onto a rotating plate. After some thought you wonder if there could be a significant amount of friction on the rotating shaft that would affect the result. You assume that this force is approximately constant, except perhaps just when the ring hits the disk. To measure that force, you drop the ring centered on the

disk that you get spinning about its center at 3.0 revolutions per second. The disk and ring go around 17 more times before coming to rest. The radii of the disk and shaft are 11 cm and 0.63 cm. The ring has an outside radius of 6.5 cm and inside radius of 5.5 cm. The moments of inertia (about the appropriate axis) for the disk, shaft, and ring are 5.1×10^{-3} kg m^2, 3.7×10^{-6} kg m^2, and 8.9×10^{-3} kg m^2 respectively.

88. (Gravitational force, circular motion, group) You found your physics course so interesting that you decided to get a job working in a research group investigating the ozone depletion at the Earth's poles. This group is planning to put an atmospheric measuring device in a satellite that will pass over both poles. To collect samples of molecules escaping from the upper atmosphere, the satellite will be put into a circular orbit 150 miles above the surface of the Earth, which has a radius of about 4000 miles. To adjust the instruments for the proper data taking rate, you need to calculate how many times per day the device will sample the atmosphere over the South pole. You do not remember the values of G or the mass of the Earth, but you do remember the acceleration with which objects fall at the surface of the Earth.

89. (Gravitational force, circular motion) While watching TV using your satellite receiver, you wonder if the satellite transmitting the signal stays over your town all of the time. If it does, how high must it be?

90. (Gravitational force, circular motion) As you notice a full moon rising, you wonder about the distance from the earth to the moon. You know that it takes about 28 days for the moon to go once around the earth and that the radius of the earth is about 4000 miles. You do not remember the universal gravitational constant, G, but you do know the acceleration of an object dropped near the surface of the earth.

91. (Gravitational force, circular motion, group) You are reading a magazine article about a satellite in orbit around the Earth that detects X-rays coming from outer space. The article states that the X-ray signal detected from one source, Cygnus X-3, has an intensity that changes with a period of 4.8 hours. This type of astronomical object emitting periodic signals is called a pulsar. One popular theory holds that the pulsar is a normal star (similar to our Sun) that orbits a much more massive neutron star. The period of the X-ray signal is then the period of the orbit. The article claims that the distance between the normal star and the neutron star is approximately the same as the distance between the Earth and our Sun. You realize that if this is correct, you can determine how much more massive the Cygnus X-3 neutron star is than our Sun. You don't know the distance from the earth to the Sun or the value of the Universal Gravitational Constant, but you do remember the period of the earth around the Sun.

92. (Gravitational force, Conservation of energy, conservation of angular momentum) You have a job with a research group investigating the destruction of the ozone layer in the atmosphere. They are planning to orbit a satellite to monitor the amount of chlorine ions in the upper atmosphere over North America. It has been determined that the satellite should collect samples at a height of 100 miles above the Earth's surface. At that height air resistance would make the amount of time the satellite would stay in orbit too short to be useful so an elliptical orbit is planned. This orbit would allow the satellite to be close to the Earth over North America, where data was desired, but farther from the Earth at a height of 1000 miles, out of most of the atmosphere, on the other side of our planet. To determine the apparatus needed to collect the data, you must calculate how fast the satellite traveling at its lowest point? You do not remember the universal gravitational constant, G, but you do know that the radius of the earth is about 4000 miles.

93. (Oscillations, Rotations) You are helping a friend build an experiment to test behavior modification techniques on rats. The design calls for an obstacle that swings across a path every second. To keep the experiment as inexpensive as possible, you decide to use a meter stick as the swinging obstacle. Where do you drill a hole in the meter stick so that, when hanging by a nail through that hole, it will do the job.

94. (Oscillations, Rotations) Your friend is trying to construct a pendulum clock for a craft show and asks you for some advice. The pendulum will be a very thin, light wooden bar with a thin, but heavy, brass ring fastened to one end. The length of the rod is 80 cm and the diameter of the ring is 10 cm. For aesthetic reasons, the plan is to drill a hole in the bar to place the axis of rotation 15 cm from one end. You have been asked to calculate the period of this pendulum.

95. (Oscillations, Torque, Rotations) You have been asked to help design a system for applying resistive paint to plastic sheeting to make containers that protect sensitive electronic components from electric charges. The object used to apply the paint is a solid cylindrical roller. The roller is pushed back and forth over the plastic sheeting by a horizontal spring attached to a yoke that is attached to an axle through the center of the roller. The other end of the spring is attached to a fixed post. To apply the paint evenly, the roller must roll without slipping over the surface of the plastic. In order to determine how fast the process can proceed, you have been assigned to calculate how the oscillation frequency of the roller depends on its mass, radius and the stiffness of the spring. You know that the moment of inertia of a solid cylinder with respect to an axis through its center is 1/2 that of a ring with the same axis.

96. (Oscillations, Conservation of Energy, Rotations, group) You have a job at a software company that is producing a program simulating accidents in a modern commuter railroad station. Your task is to determine the response of a safety system to prevent a railroad car from crashing into the station. In the simulation, a coupling fails causing a passenger car to break away from a train and roll into the station. Because the brakes on the passenger car have failed, it cannot stop. The safety system at the end of the track is a large horizontal spring with a hook that will grab onto the car when it hits preventing the car from crashing into the station platform. After the car hits the spring, your program must calculate the frequency and amplitude of the car's oscillation based on the specifications of the passenger car, the specifications of the spring, and the speed of the passenger car. In your simulation, the wheels of the car are disks with a significant mass and a moment of inertia half that of a ring of the same mass and radius. At this stage of your simulation, you ignore any energy dissipation in the car's axle or in the flexing of the spring, and the mass of the spring.

97. (Waves) You have a summer job working on an oil tanker in the waters of Alaska. Your Captain knows that the ship is near an underwater ridge that could tear the bottom out. He estimates that it is about 6 km straight ahead of the ship. The ship's instruments tell him the ship is moving through still water at a speed of 31 km/hr but the captain cannot take any chances. He asks you to use the sonar to check the ship's speed. A sonar signal is sent out with a frequency of 980 Hz, bounces off the underwater ridge, and is detected on the ship. If the ship's speed indicator is correct, what frequency should you detect? You use your trusty Physics text to find the speed of sound in seawater is 1522 m/s.

98. (Waves) You've been hired as a technical consultant to the police department to design a detector-proof device that measures the speed of vehicles. You know that a moving car emits a variety of characteristic sounds. You decide to make a very small device to be placed in the center of the road that will detect a specific frequency emitted by the car as it approaches and then measure the change in that frequency as the car moves off in the other direction. A microprocessor in the device will

then compute the speed of the vehicle. To write the program for the microprocessor you need an equation for the speed of the car using the data received by the microprocessor. Your may also include necessary physical constants.

99. (Standing waves) You have a job in a biomedical engineering laboratory working on technology to enhance hearing. You have learned that the human ear canal is essentially an air filled tube approximately 2.7 cm long that is open on one end and closed on the other. You wonder if there is a connection between hearing sensitivity and the standing waves that can exist in the ear canal. To test your idea, you calculate the lowest three frequencies of the standing waves in the ear canal. From your Physics textbook, you find that the speed of sound in air is 343 m/s.

Appendix C
Context-rich Electricity & Magnetism Problems

1. (Vector forces, Coulomb force, discrete charge, group) While working in a University research laboratory your group is given the job of testing an electrostatic scale, used to precisely measure the weight of small objects. The device consists of two light strings attached to a support so that they hang straight down. A different object is attached to the other end of each string. One of the objects has a very accurately known weight while the other object is the unknown. A power supply is slowly turned on to give each object an electric charge. This causes the objects to slowly move away from each other. When the power supply is kept at its operating value, the objects come to rest at the same horizontal level. At that time, each of the strings makes a different angle with the vertical and that angle is measured. To test your understanding of the device, you decide to calculate the weight of an unknown sphere from the measured angles and the weight of a known sphere. Your known is a standard sphere with a weight of 0.050 N supported by a string that makes an angle of 10.00^o with the vertical. The unknown sphere's string makes an angle of 20.00^o with the vertical.

2. (Coulomb force, discrete charge) While studying about the importance of hydrogen atoms in organic molecules, you wonder about the energy states of a hydrogen atom. Using the planetary model of an atom, you decide to calculate the kinetic energy of the electron in a circular orbit around the proton as a function of the radius of the orbit and the properties of the electron and proton.

3. (Coulomb force, discrete charge) You are working for a chemical company with a group trying to produce new polymers. Your boss has asked you to help determine the structure of part of a polymer chain. She wants to know where a chlorine ion of effective charge -e would situate itself near a carbon dioxide ion. The carbon dioxide ion is composed of 2 oxygen ions each with an effective charge -2e and a carbon ion with an effective charge +3e. You have been told to assume that these ions are arranged in a line with the carbon ion sandwiched midway between the two oxygen ions. The distance between each oxygen ion and the carbon ion is 3.0×10^{-11} m. Assuming that the chlorine ion is on a line perpendicular to the axis of the carbon dioxide ion and that the line goes through the carbon ion, where is its equilibrium position? For simplicity, you assume that the carbon dioxide ion does not deform in the presence of the chlorine ion. Looking in your trusty physics textbook, you find the charge of the electron is 1.60×10^{-19} C

4. (Coulomb force, continuous charge, group) You have a part time job in a research laboratory building equipment for experiments on the new space station. Because it is expensive to send heavy equipment into orbit, your group is investigating ideas for a lighter cathode ray tube. The design calls for an electron acceleration mechanism consisting of two equal radius parallel rings separated by twice their radius. Each ring is given an equal magnitude charge by a power supply. Electrons originate from a wire at the center of one of the rings and are accelerated along the axis between the centers of the two rings. To estimate the variation of the electron's motion, you decide to calculate the ratio of the electron's acceleration when it is at the center of one of the rings to its acceleration when it is midway between the two rings.

5. (Electric field, continuous charges) You are helping to design a new electron microscope to investigate the structure of the HIV virus. A new device to position the electron beam consists of a charged circle of conductor. This circle is divided into two half circles separated by a thin insulator so that half of the circle can be charged positively and half can be charged negatively. The electron beam will go through the center of the circle. To complete the design your job is to

calculate the electric field in the center of the circle as a function of the amount of positive charge on the half circle, the amount of negative charge on the half circle, and the radius of the circle.

6. (Electric field, continuous charges, calculus) You have a summer job with the telephone company investigating the vulnerability of underground telephone lines to natural disasters. Your task is to write a computer program that will be used determine the possible harm to a telephone wire from the high electric fields caused by lightning. The underground telephone wire is supported in the center of a long, straight steel pipe that protects it. When lightening hits the ground it charges the steel pipe. Since you think that the largest field on the wire will be where it leaves the end of the pipe, you calculate the electric field at that point as a function of the length of the pipe, the radius of the pipe, and the charge on the pipe.

