

# Using Think-Pair-Share (TPS) to Promote Quantitative Problem Solving



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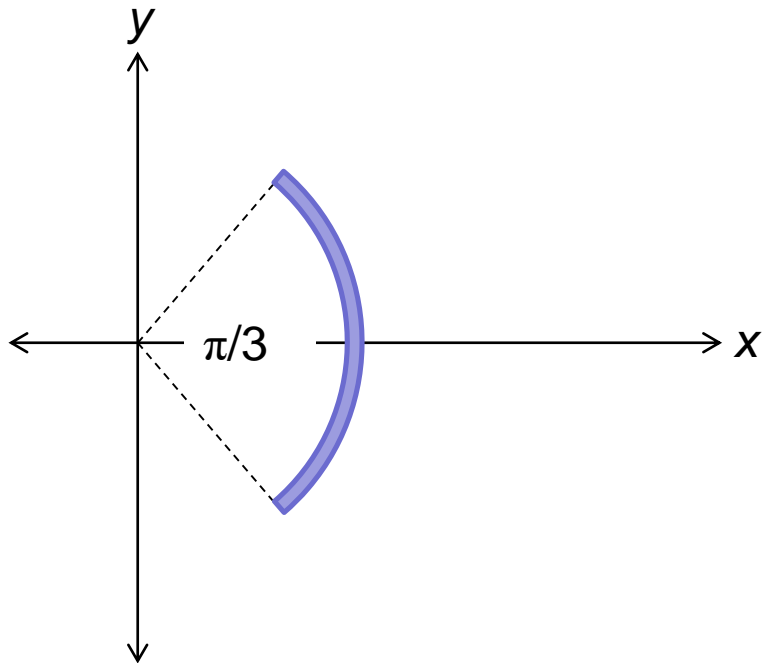
# Learning Outcomes

**Participants will be able to:**

- **Identify fundamental problem solving issues to target with TPS questions**
- **Describe how TPS question sequences can be used to promote student problem solving abilities**
- **Describe how TPS problem solving techniques can be implemented in the lecture portion of the course**



A plastic rod with a uniformly distributed charge  $-Q$  is bent into a circular arc of radius  $r$  that subtends an angle of  $\pi/3$  radians. We place coordinate axes such that the axis of symmetry of the rod lies along the  $x$ -axis and the origin is the center of curvature for the rod. What is the electric field (magnitude and direction) at the origin?

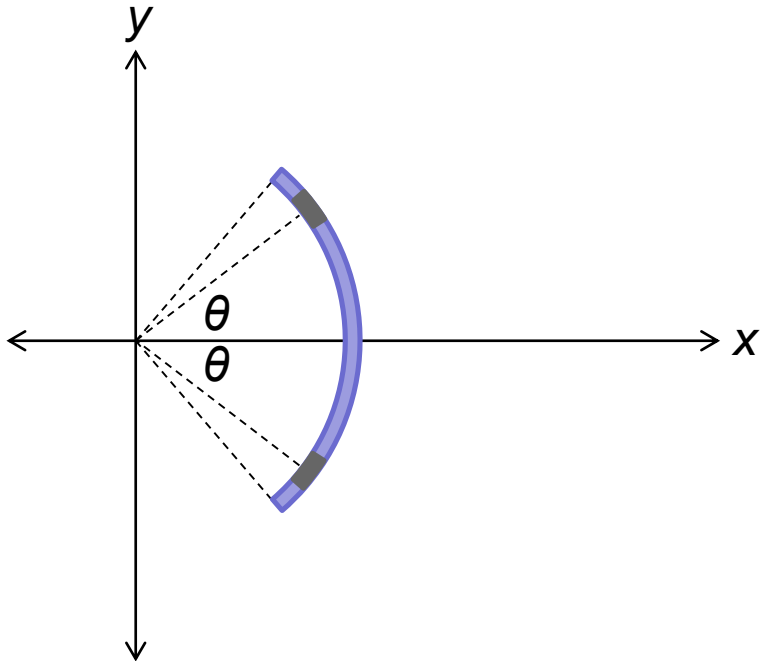


**Hint:** Imagine that the arc is made up of many infinitesimally small point charges  $dq$ . Each  $dq$  creates a differential electric field of magnitude  $dE = k dq/r^2$ .

Sum up all the  $dE$ s from all the  $dq$ s to get the magnitude of the overall electric field.

Integration is a way to add many infinitely small elements.

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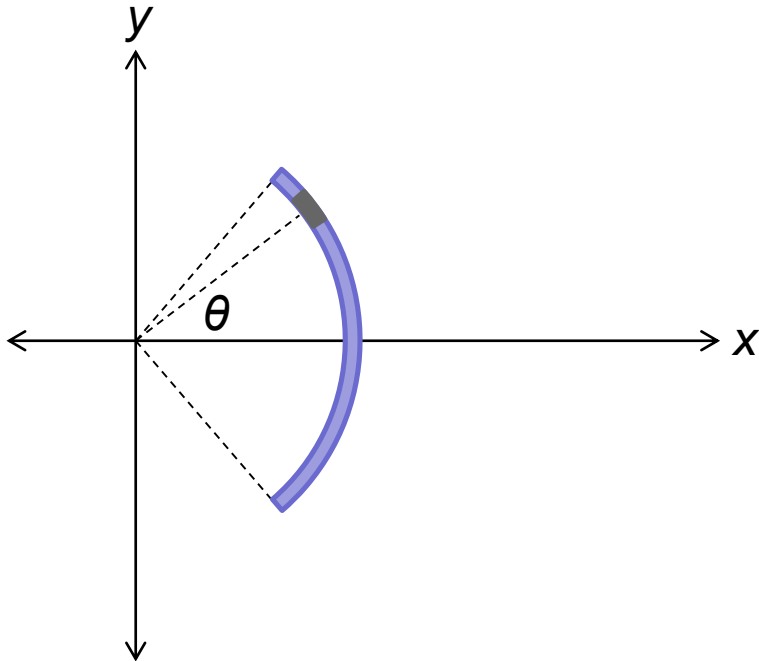


What is true about the electric fields produced by the two  $dq$  elements shown in grey on the left?

- A) Their  $x$ -components cancel out and their  $y$ -components add together.
- B) Their  $x$ -components add together and their  $y$ -components cancel out.
- C) Both their  $x$ - and  $y$ -components add together.
- D) Both their  $x$ - and  $y$ -components cancel out.

Since the  $y$ -components of the electric field cancel out, we only have to add the  $x$ -components.

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What is the magnitude of the  $x$ -component of  $dE$  due to the  $dq$  element shown in grey?