7. (Conservation of energy, Coulomb potential) While sitting in a restaurant with some friends, you notice that some "neon" signs are different in color than others. One of your friends, an art major who makes sculpture from these things, tells you that the color of the light depends on which gas is in the tube. All "neon" signs are not made using neon gas. You know that the color of light tells you its energy. Red light is a lower energy than blue light. Since the light is given off by the atoms that make up the gas, the different colors must depend on the structure of the gas atoms. Another one of your friends has read about the Bohr theory that states electrons are in uniform circular motion around a heavy, motionless nucleus in the center of the atom. This theory also states that the electrons are only allowed to have certain orbits. When an atom changes from one allowed orbit to a lower one, it radiates light as required by the conservation of energy. Since only certain orbits are allowed, only light of certain energies (colors) can be emitted. You decide to explore the theory by calculating the energy of light emitted by a hydrogen atom when an electron makes a transition from one allowed orbit to another. You remember that the proton has a mass 2000 times that of an electron. When you get home you look in your textbook and find the electron mass is 9×10^{-31} kg and its charge is 1.6×10^{-19} C. The radius of the smallest allowed electron orbit for hydrogen is 0.5×10^{-10} meters. The next allowed orbit has a radius 4 times as large as the smallest orbit.

8. (Conservation of energy, Coulomb potential) You have a job working in a cancer research laboratory. Your team is trying to construct a gas laser that will give off light of an energy that will pass through the skin but be absorbed by cancer tissue. You know that an atom emits a photon (light) when an electron goes from a higher energy orbit to a lower energy orbit. Only certain orbits are allowed in a particular atom. To begin the process, you calculate the energy of photons emitted by a Helium ion in which the electron changes from an orbit with a radius of 0.40 nanometers to another orbit with a radius of 0.26 nanometers.

9. (Conservation of energy, Coulomb potential) You have been asked to write operating instructions for a new device that deposits ions on the surface of silicon to make better semiconductors. The device accelerates the ions in a straight line from rest as they pass through a potential difference between two conductive parallel plates. The plates have a hole through them to let the ions go through. After leaving the parallel plates, the ions travel a long distance to the silicon. The small piece of silicon is at the center of circle with six small electrodes with the same charge at equal distances around the circumference. The plane of the circle is perpendicular to the ion beam. The entire device is sealed so that the ion travels in a vacuum. You need to determine the relationship between the charge on each electrode and the potential difference between the two parallel plates so that the ion stops on the silicon surface. You may need to know the distance of each electrode from

the silicon, the distance of each electrode to the next electrode, the charge of the ion, the mass of the ion, and the distance between the parallel plates and the silicon.

10. (Conservation of energy, Coulomb force, continuous charge, group, calculus) You have been asked to determine if a proposed apparatus to implant ions in silicon to make better semiconductors will work. The apparatus slows down positive He ions that have a charge twice that of an electron (He^{++}). It consists of a circular wire that is connected to a power supply so that it becomes a negatively charged circle. An ion with a velocity of 200 m/s on a trajectory perpendicular to the plane of the circle is shot out from the center of the circle. The wire circle has a radius of 3.0 cm and can have a charge up to 8.0 μC. The sample into which the ion is to be implanted is to be placed 2.5 mm from the charged circle. You look up the charge of an electron and mass of the helium and find them to be 1.6×10^{-19} C and 6.7×10^{-27} Kg.

11. (Conservation of energy, Coulomb potential, continuous charge, calculus) You have been hired by a company engaged in developing faster computer chips by implanting certain ions into the silicon that makes up the chips. Your job is to help design a new device to do this. You start with a thin bar of silicon that is given a uniform positive charge. The device will then direct negative ions from an ion source along a path aligned with the axis of the silicon bar so that the ions hit the end of the bar. To control the process, you need to know the acceleration of the ion as a function of its distance from the end of the silicon rod that the ions hit when you are given the properties of the ion and the charge and size of the silicon rod.

12. (Conservation of energy, Coulomb potential, group) You have a job in a University laboratory that is planning experiments to study the forces between nuclei in order to understand the energy output of the Sun. In one of these experiments, alpha particles are shot from a Van de Graaf accelerator at a sheet of lead. The alpha particle is the nucleus of a helium atom and is made of 2 protons and 2 neutrons. The lead nucleus is made of 82 protons and 125 neutrons. The mass of the neutron is almost the same as the mass of a proton. You have been told to calculate the potential difference between the two ends of the Van de Graaf accelerator so that the alpha particle should come into contact with a lead nucleus. The alpha particle has a radius of 1.0×10^{-13} cm and the lead nucleus has a radius 4 times larger.

13. (Conservation of energy, Coulomb potential, Gravitational potential) NASA has asked your team of rocket scientists about the feasibility of a new satellite launcher that will save rocket fuel. NASA's idea is basically an electric slingshot that consists of 4 electrodes arranged in a horizontal square 5 m on a side. The satellite is placed 15 m directly under the center of the square. A power supply will provide each of the four electrodes with the same charge and the satellite with an opposite charge 4 times larger. When the satellite is released from rest, it moves up and passes through the center of the square. At the instant it reaches the square's center, the power supply is turned off and the electrodes are grounded, giving them a zero electric charge. To test this idea, you decide to use energy considerations to calculate the electrode charge necessary to get a 100 kg satellite to an orbit height of 300 km. In your physics text you find the mass of the Earth to be 6.0×10^{24} kg.

14. (Conservation of energy, Coulomb potential, continuous charge, calculus) You have been asked to evaluate a design for accelerating electrons in a precise electron microscope. Pulses of electrons are shot into the center of a 3.0 cm diameter wire ring along the axis perpendicular to the plane of the ring. When an electron reaches the center of the ring, the ring is rapidly charged to 4.5 mC. You want to know the speed of the electron when it is very far away from the ring if it starts with a speed

of 6.0 m/s at the center. From you Physics book you find the charge of an electron is 1.6×10^{-19} C and its mass is 9.1×10^{-31} kg.

15. (Conservation of energy, Coulomb potential, continuous charge, calculus, group) You have been asked to evaluate a new electron gun design for producing low velocity electrons. The electrons have a 20 cm path from the heating element that emits them to the end of the gun. The electrons must reach the end of the gun with a speed of 10^2 m/s. After leaving the heating element, the electrons pass through a 5.0 mm diameter hole in the center of a 3.0 cm diameter charged circular disk. The disk's positive charge is kept at 3.0 μC/m^2. The heating element is a spherical electrode 0.10 mm in diameter whose negative charge can be adjusted up to -0.10 mC. There is 1.0 cm between the heating element and the hole in the disk. Your co-worker thinks that the hole in the disk is too large to ignore in your calculations. Using your physics text you find that the mass of the electron is 9.11×10^{-31} kg and its charge is 1.6×10^{-19} C.

16. (Coulomb potential, Electric force) You've been hired to design the hardware for an ink jet printer. Your printer uses a deflecting electrode to cause charged ink drops to form letters on a page. Uniform ink drops of about 30 microns radius are charged while being sprayed out towards the page at a speed of about 20 m/s. Along the way to the page, they pass into a region between two deflecting plates that are 1.6 cm long. The deflecting plates are 1.0 mm apart and charged to 1500 volts. You measure the distance from the edge of the plates to the paper and find that it is one-half inch. Assuming an uncharged droplet forms the bottom of the letter, how much charge is needed on the droplet to form the top of a letter 3 mm high

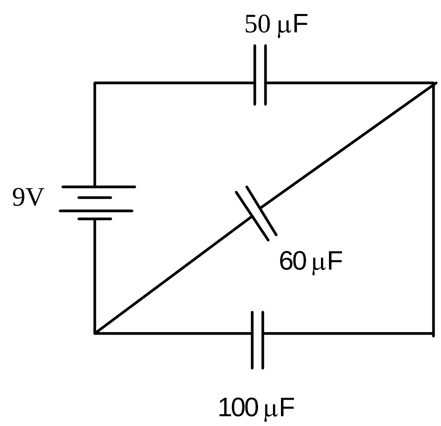
17. (Gauss' Law, conservation of energy, Coulomb potential, calculus) You are working in cooperation with the Public Health Department to design a device to measure particles from auto emissions. The average particle has a mass of 6.0×10^{-9} kg. When it enters the device it is exposed to ultraviolet radiation that knocks off electrons so that it has a charge of $+3.0 \times 10^{-8}$ C. This average particle is then moving at a speed of 900 m/s and is 15 cm from a very long negatively charged wire with a linear charge density of -8.0×10^{-6} C/m. The detector for the particle is located 7.0 cm from the wire. In order to design the proper kind of detector, you need to know the speed that an average emission particle hits the detector. They tell you that an average emission particle has a mass of 6.0×10^{-9} kg.

18. (Conservation of energy, Coulomb potential, Gauss' Law, calculus) You have a summer job in a research laboratory with a group investigating the possibility of producing power from fusion. The device being designed confines a hot gas of positively charged ions, called plasma, in a very long cylinder with a radius of 2.0 cm. The charge density of the plasma in the cylinder is 6.0×10^{-5} C/m^3. Positively charged Tritium ions are to be injected into the plasma perpendicular to the axis of the cylinder in a direction toward the center of the cylinder. Your job is to determine the speed that a Tritium ion should have when it enters the plasma cylinder so that its velocity is zero as it reaches the axis of the cylinder. Tritium is an isotope of Hydrogen with one proton and two neutrons. You look up the charge of a proton and the mass of tritium in your Physics text to be 1.6×10^{-19} C and 5.0×10^{-27} Kg.

19. (Capacitance, potential, Gauss' Law, group, calculus) Your team is designing an inexpensive emergency electrical system for a rural hospital. Someone has suggested storing energy in large capacitors made from two thin walled metal pipes. The pipes would be concentric and of different radii but the same length. The idea is to first charge the capacitor by connecting each pipe to the opposite terminals of a power supply. After each pipe has its maximum charge, the power supply is

disconnected. One of the engineers on the team has an idea to increase the energy stored in a capacitor by changing its capacitance after it is charged. After the power supply is disconnected, a mechanical device would insert a third concentric metal thin walled pipe between the original two but not touching them. To evaluate the usefulness of this idea, you decide to calculate the ratio of the capacitance of the final three pipe configuration to that of the original two pipe configuration as a function of the size of the pipes.

20. (Capacitance) As part of your summer job as a design engineer at an electronics company, you have been asked to evaluate the circuit shown below to determine whether a dangerous amount of energy is stored in the capacitors.