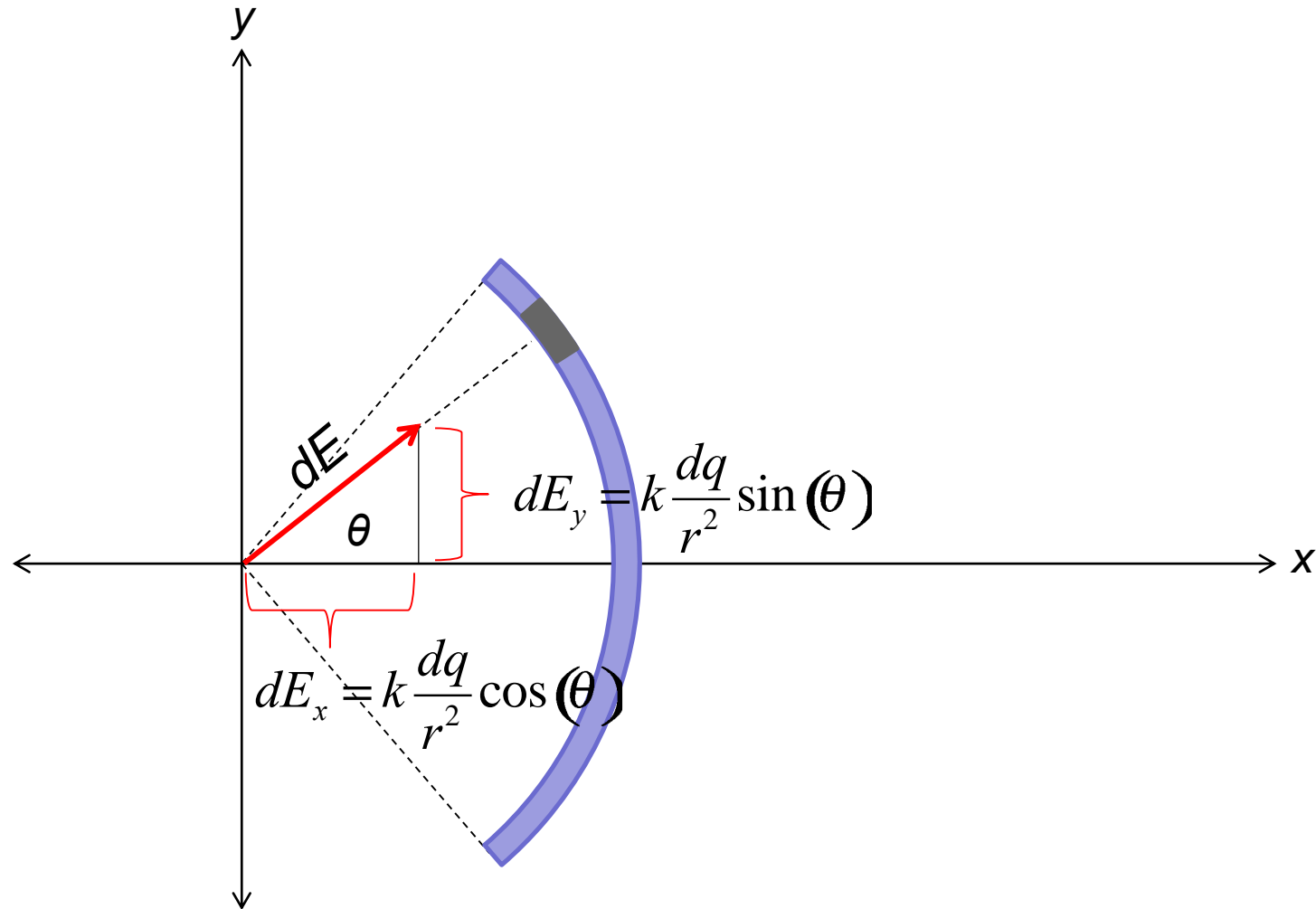
A)  $k \frac{dq}{r^2 \sin(\theta)}$

B)  $k \frac{dq}{r^2} \sin(\theta)$

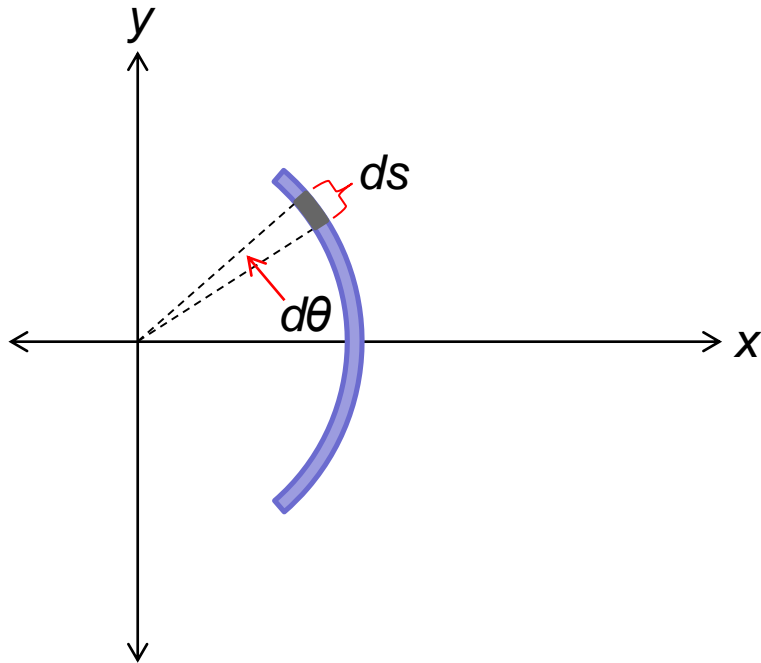
C)  $k \frac{dq}{r^2 \cos(\theta)}$

D)  $k \frac{dq}{r^2} \cos(\theta)$

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We need to integrate over all angles ( $\theta$ ), but  $dE_x$  is in terms of  $dq$ , not  $d\theta$ .

**Solution:** Relate  $dq$  to arc length  $ds$  using linear charge density  $\lambda$

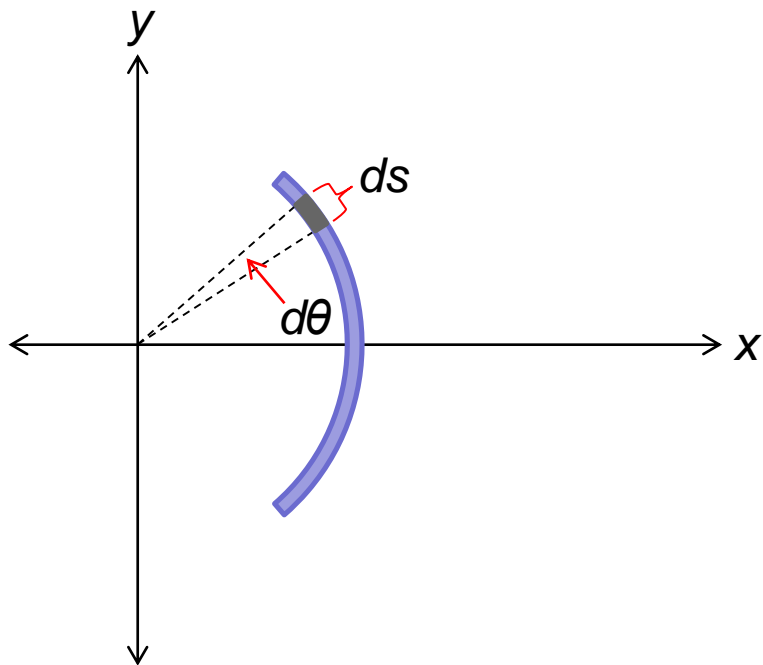
$$dq = \lambda ds$$

How is the differential arc length  $ds$  related to the differential angle  $d\theta$ ?