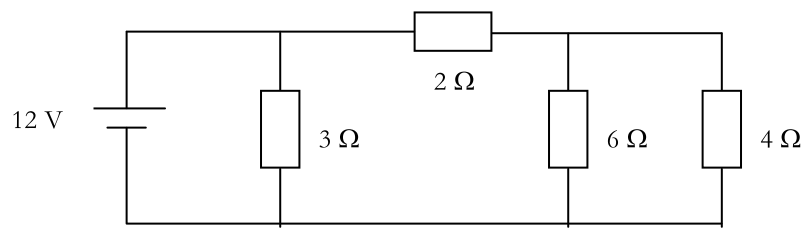
21. (Resistance, Ohm's law) You have a summer job as an assistant technician for a telephone company in California. During a recent earthquake, a 1.0-mile long underground telephone line is crushed at some point. This telephone line is made up of two parallel copper wires of the same diameter and same length, which are normally not connected. At the place where the line is crushed, the two wires make contact. Your boss wants you to find this place so that the wire can be dug up and fixed. You disconnect the line from the telephone system by disconnecting both wires of the line at both ends. You then go to one end of the line and connect one terminal of a 6.0-V battery to one wire, and the other terminal of the battery to one terminal of an ammeter. When the other terminal of the ammeter is connected to the other wire, the ammeter shows that the current through the wire is 1.0 A. You then disconnect everything and travel to the other end of the telephone line, where you repeat the process and find a current of 1/3 A.

22. (Conservation of energy, power) You are working with a company to design a new, 700-foot high, 50-story office building. The owner has discovered that it would take the 6500-lb loaded elevator one minute to rise 20 stories and thinks this is too long for these busy executives to spend in an elevator. You have been told to speed up the elevator and decide to buy a bigger power supply for it. You find a supply that it is the same as the old one except that it outputs twice the voltage. Now you must calculate the operating expenses of the new power supply. You estimate that while the elevator runs at maximum speed, the whole system, including the power supply, is 60% efficient. The cost of electricity is $0.06 per kilowatt-hour.

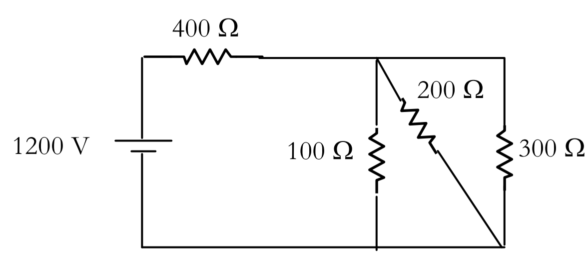
23. (Conservation of energy, power) You are having some friends over for dinner and decide to cook spaghetti. You start by boiling 5.0 kg of water. To decide when to start the water, you need to estimate how long it will take to get to its boiling point. Your electric stove is a 200-ohm device that

operates at 120 volts. Checking your physics book you find that water has a specific heat capacity of 4200 J/(kg ^{o}C) and heat of vaporization is 2.3 x 106 J/kg.

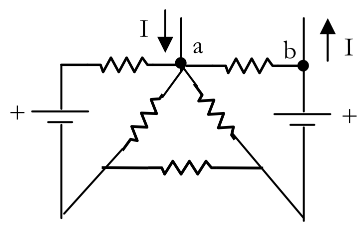
24. x(Circuits, power, Ohm's law) You are working in a group designing electronics for use in an underground neutrino experiment. One of your concerns is that the power dissipated by the circuit since will cause the laboratory temperature to increase affecting the performance of the apparatus and the comfort of the people. You have determined that most of the problem comes from the part of the circuit shown below. You decide to calculate the total power dissipated by all of the resistors together as well as the power dissipated by the 6 ohm resistor.



25. x(Circuits, power, Ohm's law, group) You are on a safety team reviewing a large chemical plant. To increase the speed of a chemical reaction, a tank of potentially explosive liquid is heated using the system shown in the diagram below. A 400 ohm heating element's temperature is controlled by the other three resistors in the circuit. The power is supplied by a 1200 volt power supply. To protect against excessive heating, the heating element and the 200 ohm resistor are monitored by infrared sensors. If something goes dangerously wrong, the ratio of the power dissipated by these resistors will become large. If that ratio goes over 10,000 an alarm will sound, all power is cut off, and the factory is evacuated. To determine the sensitivity of this safety system, you are asked to calculate that ratio during normal operations.

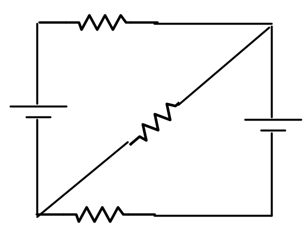


26. x(Circuits, Ohm's law, group) You have been asked to check a circuit that will become part of a new communications satellite to see if it will function as intended. This circuit contains batteries that will power the satellite when it is in the Earth's shadow. When the satellite is in the light, its solar panels are intended to charge up the batteries by supplying a constant current at points a and b as shown in the circuit below. You know the properties of the two identical batteries and the identical resistors.

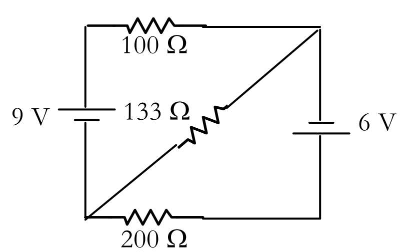


27. x(Circuits, Ohm's law, group) You are on a team designing a new electric car. The amount of current that the battery supplies to the electric motor controls the speed of the car. To make a simple current control, you connect a resistive wire in series with the battery. One terminal of the motor is then connected to one terminal of the battery. The other terminal of the motor is connected to a point on the resistive wire that can be adjusted. This connection divides the resistive wire into two resistors whose ratio can be changed by moving the point of contact on the resistive wire. To see if you can get enough power from this arrangement, you decide to determine how the current through the motor depends on the properties of the battery and the electric motor, the total resistance of the resistive wire, and the ratio of resistances into which it is divided.

28. x(Circuits, power, Ohm's law) You have been assign to determine the rate that batteries run down when operating a new portable amplifier for cell phones. You have analyzed that part of the circuit and find it is equivalent to the one below consisting of three identical resistors and two identical batteries. You decide to calculate the rate that each battery runs down as a function of the properties of the batteries and the resistors.



29. x(Circuits, power, Ohm's law) As part of your summer job at an electronics company, you have been asked to evaluate the circuit shown below. The resistors are rated at 0.5 Watts, which means they burn-up if their power output exceeds 0.5 Watts. Is the 100W resistor safe?



30. (Magnetic force, electric potential) You decide to relax by watching some TV. After a few minutes, you are bored and your mind starts drifting. You think about how the TV picture is generated. In a typical color picture tube for a TV, the electrons start from a cathode at the back of the tube. Near that cathode the electrons are accelerated through about 20,000 volts then go at a constant speed to hit the picture tube screen about 1.5 ft away. On the screen is a grid of color dots about 1/100 inch apart. When electrons hits one of them, the dot glows the appropriate color producing the color

picture. You know that the Earth's magnetic field is different at different places in the world and wonder if the TV's location has an effect on the picture. You remember that a typical value for the Earth's magnetic field is 0.5 Gauss. In your Physics text you find that the mass of a proton is 1.67 x 10^{-27} kg and its charge is 1.6 x 10^{-19} C.

31. (Magnetic force, group) You have a summer job in the medical school research group investigating short lived radioactive isotopes to use in fighting cancer. Your group is working on a way of transporting alpha particles (Helium nuclei) from their source to another room where they will collide with other material to form the isotopes. Your job is to design that part of the transport system that will deflect the beam of alpha particles (mass of 6.64 x 10^{-27} kg, charge of 3.2 x 10^{-19} C) through an angle of 90^{o} by using a magnetic field. The beam will be traveling horizontally in an evacuated tube. To make the alpha particles turn 90 degrees, you decide to use a dipole magnet that provides a uniform vertical magnetic field of 0.030 T. Your design has a tube of the appropriate shape between the poles of the magnet. Before you submit your design for consideration, you must determine how long the alpha particles will spend in the uniform magnetic field while making the 90^{o}-turn.

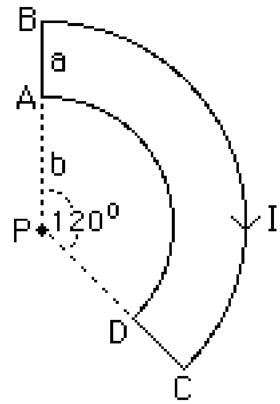
32. (Magnetic force) Your team has been told to develop a method for keeping charged particles from damaging the Space Telescope's sensitive electronic equipment. The Space Telescope is essentially a hollow cylinder of diameter of 2.0 meters with a cover on one end. Any particle entering parallel to the axis of the telescope could damage the equipment. Your plan is to turn on a uniform magnetic field outside the end of the telescope whenever its cover is opened. The design goal is for a 0.010 Tesla field to deflect a particle with a charge of 79 protons, a mass 197 times that of a proton, and a velocity of 1.0 x 10^{6} $^{m}/_{s}$ from a path down the axis of the cylinder to hit the wall at an angle of 20 degrees. Assuming no magnetic field enters the telescope, how far from the telescope opening must the magnetic field extend into space to achieve this goal? In your Physics text you find that the mass of a proton is 1.67 x 10^{-27} kg and its charge is 1.6 x 10^{-19} C.

33. (Magnetic force, calculus, group) You have been asked to evaluate the design for a new electric train that uses the Earth's magnetic field to propel it. A current goes through one rail of the track, up through one of the train's wheels, through the wheel's horizontal axle, through the wheel at the other end of the axle, and then through the other rail of the track. To check the feasibility of this design, you decide to calculate the speed of the train as a function of the mass of the train, the current through the rails, the length of the axle, the radius of the wheels, and the time elapsed since the train started up. You also look up the magnitude of the vertical component of the Earth's magnetic field.

34. (Magnetic force, Ampere's Law, calculus) You have been asked to review a design for an electron beam device for making electronic microcircuits. During part of its trajectory, the electron beam runs parallel to a section of a long wire at a distance of 5.0 cm from the wire. To determine the effect on the beam, you decide to calculate the force on an electron when a current of 40 amps is switched through the wire. You know that the mass of an electron is 9.1 x 10^{-31} kg, and it is moving at 3.0 x 10^{7} m/s.

35. (Magnetic force, Ampere's Law, calculus, group) You are designing the supports for a high voltage power line to bring electricity to the city from a dam. The two copper cables comprising the power line will run side by side. Each cable hangs from light-weight vertical non-conductive straps, 80 cm long, attached to concrete poles. Before current is turned on, the straps hang straight down supporting the cables so that they are separated by 10 cm. For structural reasons, the cables cannot be separated by more than 15 cm or less than 5 cm. You need to specify the maximum current that your design will allow. The copper cables that will be used have a weight of 100 N per meter.

36. (Biot-Savart Law, calculus) You are continually having troubles with the CRT screen of your computer and wonder if it is due to magnetic fields from the power lines running in your building. A blueprint of the building shows that the nearest power line is as shown below. Your CRT screen is located at point P. You decide to calculate the magnetic field at P as a function of the current I and the distances a and b. Segments BC and AD are arcs of concentric circles. Segments AB and DC are straight line segments.



37. (Magnetic force, Biot-Savart Law, torque, calculus) You are investigating ways to exert torque on parts of nanomachines. One possible device is a large multi-turn circular coil of wire that conducts a current. At the center of that large coil, a very small wire square that is mounted so that it rotates about one of its sides. The current in the small coil will be different than the current in the large coil. Determine the maximum torque on the square in terms of the size and number of turns of the large coil, the size of the wire square, and the current through each.

38. (Hall Effect, or Magnetic force, Electric force, potential, calculus, group) You are helping to design a new device to measure the blood flow through arteries as a diagnostic device for heart surgery. The idea is to use microsurgery to attach small wires to opposite sides of an artery. The wires are brought outside of the body and attached to a voltmeter. Helmholz coils outside the body are then used to create a uniform magnetic field at the artery that is perpendicular to the blood flow and to the direction between the wire attachment points. After the magnetic field is turned on, the ions in the blood come to equilibrium such that there is a constant electric field in the artery. You decide to calculate the smallest velocity of blood flow that this device can measure for an artery of diameter 3 mm in a magnetic field of 0.5 T if the voltmeter is accurate to 1 microvolt.

39. (Magnetic force, Faraday's law, Ohm's law, calculus, group) You have a summer job working at a company developing systems to safely move large loads down ramps. The safety system your team is investigating consists of a conducting bar sliding on two parallel conducting rails that run down the ramp. The bar is perpendicular to the rails and is in contact with them. At the bottom of the ramp, the two rails are connected together. A magnet with poles above and below the ramp creates a vertical uniform magnetic field. Before setting up a laboratory test, you decide to calculate the velocity of the bar sliding down the ramp as a function of the mass of the bar, the strength of the magnetic field, the angle of the ramp from the horizontal, the distance between the tracks, and the resistance of the bar. Assume that all of the other conductors in the system have a much smaller resistance than the bar.

40. (LC circuit, calculus) You are evaluating the design of a circuit to use a coil of wire to generate a magnetic field. In this circuit, a coil of wire, a capacitor, and a battery with known properties are connected together in series. You worry how the current through the coil varies with time. For simplicity, you assume that the coil and all wires in the circuit have a negligible resistance.

APPENDIX D
PROBLEM SOLVING LABORATORIES

The following are examples of laboratories that emphasize problem solving integrated with the measurement of physical systems. These examples are taken from the first year physics course.

LABORATORY I:
DESCRIPTION OF MOTION IN ONE DIMENSION

In this laboratory you will study the motion of objects that can go in only one dimension; that is, along a straight line. You will be able to measure the position of these objects by capturing video images on a computer. Through these measurements and their analysis you can investigate the relationship of quantities that are useful to describe the motion of objects. Determining these kinematic quantities (position, time, velocity, and acceleration) under different conditions allows you to improve your intuition about their quantitative relationships. In particular, you should be able to determine which relationships depend on specific situations and which apply to all situations.

There are many possibilities for the motion of an object. It might either move at a constant speed, speed up, slow down, or have some combination of these motions. In making measurements in the real world, you need to be able to quickly understand your data so that you can tell if your results make sense to you. If your results don't make sense, then either you have not set up the situation properly to explore the physics you desire, you are making your measurements incorrectly, or your ideas about the behavior of objects in the physical world are incorrect. In any of the above cases, it is a waste of time to continue making measurements. You must stop, determine what is wrong and fix it. If it is your ideas that are wrong, this is the time to correct them by discussing the inconsistencies with your partners, rereading your text, or talking with your instructor. Remember, one of the reasons for doing physics in a laboratory setting is to help you confront and overcome your incorrect preconceptions about physics, measurements, calculations, and technical communications. Because most people are much faster at recognizing patterns in pictures than in numbers, the computer will graph your data **as you go along**.

OBJECTIVES:

After you successfully complete this laboratory, you should be able to:

- Describe completely the motion of any object moving in one dimension using the concepts of position, time, velocity, and acceleration.
- Distinguish between average quantities and instantaneous quantities when describing the motion of an object.
- Express mathematically the relationships among position, time, velocity, average velocity, acceleration, and average acceleration for different situations.
- Graphically analyze the motion of an object.
- Begin using technical communication skills such as keeping a laboratory journal and writing a laboratory report.

PREPARATION:

Read Tipler: Chapter 2. Also read Appendix D, the instructions for doing video analysis. Before coming to the lab you should be able to:

- Define and recognize the differences among these concepts:
    - Position, displacement, and distance.
    - Instantaneous velocity and average velocity.
    - Instantaneous acceleration and average acceleration.
- Find the slope and intercept of a straight-line graph. If you need help, see Appendix C.
- Determine the slope of a curve at any point on that curve. If you need help, see Appendix C.
- Determine the derivative of a quantity from the appropriate graph.
- Use the definitions of sin θ, cos θ, and tan θ with a right triangle.

PROBLEM #1:
CONSTANT VELOCITY MOTION

These laboratory instructions may be unlike any you have seen before. You will not find worksheets or step-by-step instructions. Instead, each laboratory consists of a set of problems that you solve **before coming to the laboratory** by making an organized set of decisions (a problem solving strategy) based on your initial knowledge. The instructions are designed to help you examine your thoughts about physics. These labs are your opportunity to compare your ideas about what "should" happen with what really happens. The labs will have little value in helping you learn physics unless you take time to predict what will happen before you do something. While in the laboratory, <u>take your time</u> and try to answer all the questions in this lab manual. In particular, the **exploration** questions are important to answer before you make measurements. Make sure you complete the laboratory problem, including all analysis and conclusions, before moving on to the next one.

Since this design may be new to you, this first problem contains both the instructions to explore constant velocity motion and an explanation of the various parts of the instructions. The explanation of the instructions is in this font and is preceded by the double, vertical lines seen to the left.

Why are we doing this lab problem? How is it related to the real world? In the lab instructions, the first paragraphs describe a possible situation that raises the problem you are about to solve. This emphasizes the application of physics in solving real-world problems.

To earn some extra money, you have taken a job as a camera operator for the Minneapolis Grand Prix automobile race. Since the race will be simulcast on the Internet, you will be using a digital video camera that stores the images directly on a computer. You notice that the image is distorted near the edges of the picture and wonder if this affects the measurement of a car's speed from the video image. You decide to model the situation using a toy car, which moves at a constant velocity.

?

> Does the measured speed of a car moving with a constant velocity depend on the position of the car in a video picture?

The question, framed in a box and preceded by a question mark, defines the experimental problem you are trying to solve. You should keep the question in mind as you work through the problem.

EQUIPMENT

To make a prediction about what you expect to happen, you need to have a general understanding of the apparatus you will use before you begin. This section contains a **brief** description of the apparatus and the kind of measurements you can make to solve the laboratory problem. The details should become clear to you as you use the equipment.

For this problem, you will use a motorized toy car, which moves with a constant velocity on an aluminum track. You will also have a stopwatch, a meter stick, a video camera and a computer with a video analysis application written in LabVIEW™ (described in Appendix D) to help you analyze the motion. In the computer the LabVIEW™ application programs include VIDEOPLAYER and VIDEOTOOL.

PREDICTION

Everyone has his/her own "personal theories" about the way the world works. One purpose of this lab is to help you clarify your conceptions of the physical world by testing the predictions of your personal theory against what really happens. For this reason, you will always predict what will happen *before* collecting and analyzing the data. **Your prediction should be completed and written in your lab journal *before* you come to lab.** The "Method Questions" in the next section are designed to help you determine your prediction and should also be completed before you come to lab. This may seem a little backwards. **Although the prediction question is given before the method questions, you should complete the method questions before making the prediction.** The prediction question is given first so you know your goal.

Spend the first few minutes at the beginning of the lab session comparing your prediction with those of your partners. Discuss the reasons for any differences in opinion. It is not necessary that your predictions are correct, but it is necessary that you understand the basis of your prediction.

How would each of the graphs of position-versus-time, velocity-versus-time, and acceleration-versus-time show a distortion of the position measurement? Sketch these graphs to illustrate your answer. How would you determine the speed of the car from each of the graphs? Which method would be the most sensitive technique for determining any distortions? Appendix B might help you answer this question.

Sometimes, as with this problem, your prediction is an "educated guess" based on your knowledge of the physical world. There is no way to calculate an exact answer to this problem. For other problems, you will be asked to use your knowledge of the concepts and principles of physics to calculate a mathematical relationship between quantities in the experimental problem.

WARM UP QUESTIONS

Method Questions are a series of questions intended to help you solve the experimental problem. They either help you make the prediction or help you plan how to analyze data. **Warm Up Questions should be answered and written in your lab journal *before* you come to lab.**

To determine if the measured speed is affected by distortion, you need to think about how to measure and represent the motion of an object. The following questions should help with the analysis of your data.

1. How would you expect an *instantaneous velocity-versus-time graph* to look for an object moving with a constant velocity? Make a rough sketch and explain your reasoning. Write down the equation that describes this graph. If this equation has any constant quantities in it, what are the units of those constant quantities? What parts of the motion of the object does each of these constant quantities represent? For a toy car, what do you estimate should be the magnitude of those quantities? How would a distortion affect this graph? How would it affect the equation that describes the graph? How will the uncertainty of your position measurements affect this graph? How might you tell the difference between uncertainty and distortion?

2. How would you expect a *position-versus-time graph* to look for an object moving with a constant velocity? Make a rough sketch and explain your reasoning. What is the relationship between this graph and the instantaneous velocity versus time graph? Write down the equation that describes this graph. If this equation has any constant quantities in it, what are the units of those constant quantities? What parts of the motion of the object does each of these constant quantities represent? For a toy car, what do you estimate should be the magnitude of those quantities? How would a distortion affect this graph? How would it affect the equation that describes the graph? How will the uncertainty of your position measurements affect this graph? How might you tell the difference between uncertainty and distortion?

3. How would you expect an *instantaneous acceleration-versus-time graph* to look for an object moving with a constant velocity? Make a rough sketch and explain your reasoning. Write down the equation that describes this graph. If this equation has any constant quantities in it, what are the units of those constant quantities? What parts of the motion of the object does each of these constant quantities represent? For a toy car, what do you estimate should be the magnitude of those quantities? How would a distortion affect this graph? How would it affect the equation that describes the graph? How will the uncertainty of your position measurements affect this graph? How might you tell the difference?