- A)  $ds = r d\theta$
- B)  $ds = r^2 d\theta$
- C)  $ds = d\theta/r$
- D)  $ds = d\theta/r^2$



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Find the magnitude of the electric field at the origin in terms of  $\lambda$ .

$$E = \int_{-\pi/6}^{\pi/6} k \frac{\lambda}{r^2} \cos(\theta) r d\theta = \frac{k\lambda}{r}$$

What is  $\lambda$ ?

A)  $\frac{Q}{r(\pi/6)}$

C)  $\frac{Q(\pi/6)}{r}$

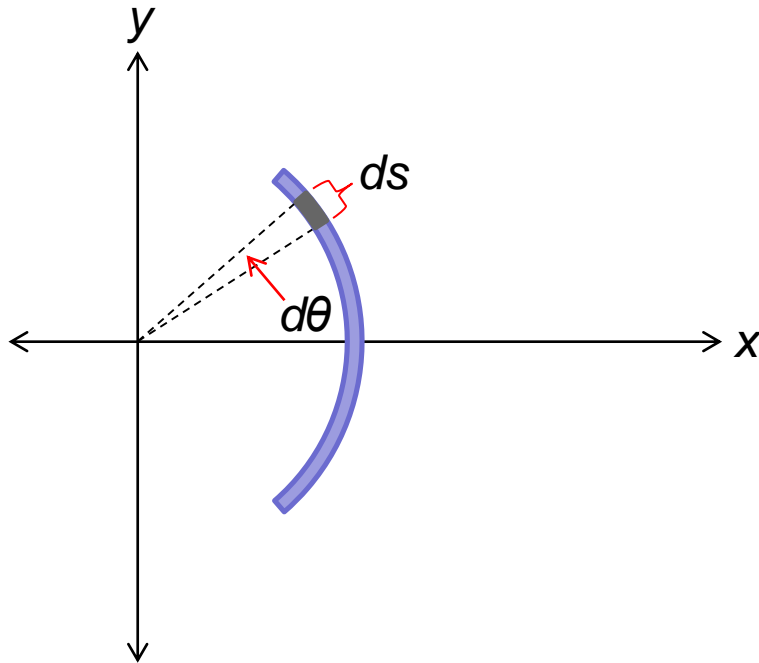
B)  $\frac{Q}{r(\pi/3)}$

D)  $\frac{Q(\pi/3)}{r}$

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Magnitude of the electric field at the origin:

$$E = \frac{3kQ}{\pi r^2}$$



Electric field at the origin:  $E = \frac{3kQ}{\pi r^2} \hat{i}$

What is the direction of the electric field at the origin?

- A)  $+\hat{i}$
- B)  $-\hat{i}$
- C)  $+\hat{j}$
- D)  $-\hat{j}$

# Idealized Implementation

- **Give a tightly-focused mini-lecture and then present the quantitative problem to your students.**
- **Give students 2-5 minutes to work on the problem before asking the first TPS question.**
  - Students need time to interpret the question and realize they are stuck.
- **Circulate around the room while students are working and engaged in discussions with their neighbors.**
  - Listen to what students are saying to each other.
  - Students are more likely to ask you questions if you're nearby.
- **Give students time to finish the problem after the last TPS question and debrief interactively.**

# Using Voting Questions

- **Choose a quantitative problem that requires students to use multiple pieces of physics and astronomy knowledge.**
- **Turn specific student difficulties into TPS questions.**
- **Answer choices are mathematical expressions/quantitative relationships.**
- **Wrong answers (“distractors”) represent real errors students frequently make.**

# Where Will Students Struggle?

- Students will likely struggle when they have to do more than plugging in a number, performing an algebraic manipulation, or executing a well known algorithm.
- ACER Framework for Problem Solving (Caballero *et al.* 2015):
  - **Activation** of mathematical tool
  - **Construction** of mathematical model
  - **Execution** of the math
  - **Reflection**

“...the majority of execution errors observed in this study were made by students who had already made one or more significant mistakes in the activation or construction components of the their solution.” (Wilcox and Corsiglia 2019)