EXPLORATION

This section is extremely important—many instructions will not make sense, or you may be lead astray, if you do not take the time to carefully explore your experimental plan.

In this section you practice with the apparatus before you make time-consuming measurements which may not be valid. This is where you carefully observe the behavior of your physical system, before you begin making measurements. You will also need to explore the range over which your apparatus is reliable. Remember to always treat the apparatus with **care and respect**. Your fellow students in the next lab section will need to use the equipment after you are finished with it. If you are unsure about how the apparatus works, ask your lab instructor.

Most apparatus has a range in which its operation is simple and straightforward. This is its range of reliability. Outside of that range, complicated corrections need to be applied. You can quickly determine the range of reliability by making **qualitative** observations at what you consider to be the extreme ranges of your measurements. Record your observations in your lab journal. If you observe that the apparatus does not function properly for the range of quantities you were considering measuring, you can modify your experimental plan before you have wasted time taking an invalid set of measurements.

The result of the exploration should be a plan for doing the measurements that you need. **Record your measurement plan in your journal.**

Place one of the metal tracks on your lab bench and place the toy car on the track. Turn on the car and observe its motion. Determine if it actually moves with a constant velocity. Use the meter stick and stopwatch to determine the speed of the car.

Turn on the video camera and look at the motion as seen by the camera on the computer screen. Go to Appendix D for instructions about using the video recorder.

Do you need to focus the camera to get a clean image? How do the room lights affect the image? Which controls help sharpen the image? Record your camera adjustments in your lab journal.

Move the position of the camera closer to the car. How does this affect the video image on the screen? Try moving it farther away. Raise the height of the camera tripod. How does this affect the image? Decide where you want to place the camera to minimize the distortion.

Practice taking a video of the toy car. Looking at the video frame by frame allows you to check whether the computer has missed any frames (the motion should be smooth). The capacity of the computer to take in all of the data from the video camera depends on the amount of data. If your computer is dropping too many frames, you will not have enough data to analyze. You can minimize the number of frames dropped by decreasing the amount of data in the video picture by adjusting the picture size and keeping the picture as feature free as possible. Check out these effects. Write down the best situation for taking a video in your journal for future reference. You will be doing a lot of this. When you have the best movie possible, save it and open the video analysis application.

Make sure everyone in your group gets the chance to operate the camera and the computer.

MEASUREMENT

Now that you have predicted the result of your measurement and have explored how your apparatus behaves, you are ready to make careful measurements. To avoid wasting time and effort, make the minimal measurements necessary to convince yourself and others that you have solved the laboratory problem.

Measure the speed of the car using a stopwatch as it travels a known distance. How many measurements should you take to determine the car's speed? (Too few measurements may not be convincing to others, too many and you may waste time and effort.) How much accuracy do you need from your meter stick and stopwatch to determine a speed to at least two significant figures? Make the number of measurements you need and record them in a neat and organized manner so that you can understand them a month from now if you must. Also make sure to record precisely how you make these measurements. Some future lab problems will require results from earlier ones.

Take a video of the motion of the car to determine its speed. Measure some object you can see in the video so that you can tell the analysis program the real size of the video images when it asks you to calibrate. The best object to measure is the car itself. When you digitize the video, why is it important to click on the same point on the car's image? Estimate your accuracy in doing so. Be sure to take measurements of the motion of the car in the distorted regions (edges) of the video.

Make sure you set the scale for the axes of your graph so that you can see the data points as you take them. Use your measurements of total distance the car travels and total time to determine the maximum and minimum value for each axis before taking data.

Are any points missing from the position versus time graph? Missing points result from more data being transmitted from the camera than the computer can write to its memory. If too many points are missing, make sure that the size of your video frame is optimal (see Appendix D). It may also be that your background is too busy. Try positioning your apparatus so that the background has fewer visual features.

Note: Be sure to record your measurements with the appropriate number of significant figures (see Appendix A) and with your estimated uncertainty (see Appendix B). Otherwise, the data is nearly meaningless.

ANALYSIS

Data by itself is of very limited use. Most interesting quantities are those *derived* from the data, not direct measurements themselves. Your predictions may be qualitatively correct but quantitatively very wrong. To see this you must process your data.

Always complete your data processing (analysis) before you take your next set of data. If something is going wrong, you shouldn't waste time taking a lot of useless data. After analyzing the first data, you may need to modify your measurement plan and re-do the measurements. If you do, be sure to **record the changes in your plan in your journal**.

Calculate the average speed of the car from your stopwatch and meter stick measurements. Determine if the speed is constant within your measurement uncertainties. Can you determine the instantaneous speed of your car as a function of time?

Analyze your video to find the instantaneous speed of the car as a function of time. Determine if the speed is constant within your measurement uncertainties. See Appendix D for instructions on how to do video analysis.

Why do you have less data points for the velocity versus time graph compared to the position versus time graph? Use the data tables generated by the computer to explain how the computer generates the velocity graphs.

CONCLUSIONS

After you have analyzed your data, you are ready to answer the experimental problem. State your result in the most general terms supported by your analysis. **This should all be recorded in your journal in one place before moving on to the next problem assigned by your lab instructor. Make sure you compare your result to your prediction.**

Compare the car's speed measured with video analysis to the measurement using a stopwatch. How do they compare? Did your measurements and graphs agree with your answers to the Method Questions? If not, why? What are the limitations on the accuracy of your measurements and analysis?
Do measurements near the edges of the video give the same speed as that as found in the center of the image within the uncertainties of your measurement? What will you do for future measurements?

PROBLEM #3:
MOTION UP AND DOWN AN INCLINE

A proposed ride at the Valley Fair amusement park launches a roller coaster car up an inclined track. Near the top of the track, the car reverses direction and rolls backwards into the station. As a member of the safety committee, you have been asked to compute the acceleration of the car throughout the ride. To check your results, you decide to build a laboratory model of the ride.

> What is the acceleration of an object moving up and down a ramp at all times during its motion?

EQUIPMENT

For this problem you will have a stopwatch, a meter stick, an end stop, a wood block, a video camera and a computer with a video analysis application written in LabVIEW™. You will also have a cart to roll up the inclined track.

PREDICTION

Make a rough sketch of how you expect the acceleration-versus-time graph to look for a cart given an initial velocity up along an inclined track. The graph should be for the entire motion of going up the track, reaching the highest point, and then coming down the track.

Do you think the acceleration of the cart moving up an inclined track will be ***greater than, less than****, or* ***the same as*** *the acceleration of the cart moving down the track? What is the acceleration of the cart at its highest point? Explain your reasoning.*

WARM UP QUESTIONS

The following questions should help with your prediction and the analysis of your data.

1. Sketch a graph of how you expect an *instantaneous acceleration-versus-time graph* to look if the cart moved down the track with the direction of a constant acceleration always down along the track. Sketch a graph of how you expect an *instantaneous acceleration-versus-time graph* to look if the cart moved up the track with the direction of a constant acceleration always down along the track **after** an initial push. When the cart moves up the track and then down the track, graph how would you expect an *instantaneous acceleration-versus-time graph* to look for the entire motion **after** an initial push? Explain your reasoning for each graph. To make the comparison easier, it is useful to draw these graphs next to each other. Write down the equation(s) that best represents each of these graphs. If there are constants in your equation, what kinematic quantities do they represent? How would you determine these constants from your graph?

2. Write down the relationship between the acceleration and the velocity of the cart. Use that relationship to construct an instantaneous velocity versus time graph just below each of your acceleration versus time graphs from question 1. The connection between the derivative of a function and the slope of its graph will be useful. Use the same scale for your time axes. Write down the equation that best represents each of these graphs. If there are constants in your equation, what kinematic quantities do they represent? How would you determine these constants from your graph? Can any of these constants be determined from the constants in the equation representing the acceleration versus time graphs? Which graph do you think best represents how velocity of the cart changes with time? Change your prediction if necessary.

3. Write down the relationship between the velocity and the position of the cart. Use that relationship to construct a position versus time graph just below each of your velocity versus time graphs from question 2. The connection between the derivative of a function and the slope of its graph will be useful. Use the same scale for your time axes. Write down the equation that best represents each of these graphs. If there are constants in your equation, what kinematic quantities do they represent? How would you determine these constants from your graph? Can any of these constants be determined from the constants in the equation representing the velocity versus time graphs? Which graph do you think best represents how position of the cart changes with time? Change your prediction if necessary.

EXPLORATION

What is the best way to change the angle of the inclined track in a reproducible way? How are you going to measure this angle with respect to the table? Think about trigonometry. How steep of an incline do you want to use?

Start the cart up the track with a gentle push. **BE SURE TO CATCH THE CART BEFORE IT HITS THE END STOP ON ITS WAY DOWN!** Observe the cart as it moves up the inclined track. At the instant the cart reverses direction, what is its velocity? Its acceleration? Observe the cart as it moves down the inclined track. Do your observations agree with your prediction? If not, this is a good time to change your prediction.

Where is the best place to put the camera? Is it important to have most of the motion in the center of the picture? Which part of the motion do you wish to capture?

Try several different angles. **Be sure to catch the cart before it collides with the end stop at the bottom of the track.** If the angle is too large, the cart may not go up very far and give you too few video frames for the measurement. If the angle is too small it will be difficult to measure the acceleration. Determine the useful range of angles for your track. Take a few practice videos and play them back to make sure you have captured the motion you want.

Choose the angle that gives you the best video record.

What is the total distance through which the cart rolls? How much time does it take? These measurements will help you set up the graphs for your computer data taking.

Write down your measurement plan.

MEASUREMENT

Using the plan you devised in the exploration section, make a video of the cart moving up and then down the track at your chosen angle. Make sure you get enough points for each part of the motion to determine the behavior of the acceleration. *Don't forget to measure and record the angle (with estimated uncertainty).*

Choose an object in your picture for calibration. Choose your coordinate system. Is a rotated coordinate system the easiest to use in this case?

Why is it important to click on the same point on the car's image to record its position? Estimate your accuracy in doing so.

Make sure you set the scale for the axes of your graph so that you can see the data points as you take them. Use your measurements of total distance the cart travels and total time to determine the maximum and minimum value for each axis before taking data.

Are any points missing from the position versus time graph? Missing points result from more data being transmitted from the camera than the computer can write to its memory. If too many points are missing, make sure that the size of your video frame is optimal (see Appendix D). It may also be that your background is too busy. Try positioning your apparatus so that the background has fewer visual features.

ANALYSIS

Choose a function to represent the position versus time graph. How can you estimate the values of the constants of the function from the graph? You can waste a lot of time if you just try to guess the constants. What kinematic quantities do these constants represent? Can you tell from your graph where the cart reaches its highest point?

Choose a function to represent the velocity versus time graph. How can you calculate the values of the constants of this function from the function representing the position versus time graph? Check how well this works. You can also estimate the values of the constants from the graph. Just trying to guess the constants can waste a lot of your time. What kinematic quantities do these constants represent? Can you tell from your graph where the cart reaches its highest point?

From the velocity versus time graph determine if the acceleration changes as the cart goes up and then down the ramp. Use the function representing the velocity versus time graph to calculate the acceleration of the cart as a function of time. Make a graph of that function. Can you tell from your graph where the cart reaches its highest point? Is the average acceleration of the cart equal to its instantaneous acceleration in this case?

As you analyze your video, *make sure everyone in your group gets the chance to operate the computer.*

CONCLUSION

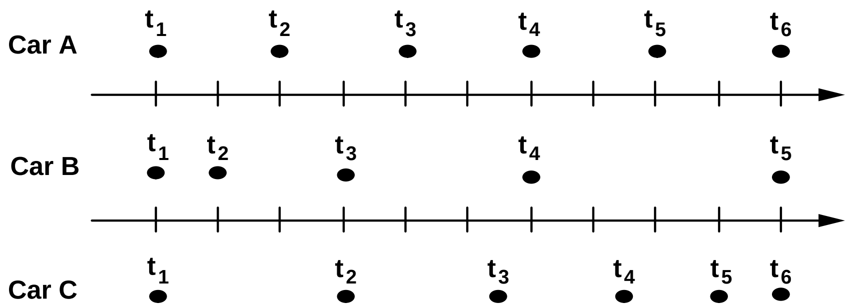
How do your position-versus-time and velocity-versus-time graphs compare with your answers to the method questions and the prediction? What are the limitations on the accuracy of your measurements and analysis?

Did the cart have the same acceleration throughout its motion? Did the acceleration change direction? Was the acceleration zero at the top of its motion? Did the acceleration change direction? Describe the acceleration of the cart through its entire motion **after** the initial push.

Justify your answer. What are the limitations on the accuracy of your measurements and analysis?

☑ CHECK YOUR UNDERSTANDING

1. Suppose you are looking down from a helicopter at three cars traveling in the same direction along the freeway. The positions of the three cars every 2 seconds are represented by dots on the diagram below. The positive direction is to the right.



a. At what clock reading (or time interval) do Car A and Car B have very nearly the same speed? Explain your reasoning.

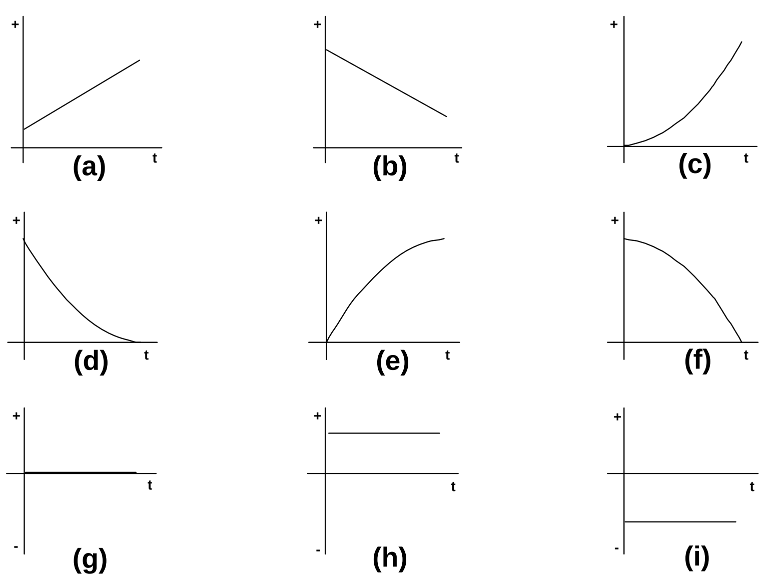
b. At approximately what clock reading (or readings) does one car pass another car? In each instance you cite, indicate which car, A, B or C, is doing the overtaking. Explain your reasoning.

c. Suppose you calculated the average velocity for Car B between t_1 and t_5. Where was the car when its instantaneous velocity was equal to its average velocity? Explain your reasoning.

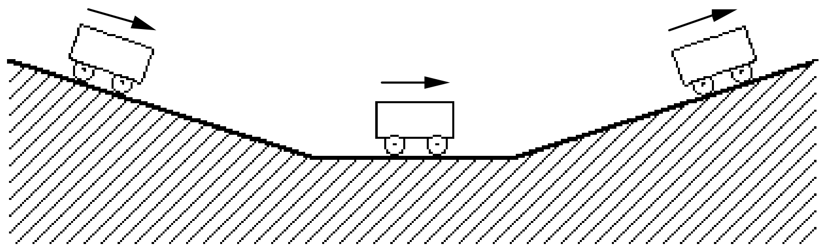
d. Which graph below best represents the position-versus-time graph of Car A? Of Car B? Of car C? Explain your reasoning.

e. Which graph below best represents the instantaneous velocity-versus-time graph of Car A? Of Car B? Of car C? Explain your reasoning. (HINT: Examine the distances traveled in successive time intervals.)

Which graph below best represents the instantaneous acceleration-versus-time graph of Car A? Of Car B? Of car C? Explain your reasoning.



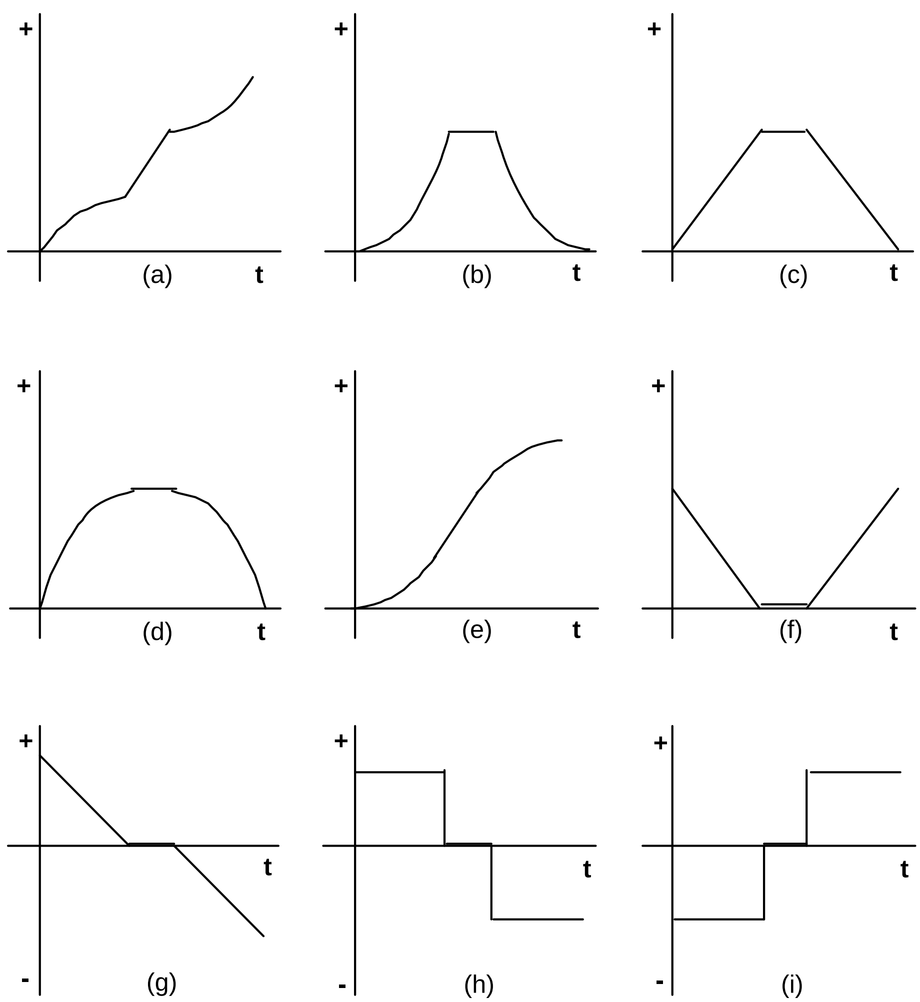
2. A cart starts from rest at the top of a hill, rolls down the hill, over a short flat section, then back up another hill, as shown in the diagram above. Assume that the friction between the wheels and the rails is negligible. The positive direction is to the right.



a. Which graph below best represents the position-versus-time graph for the motion along the track? Explain your reasoning. (Hint: Think of motion as one-dimensional.)

b. Which graph below best represents the instantaneous velocity-versus-time graph? Explain your reasoning.

c. Which graph below best represents the instantaneous acceleration-versus-time graph? Explain your reasoning.



PROBLEM #4: BOUNCING

You have a summer job working for NASA to design a low cost landing system for a Mars mission. The payload will be surrounded by a big padded ball and dropped onto the surface. When it reaches the surface, it will simply bounce. The height and the distance of the bounce will get smaller with each bounce so that it finally comes to rest on the surface. Your task is to determine how the ratio of the horizontal distance covered by two successive bounces depends on the ratio of the heights of each bounce and the ratio of the horizontal components of the velocity of each bounce. After making the calculation you decide to check it in your laboratory on Earth.

?

How does the ratio of the horizontal distance covered by two successive bounces depend on the ratio of the heights of each bounce and the ratio of the horizontal components of the velocity of each bounce?

EQUIPMENT

For this problem, you will have a ball, a stopwatch, a meter stick, and a computer with a video camera and an analysis application written in LabVIEW™.

PREDICTION

Calculate the ratio of the horizontal distance of two successive bounces if you know the ratio of the heights of the bounces and the ratio of the horizontal components of the initial velocity of each bounce.

Be sure to state your assumptions so your boss can tell if they are reasonable for the Mars mission.

WARM UP QUESTIONS

The following questions should help you make the prediction.

1. Draw a good picture of the situation including the velocity and acceleration vectors at all relevant times. Decide on a coordinate system to use. Make sure you define the positive and negative directions. During what time interval does the ball have a motion that is easiest to calculate? Is the acceleration of the ball during that time interval constant or is it changing? Why? What is the relationship between the acceleration of the ball before and after a bounce? Are the time intervals for two successive bounces equal? Why or why not? Clearly label the horizontal distances and the heights for each of those time intervals. What reasonable assumptions will you probably need to make to solve this problem? How will you check these assumptions with your data?

2. Write down all of the kinematic equations that apply to the time intervals you selected under the assumptions you have made. Make sure you clearly distinguish the equations describing the horizontal motion and those describing the vertical motion. These equations are the tools you will use to solve the problem. Start the problem by calculating the quantity you wish to find: the horizontal distance covered between two successive bounces.

3. Write down an equation that gives the horizontal distance that the ball travels during the time between the first and second bounce. For that time interval, select an equation that gives the horizontal distance that the ball travels between bounces as a function of the initial horizontal velocity of the ball, its horizontal acceleration, and the time between bounces.