# Time

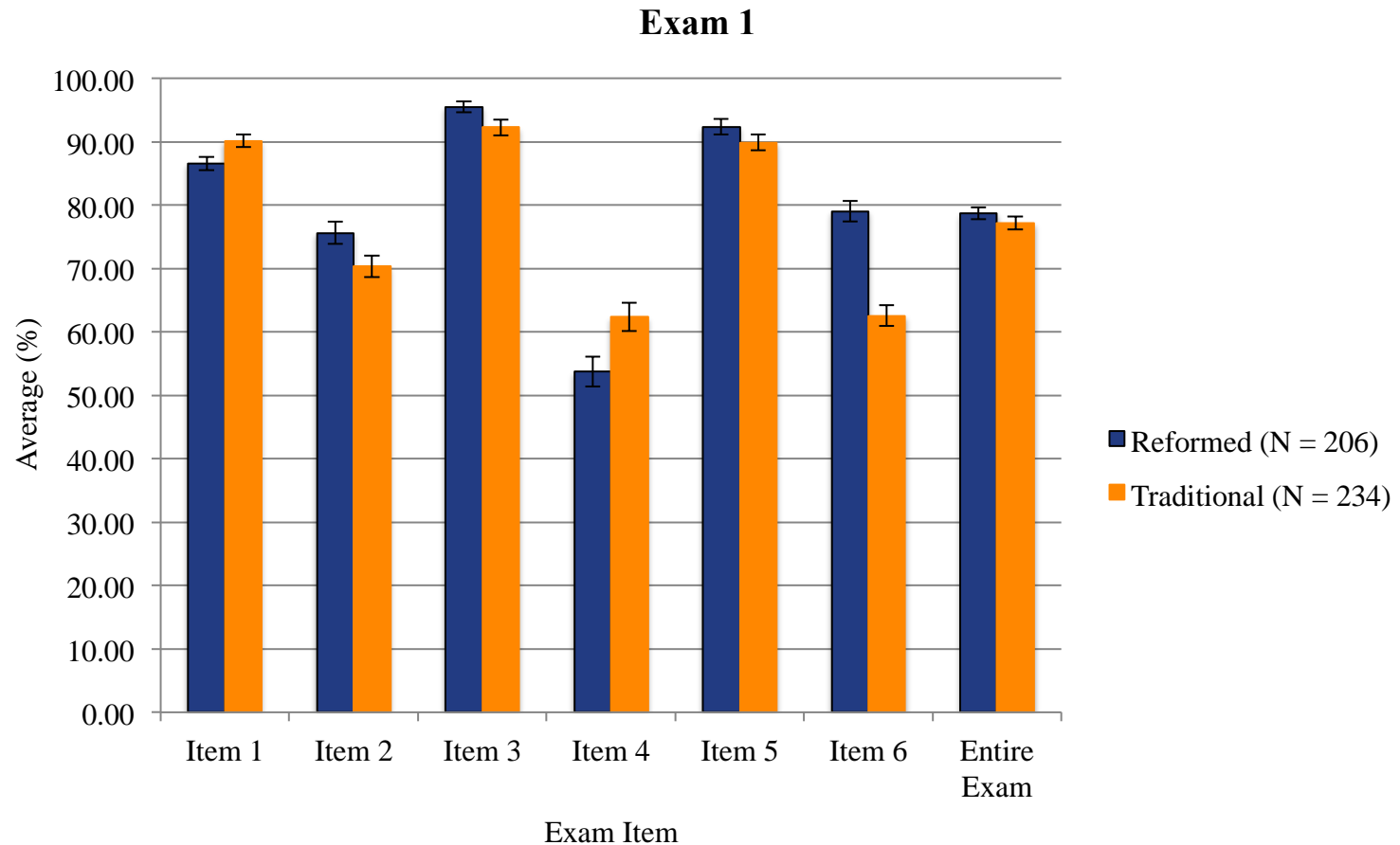
Week	Day	Date	Lecture Topic	Studio
1	Tue	20-Aug		<b>First day of classes - No studio</b>
	Wed	21-Aug	Lecture 1 - Introduction	Studio 1 - Common Cents
	Thurs	22-Aug		Studio 1 - Common Cents
2	Mon	26-Aug	Lecture 2 - Scaling 1	Studio 2 - Scaling 1
	Tues	27-Aug		Studio 2 - Scaling 1
	Wed	28-Aug	Lecture 3 - Scaling 2	Studio 3 - Scaling 2
	Thurs	29-Aug		Studio 3 - Scaling 2
3	Mon	2-Sep	<b>Holiday - Labor Day</b>	<b>No studios held today</b>
	Tues	3-Sep		<b>No studios held today</b>
	Wed	4-Sep	Lecture 4 - Kinematics 1	Studio 4 - Kinematics 1
	Thurs	5-Sep		Studio 4 - Kinematics 1
4	Mon	9-Sep	Lecture 5 - Kinematics 2	Studio 5 - Kinematics 2
	Tues	10-Sep		Studio 5 - Kinematics 2
	Wed	11-Sep	Lecture 6 - Dynamics 1: Newton's 1st and 3rd	Studio 6 - Dynamics 1: Newton's 1st and 3rd Laws
	Thurs	12-Sep		Studio 6 - Dynamics 1: Newton's 1st and 3rd Laws
5	Mon	16-Sep	Lecture 7 - Dynamics 2: Newton's 2nd Law	Studio 7 - Dynamics 2: Newton's 2nd Law
	Tues	17-Sep		Studio 7 - Dynamics 2: Newton's 2nd Law
	Wed	18-Sep	Lecture 8 - Dynamics 3: Applications of Newton's Laws	Studio 8 - Dynamics 3: Jumping Grasshoppers 1
	Thu	19-Sep		Studio 8 - Dynamics 3: Jumping Grasshoppers 1
	Fri	20-Sep	<b>EXAM 1 (Modules 1-7)</b>	
6	Mon	23-Sep	Lecture 9 - Dynamics 4: Applications of Newton's Laws	Studio 9 - Dynamics 4: Jumping Grasshoppers 2
	Tues	24-Sep		Studio 9 - Dynamics 4: Jumping Grasshoppers 2
	Wed	25-Sep	Lecture 10 - Impulse and Momentum	Studio 10 - Impulse and Momentum
	Thurs	26-Sep		Studio 10 - Impulse and Momentum
7	Mon	30-Sep	Lecture 11 - Stress and Strain	Studio 11 - Stress and Strain
	Tues	1-Oct		Studio 11 - Stress and Strain
	Wed	2-Oct	Lecture 12 - Torque 1	Studio 12 - Torque 1
	Thurs	3-Oct		Studio 12 - Torque 1
8	Mon	7-Oct	Lecture 13 - Torque 2	Studio 13 - Torque 2
	Tues	8-Oct		Studio 13 - Torque 2
	Wed	9-Oct	Lecture 14 - Energy 1: Forces, Work, and Kinetic Energy	Studio 14 - Energy 1: Forces, Work, and Kinetic Energy
	Thurs	10-Oct		Studio 14 - Energy 1: Forces, Work, and Kinetic Energy
	Fri	11-Oct	<b>EXAM 2 (Modules 8-13)</b>	

Week	Day	Date	Lecture Topic	Studio
9	Mon	14-Oct	Lecture 15 - Energy 2: Forces, Work, and Kinetic Energy II	Studio 15 - Energy 2: Forces, Work, and Kinetic Energy II
	Tues	15-Oct		Studio 15 - Energy 2: Forces, Work, and Kinetic Energy II
	Wed	16-Oct	<b>Lecture X</b>	<b>No studios held today</b>
	Thurs	17-Oct	<b>Holiday - Fall Break</b>	
10	Mon	21-Oct	Lecture 16 - Energy 3: Potential Energy	Studio 16 - Gravitational Potential Energy
	Tues	22-Oct		Studio 16 - Gravitational Potential Energy
	Wed	23-Oct	Lecture 17 - Energy 4: Walking and Running	Studio 17 - Energy 4: Walking and Running
	Thurs	24-Oct		Studio 17 - Energy 4: Walking and Running
11	Mon	28-Oct	Lecture 18 - Resilience	Studio 18 - Resilience
	Tues	29-Oct		Studio 18 - Resilience
	Wed	30-Oct	Lecture 19 - Potential Energy Curves	Studio 19 - Potential Energy Curves
	Thurs	31-Oct		Studio 19 - Potential Energy Curves
12	Mon	4-Nov	Lecture 20 - Chemical Energy	Studio 20 - Chemical Energy
	Tues	5-Nov		Studio 20 - Chemical Energy
	Wed	6-Nov	Lecture 21 - Oscillations 1	Studio 21 - Oscillations 1
	Thu	7-Nov		Studio 21 - Oscillations 1
	Fri	8-Nov	<b>EXAM 3 (Modules 14-20)</b>	
13	Mon	11-Nov	Lecture 22 - Oscillations 2	Studio 22 - Oscillations 2
	Tues	12-Nov		Studio 22 - Oscillations 2
	Wed	13-Nov	Lecture 23 - Thermodynamics 1	Studio 21 - Thermodynamics 1
	Thu	14-Nov		Studio 21 - Thermodynamics 1
14	Mon	18-Nov	Lecture 24 - Thermodynamics 2	Studio 24 - Thermodynamics 2
	Tues	19-Nov		Studio 24 - Thermodynamics 2
	Wed	20-Nov	Lecture 25 - Diffusion 1	Studio 25 - Diffusion 1
	Thurs	21-Nov		Studio 25 - Diffusion 1
15	Mon	25-Nov	Lecture 26 - Diffusion 2	Studio 26 - Diffusion 2
	Tues	26-Nov		Studio 26 - Diffusion 2
	Wed	27-Nov	<b>Holiday - Thanksgiving</b>	<b>No studios held today</b>
	Thurs	28-Nov	<b>Holiday - Thanksgiving</b>	<b>No studios held today</b>
16	Mon	2-Dec	Lecture 27 - Heat Transfer	Studio 27 - Heat Transfer
	Tues	3-Dec		Studio 27 - Heat Transfer
	Wed	4-Dec	Lecture Y - Review	<b>No studios held today</b>
	Fri	6-Dec	<b>FINAL EXAM (Section 001)</b>	
	Sat	7-Dec	<b>FINAL EXAM (Section 002)</b>	