4. The only additional unknown in your equation is the time interval between bounces. You can determine it from the vertical motion of the ball. Select an equation that gives the change of vertical position of the ball between the first and second bounce as a function of the initial vertical velocity of the ball, its vertical acceleration, and the time between bounces.

5. An additional unknown, the ball's initial vertical velocity, has been introduced. Determine it from another equation giving the height that the ball bounces as a function of the initial vertical velocity of the ball, its vertical acceleration, and the time from the bounce until it reaches that position. How is that time interval related to the time between bounces? Show that this is true.

6. Combining the previous steps gives you an equation for the horizontal distance of a bounce in terms of the ball's horizontal velocity, the height of the bounce, and the vertical acceleration of the ball.

7. Repeat the above process for the next bounce and take the ratio of horizontal distances to get your prediction.

EXPLORATION

Review your lab journal from any previous problem requiring analyzing a video of a falling ball.

Position the camera and adjust it for optimal performance. *Make sure everyone in your group gets the chance to operate the camera and the computer.*

Practice bouncing the ball without spin until you can get at least two full bounces to fill the video screen (or at least the undistorted part of the video screen). Three is better so you can check your results. It will take practice and skill to get a good set of bounces. Everyone in the group should try to determine who is best at throwing the ball.

Determine how much time it takes for the ball to have the number of bounces you will video and estimate the number of video points you will get in that time. Is that enough points to make the measurement? Adjust the camera position and screen size to give you enough data points without dropping too many.

Although the ball is the best item to use to calibrate the video, the image quality due to its motion might make this difficult. Instead, you might need to place an object of known length in the plane of motion of the ball, near the center of the ball's trajectory, for calibration purposes. Where you place your reference object does make a difference to your results. Determine the best place to put the reference object for calibration.

Step through the video and determine which part of the ball is easiest to consistently determine. When the ball moves rapidly you may see two images of the ball due to the interlaced scan of a TV camera. You should only use one of the images.

Write down your measurement plan.

MEASUREMENT

Make a video of the ball being tossed. Make sure you can see the ball in every frame of the video.

Digitize the position of the ball in enough frames of the video so that you have the sufficient data to accomplish your analysis. Make sure you set the scale for the axes of your graph so that you can see the data points as you take them. Use your measurements of total distance the ball travels and total time to determine the maximum and minimum value for each axis before taking data.

ANALYSIS

Analyze the video to get the horizontal distance of two successive bounces, the height of the two bounces, and the horizontal components of the ball's velocity for each bounce. You will probably want to calibrate the video independently for each bounce so you can begin your time as close as possible to when the ball leaves the ground. The point where the bounce occurs will usually not correspond to a video frame taken by the camera so some estimation is necessary to determine this position.

Choose a function to represent the horizontal position-versus -time graph and another for the vertical position graph for the first bounce. How can you estimate the values of the constants of the functions? You can waste a lot of time if you just try to guess the constants. What kinematic quantities do these constants represent? How can you tell where the bounce occurred from each graph? Determine the height and horizontal distance for the first bounce.

Choose a function to represent the velocity-versus-time graph for each component of the velocity for the first bounce. How can you calculate the values of the constants of these functions from the functions representing the position-versus-time graphs? Check how well this works. You can also estimate the values of the constants from the graph. Just trying to guess the constants can waste a lot of your time. What kinematic quantities do these constants represent? How can you tell where the bounce occurred from each graph? Determine the initial horizontal velocity of the ball for the first bounce. What is the horizontal and vertical acceleration of the ball between bounces? Does this agree with your expectations?

Repeat this analysis for the second bounce.

What kinematic quantities are approximately the same for each bounce? How does that simplify your prediction equation?

CONCLUSION

How do your graphs compare to your predictions and method questions? What are the limitations on the accuracy of your measurements and analysis?

Will the ratio you calculated be the same on Mars as on Earth? Why?

What additional kinematic quantity, whose value you know, can be determined with the data you have taken to give you some indication of the precision of your measurement? How close is this quantity to its known value?

PROBLEM #2:
FORCES IN EQUILIBRIUM

You have a summer job with a research group studying the ecology of a rain forest in South America. To avoid walking on the delicate rain forest floor, the team members walk along a rope walkway that the local inhabitants have strung from tree to tree through the forest canopy. Your supervisor is concerned about the maximum amount of equipment each team member should carry to safely walk from tree to tree. If the walkway sags too much, the team member could be in danger, not to mention possible damage to the rain forest floor. You are assigned to set the load standards.

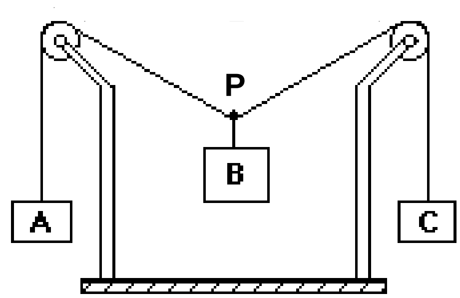
Each end of the rope supporting the walkway goes over a branch and then is attached to a large weight hanging down. You need to determine how the sag of the walkway is related to the mass of a team member plus equipment when they are at the center of the walkway between two trees. To check your calculation, you decide to model the situation using the equipment shown below.

?

How does the vertical displacement of an object suspended on a string halfway between two branches, depend on the mass of that object?

EQUIPMENT

The system consists of a central object, B, suspended halfway between two pulleys by a string. The whole system is in equilibrium. The picture below is similar to the situation with which you will work. The objects A and C, which have the same mass, allow you to determine the force exerted on the central object by the string.
You do need to make some assumptions about what you can neglect. For this investigation, you will also need a meter stick, two pulley clamps, three mass hangers and a mass set to vary the mass of objects.



PREDICTION

Calculate the change in the vertical displacement of the central object B as you increase its mass. You should obtain an equation that predicts how the vertical displacement of central object B depends on its mass, the mass of objects A and C, and the horizontal distance between the two pulleys.

Use your equation to make a graph of the vertical displacement of object B as a function of its mass.

WARM UP QUESTIONS

To solve this problem it is useful to have an organized problem-solving strategy such as the one outlined in the following questions. You should use a technique similar to that used in Problem 1 (where a more detailed set of Warm Up Questions is given) to solve this problem. You might also find the Problem Solving section 4-6 of your textbook is useful.

1. Draw a sketch similar to the one in the Equipment section. Draw vectors that represent the forces on objects A, B, C, and point P. Use trigonometry to show how the vertical displacement of object B is related to the horizontal distance between the two pulleys and the angle that the string between the two pulleys sags below the horizontal.

2. The "known" (measurable) quantities in this problem are the masses of the objects and the distance between the pulleys; the unknown quantity is the vertical displacement of object B.

3. Use Newton's laws to solve this problem. Write down the acceleration for each object. Draw separate force diagrams for objects A, B, C and for point P (if you need help, see your text). What assumptions are you making?

Which angles between your force vectors and your horizontal coordinate axis are the same as the angle between the strings and the horizontal?

4. For each force diagram, write Newton's second law along each coordinate axis.

5. Solve your equations to predict how the vertical displacement of object B depends on its mass, the mass of objects A and C, and the horizontal distance between the two pulleys. Use this resulting equation to make a graph of how the vertical displacement changes as a function of the mass of object B.

6. From your resulting equation, analyze what is the limit of mass of object B corresponding to the fixed mass of object A and C. What will happen if the mass of object B is larger than twice the mass of A?

EXPLORATION

Start with just the string suspended between the pulleys (no central object), so that the string looks horizontal. Attach a central object and observe how the string sags. Decide on the origin from which you will measure the vertical position of the object.

Try changing the mass of objects A and C (keep them equal for the measurements but you will want to explore the case where they are not equal).

Do the pulleys behave in a frictionless way for the entire range of weights you will use? How can you determine if the assumption of frictionless pulleys is a good one?

Add mass to the central object to decide what increments of mass will give a good range of values for the measurement. Decide how measurements you will need to make.

MEASUREMENT

Measure the vertical position of the central object as you increase its mass. Make a table and record your measurements.

ANALYSIS

Make a graph of the vertical displacement of the central object as a function of its mass based on your measurements. On the same graph, plot your predicted equation.

Where do the two curves match? Where do the two curves start to diverge from one another? What does this tell you about the system?

What are the limitations on the accuracy of your measurements and analysis?

CONCLUSION

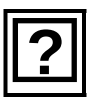
What will you report to your supervisor? How does the vertical displacement of an object suspended on a string between two pulleys depend on the mass of that object? Did your measurements of the vertical displacement of object B agree with your initial predictions? If not, why? State your result in the most general terms supported by your analysis.

What information would you need to apply your calculation to the walkway through the rain forest?

Estimate reasonable values for the information you need, and solve the problem for the walkway over the rain forest.

PROBLEM #5: CONSERVATION OF ANGULAR MOMENTUM

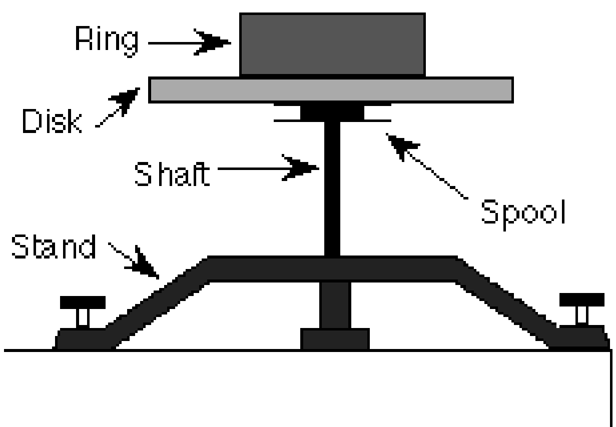
While driving around the city, your car is constantly shifting gears. You wonder how the gear shifting process works. Your friend tells you that there are gears in the transmission of your car that are rotating about the same axis. When the car shifts, one of these gear assemblies is brought into connection with another one that drives the car's wheels. Thinking about a car starting up, you decide to calculate how the angular speed of a spinning object changes when it is brought into contact with another object at rest. To keep your calculation simple, you decide to use a disk for the initially spinning object and a ring for the object initially at rest. Both objects will be able to rotate freely about the same axis that is centered on both objects. To test your calculation you decide to build a laboratory model of the situation.



What is the final angular velocity of a disk with an initial angular speed after being connected with a ring initially at rest?

EQUIPMENT

You will use the same basic equipment in the previous problems.



PREDICTION

Calculate, in terms of the initial angular velocity of the disk before the ring is dropped onto the spinning disk and the characteristics of the system, the angular velocity of the disk after the ring is dropped onto the initially spinning disk.

WARM UP QUESTIONS

To complete your prediction, it is useful to use a problem solving strategy such as the one outlined below:

1. Make two side view drawings of the situation (similar to the diagram in the Equipment section), one just as the ring is released, and one after the ring lands on the disk. Label all relevant kinematic quantities and write down the relationships that exist between them. Label all relevant forces.

2. Determine the basic principles of physics that you will use and how you will use them. Determine your system. Are any objects from outside your system interacting with your system? Write down your assumptions and check to see if they are reasonable.

3. Use conservation of angular momentum to determine the final angular speed of the rotating objects. Why not use conservation of energy or conservation of momentum? Define your system and write the conservation of angular momentum equation for this situation:

What is the angular momentum of the system as the ring is released? What is its angular momentum after the ring connects with the disk? Is any significant angular momentum transferred to or from the system? If so, can you determine it or redefine your system so that there is no transfer?

4. Identify the target quantity you wish to determine. Use the equations collected in steps 1 and 3 to plan a solution for the target. If there are more unknowns than equations, reexamine the previous steps to see if there is additional information about the situation that can be expressed in an addition equation. If not, see if one of the unknowns will cancel out.

EXPLORATION

Practice dropping the ring into the groove on the disk as gently as possible to ensure the best data. What happens if the ring is dropped off-center? What happens if the disk does not fall smoothly into the groove? Explain your answers.

Decide what measurements you need to make to check your prediction. If any major assumptions are used in your calculations, decide on the additional measurements that you need to make to justify them.

Outline your measurement plan.

Make some rough measurements to be sure your plan will work.

MEASUREMENTS

Follow your measurement plan. What are the uncertainties in your measurements?

ANALYSIS

Determine the initial and final angular velocity of the disk from the data you collected. Be sure to use an analysis technique that makes the most efficient use of your data and your time. Using your prediction equation and your measured initial angular velocity, calculate the final angular velocity of the disk. If your calculation incorporates any assumptions, make sure you justify these assumptions based on data that you have analyzed.

CONCLUSION

Did your measurement of the final angular velocity agree with your calculated value by prediction? Why or why not? What are the limitations on the accuracy of your measurements and analysis?

Could you have used conservation of energy to solve this problem? Why or why not? Use your data to check your answer.

PROBLEM #3: MEASURING THE MAGNETIC FIELD OF PERMANENT MAGENTS

You are now ready to measure the magnetic field near high-voltage power lines. Before making this measurement, you decide to practice by using your Hall probe on a bar magnet. Since you already know the map of the magnetic field of a bar magnet, you decide to use the Hall probe to determine how the magnitude of the magnetic field varies as you move away from the magnet along each of its axes. While thinking about this measurement you wonder if a bar magnet's magnetic field might be the result of the sum of the magnetic field of each pole. Although, to date, no isolated magnetic monopoles have ever been discovered, you wonder if you can model the situation as two magnetic monopoles, one at each end of the magnet. Is it possible that the magnetic field from a single magnetic pole, a monopole, if they exist, has the same behavior as the electric field from a point charge? You decide to check it out.

How does the magnitude of the magnetic field from a bar magnet along each of its axes depend on the distance from the magnet? Is that behavior consistent with the dependence of the magnetic field on the distance away from a single pole, being the same as the electric field from a point charge?

EQUIPMENT

You will have a bar magnet, a meter stick, a Hall probe (see Appendix D), and a computer data acquisition system (see Appendix E). You will also have a Taconite plate and a compass.

PREDICTION

Calculate the magnetic field strength as a function of distance along each axis of a bar magnet. Make a graph of this function for each axis. How do you expect these graphs to compare to similar graphs of the electric field along each axis of an electric dipole?

WARM UP QUESTIONS

1. Draw a bar magnet as a magnetic dipole consisting of two magnetic monopoles of equal strength but opposite sign, separated by some distance. Label each monopole with its strength and sign. Label the distance. Choose a convenient coordinate system.

2. Select a point along one of the coordinate axes, outside the magnet, at which you will calculate the magnetic field. Determine the position of that point with respect to your coordinate system. Determine the distance of your point to each pole of the magnet, in terms of the position of your point with respect to your coordinate system.

3. Assume that the magnetic field from a magnetic monopole is analogous to the electric field from a point charge, i.e. the magnetic field is proportional to g/r^2 where g is a measure of the strength of the monopole. Determine the direction of the magnetic field from each pole at the point of interest.

4. Calculate the magnitude of the each component of the magnetic field from each pole at the point of interest. Add the magnetic field (remember it is a vector) from each pole at that point to get the magnetic field at that point.

5. Graph your resulting equation for the magnetic field strength along that axis as a function of position along the axis.

6. Repeat the above steps for the other axis.

EXPLORATION

Using either a Taconite plate or a compass check that the magnetic field of the bar magnet appears to be a dipole.

Start the Hall probe program and go through the Hall probe calibration procedure outlined in Appendix E. Be sure the switch on your amplification box agrees with the value on the computer.
Take one of the bar magnets and use the probe to check out the variation of the magnetic field. Based on your previous determination of the magnetic field map, be sure to orient the Hall probe correctly. Where is the field the strongest? The weakest? How far away from the bar magnet can you still measure the field with the probe?

Write down a measurement plan.

MEASUREMENT

Based on your exploration, choose a scale for your graph of magnetic field strength against position that will include all of the points you will measure.

Choose an axis of the bar magnet and take measurements of the magnetic field strength in a straight line along the axis of the magnet. Be sure that the field is always perpendicular to the probe. Make sure a point appears on the graph of magnetic field strength versus position every time you enter a data point. Use this graph to determine where you should take your next data point to map out the function in the most efficient manner.

Repeat for each axis of the magnet.

ANALYSIS

Compare the graph of your calculated magnetic field to that which you measured for each axis of symmetry of your bar magnet. Can you fit your prediction equation to your measurements by adjusting the constants?

CONCLUSION

Along which axis of the bar magnet does the magnetic field fall off faster? Did your measured graph agree with your predicted graph? If not, why? State your results in the most general terms supported by your analysis.

How does the shape of the graph of magnetic field strength versus distance compare to the shape of the graph of electric field strength versus distance, for an electric dipole along each axis? Is it reasonable to assume that the functional form of the magnetic field of a monopole is the same as that of an electric charge? Explain your reasoning.

PROBLEM #6: MEASURING THE MAGNETIC FIELD OF TWO PARALLEL COILS

You have a part time job working in a laboratory developing large liquid crystal displays that could be used for very thin TV screens and computer monitors. The alignment of the liquid crystals is very sensitive to magnetic fields. It is important that the material sample be in a fairly uniform magnetic field for some crystal alignment tests. The laboratory has two nearly identical large coils of wire mounted so that the distance between them equals their radii. You have been asked to determine the magnetic field between them to see if it is suitable for the test.

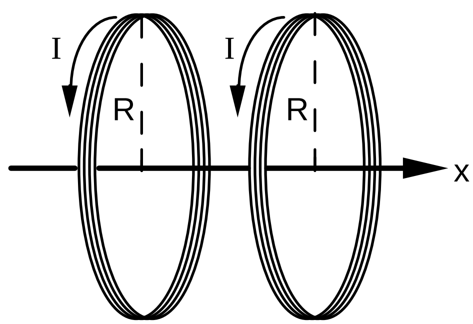
?

For two large, parallel coils, what is the magnetic field on the axis, as a function of the distance from the middle of the two coils?

EQUIPMENT

Connect two large coils to a power supply so that each coil has the same current. Each coil has 150 turns.

You will have a digital Multimeter (DMM), a compass, a meter stick, and a Hall probe. A computer is used for data acquisition.



PREDICTION

Calculate the magnitude of the magnetic field for two coils as a function of the position along their central axis, for the special case where the distance between the coils is the same as the radius of the coils. Use this expression to graph the magnetic field strength versus position along the axis.

WARM UP QUESTIONS

1. Draw a picture of the situation showing the direction of the current through each coil of wire. Establish a single convenient coordinate system for both coils.

Label all of the relevant quantities.

2. Select a point along the axis of the two coils at which you will determine an equation for the magnetic field. In the previous problem, you calculated the magnetic field caused by one coil as a function of the position along its axis. To solve this problem, add the magnetic field from each coil at the selected point along the axis. Remember to pay attention to the geometry of your drawing. The origin of your coordinate system for this problem cannot be at the center of both coils at once. Also remember that the magnetic field is a vector.

3. Use your equation to graph the magnetic field strength as a function of position from the common origin along the central axis of the coils. Describe the qualitative behavior of the magnetic field between the two coils. What about the region outside the coils?

EXPLORATION

WARNING: You will be working with a power supply that can generate large electric voltages. Improper use can cause painful burns. **To avoid danger, the power should be turned OFF and you should WAIT at least one minute before any wires are disconnected from or connected to the power supply. Never grasp a wire by its metal ends.**

Connect the large coils to the power supply with *the current flowing in the opposite direction in both coils*, using the adjustable voltage. Using your compass, explore the magnetic field produced. Be sure to look both between the coils and outside the coils.

Now connect the large coils to the power supply *with the current flowing in the same direction in both coils*, using the adjustable voltage. Using your compass, explore the magnetic field produced. Be sure to look both between the coils and outside the coils.

Based on your observations, should the currents be in the same direction or in opposite directions to give the most uniform magnetic field between the coils?

Connect the Hall probe according to the directions in Appendices D and E. For the current configuration that gives the most uniform magnetic field between the coils, explore the strength of the magnetic field along the axis between the coils. Follow the axis through the coils. Is the field stronger between or outside the coils? Where is the field strongest between the coils? The weakest?

See how the field varies when you are between the two coils but move off the axis. How far from the axis of the coils can you measure the field? Is it the same on both sides of the coils? Decide whether you should set the amplifier to high or low sensitivity.

When using the Hall probe program, consider where you want your zero position to be, so that you can compare to your prediction.

Write down a measurement plan.

MEASUREMENT

Based on your exploration, choose a scale for your graph of magnetic field strength against position that will include all of the points you will measure.

Use the Hall probe to measure the magnitude of the magnetic field along the axis of the coils of wire. Be sure to measure the field on both sides of the coils.

What are the units of your measured magnetic fields? How do these compare to the units of your prediction equations?
Use the DMM to measure the current in the two coils.

As a check, repeat these measurements with the other current configuration.

ANALYSIS

Graph the measured magnetic field of the coil along its axis as a function of position and compare to your prediction.

CONCLUSION

For two large, parallel coils, how does the magnetic field on the axis vary as a function of distance along the axis? Did your measured values agree with your predicted values? If not, why not? What are the limitations on the accuracy of your measurements and analysis?

Does this two-coil configuration satisfy the requirement of giving a fairly uniform field? Over how large a region is the field constant to within 20%? This very useful geometric configuration of two coils (distance between them equals their radius) is called a Helmholtz coil