# Time

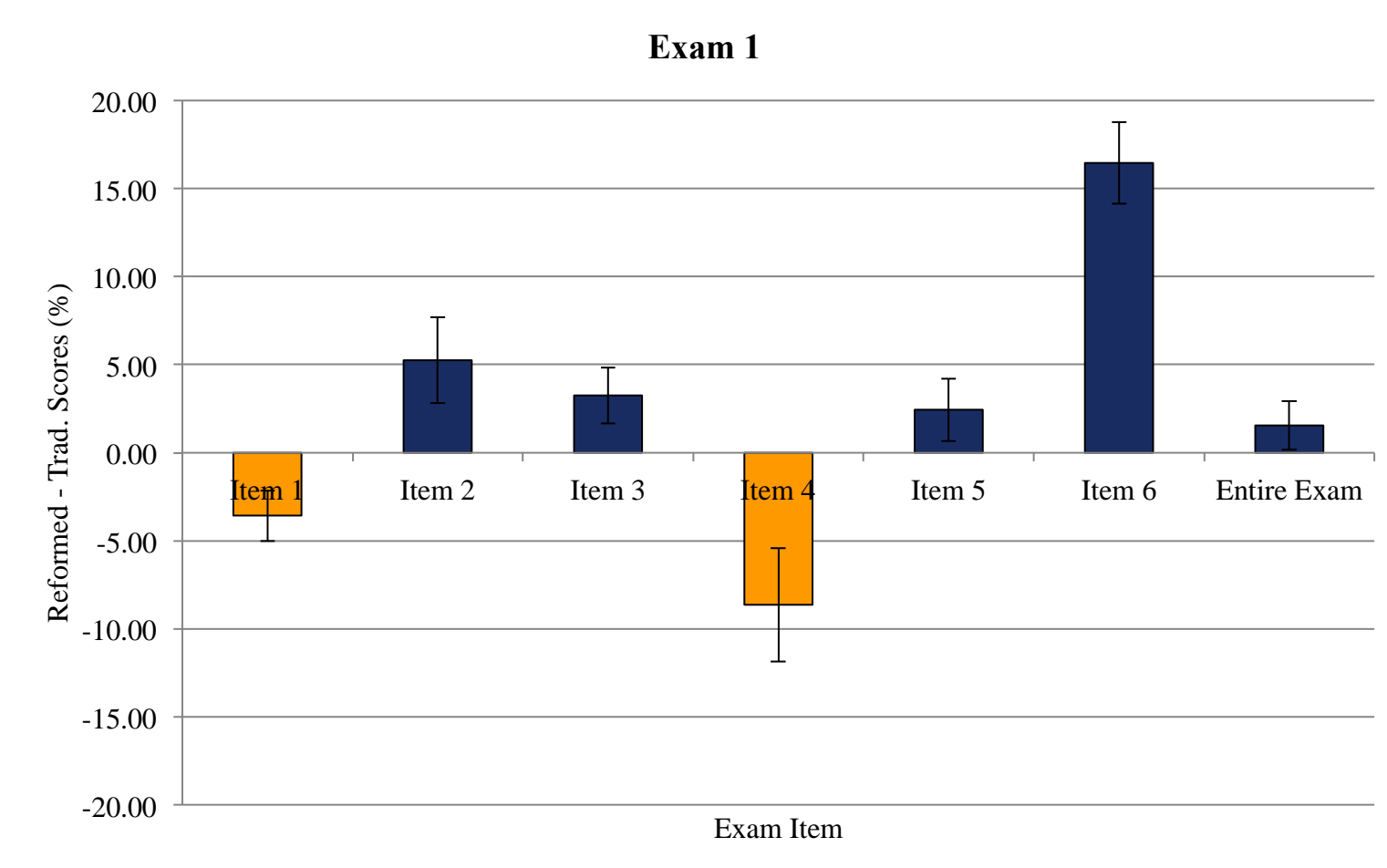
- **Lecture should provide students with just enough information.**
- **Avoid long derivations, especially if they can be found in the book.**
- **Choose 1-2 complex problems for students to do in a class period (with accompanying TPS questions).**

# Data from two different physics classes

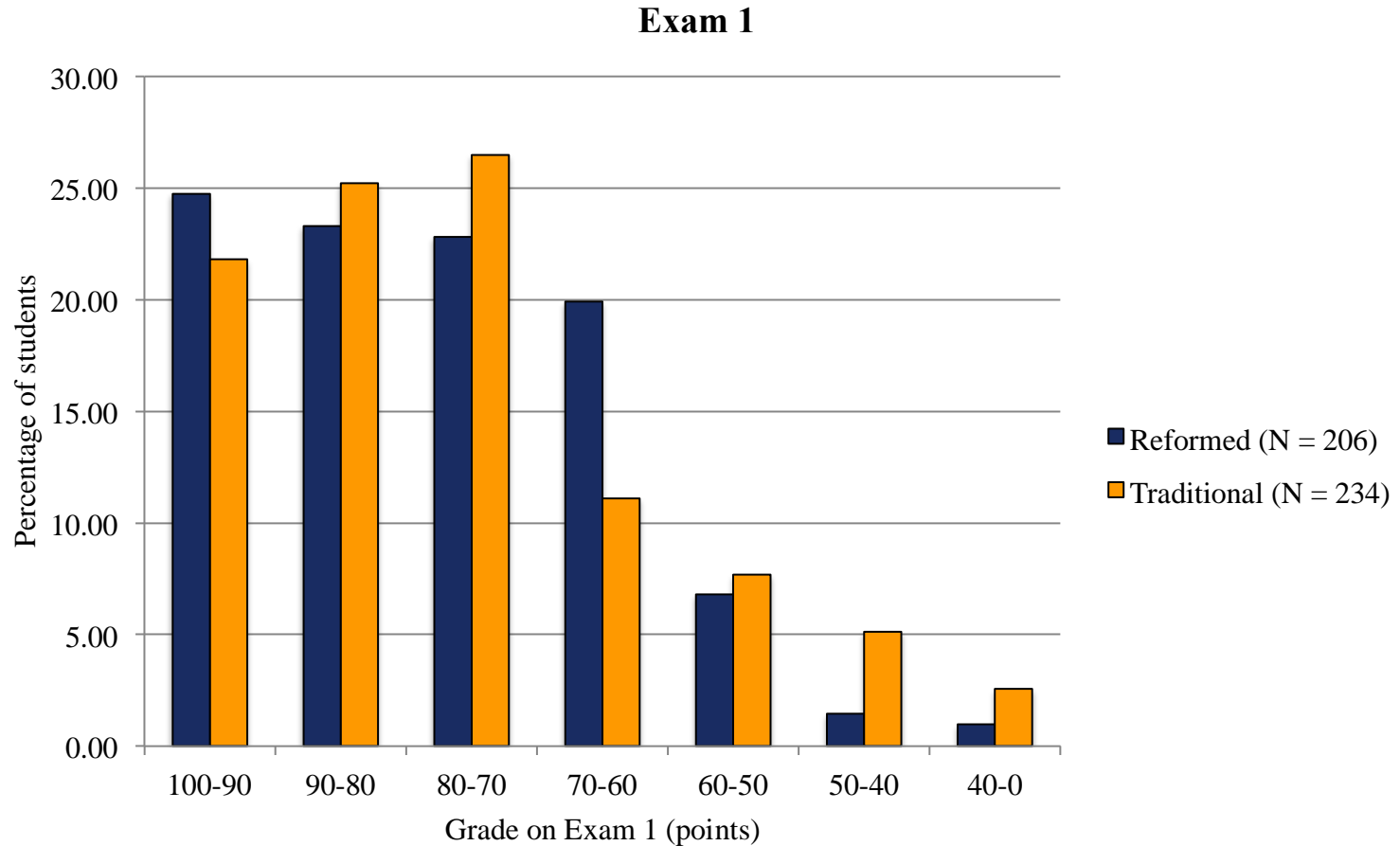




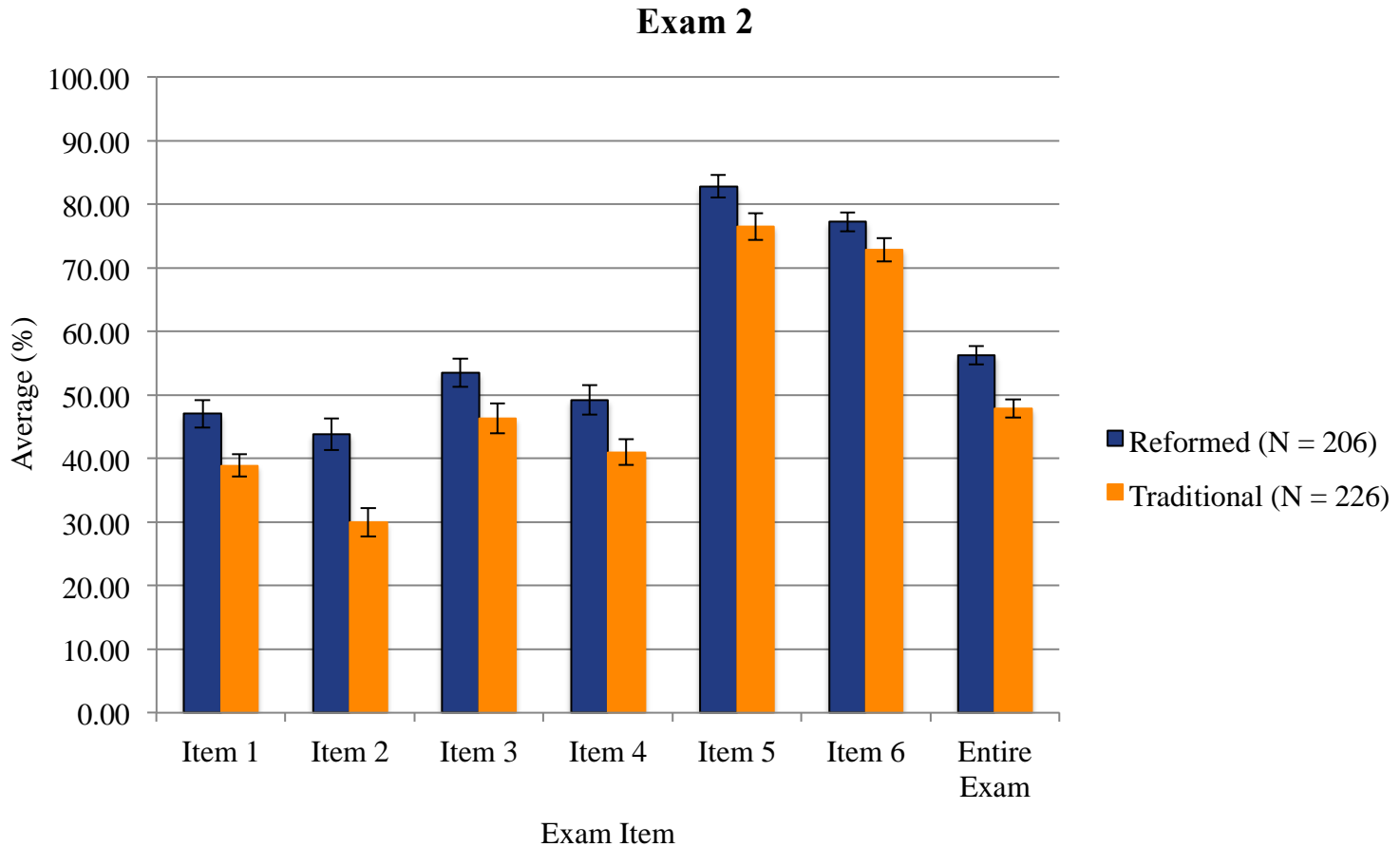
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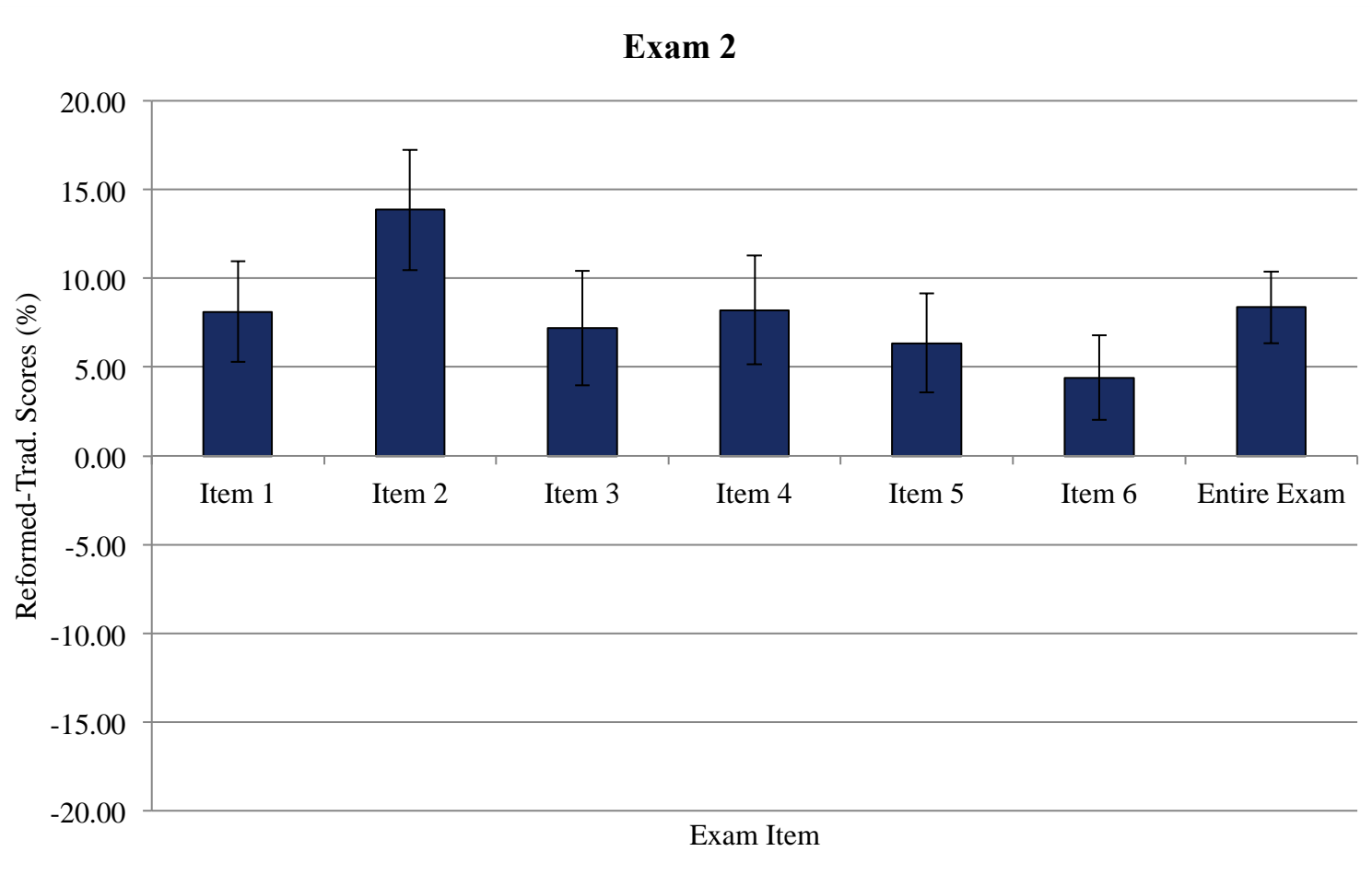
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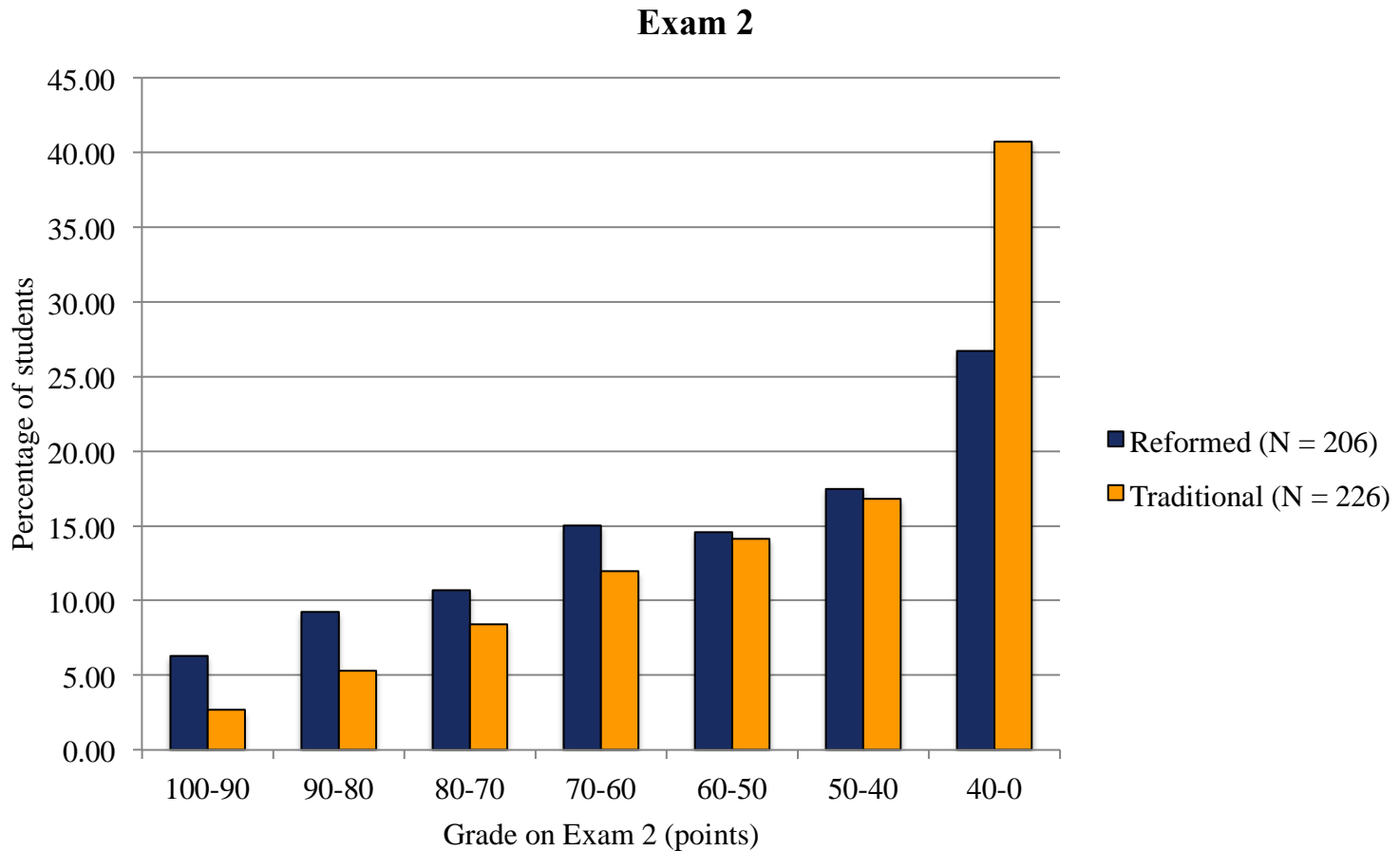
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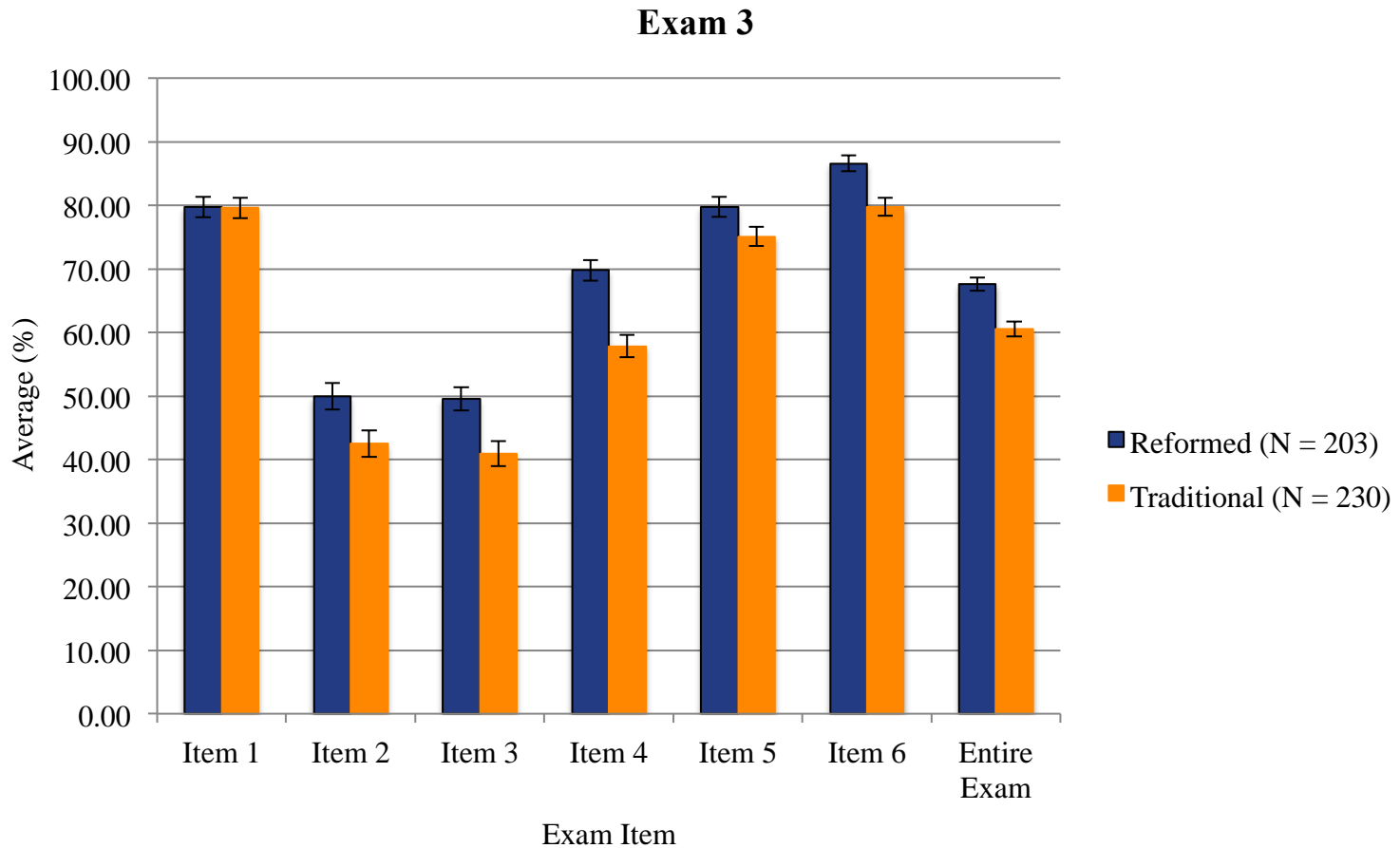
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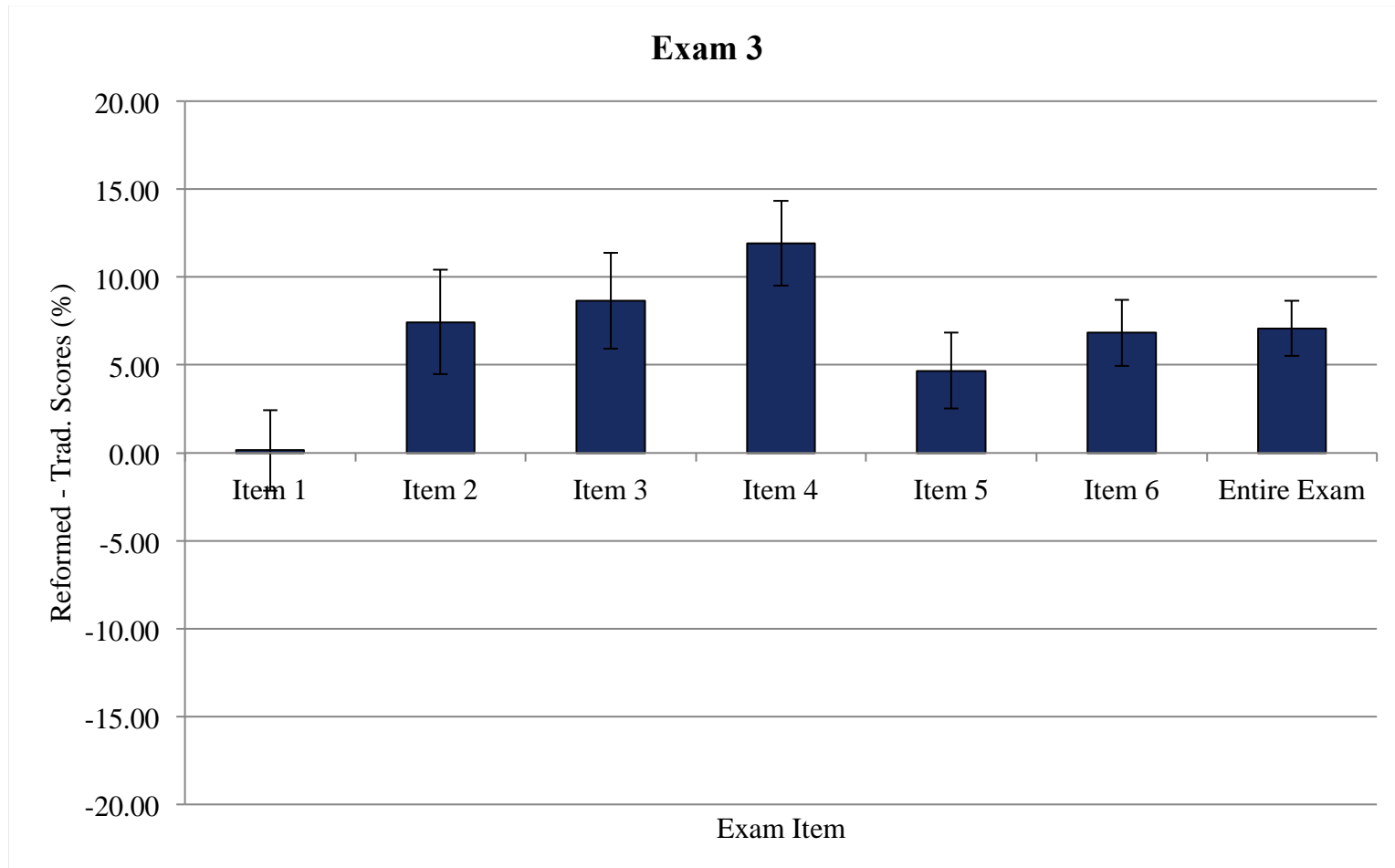
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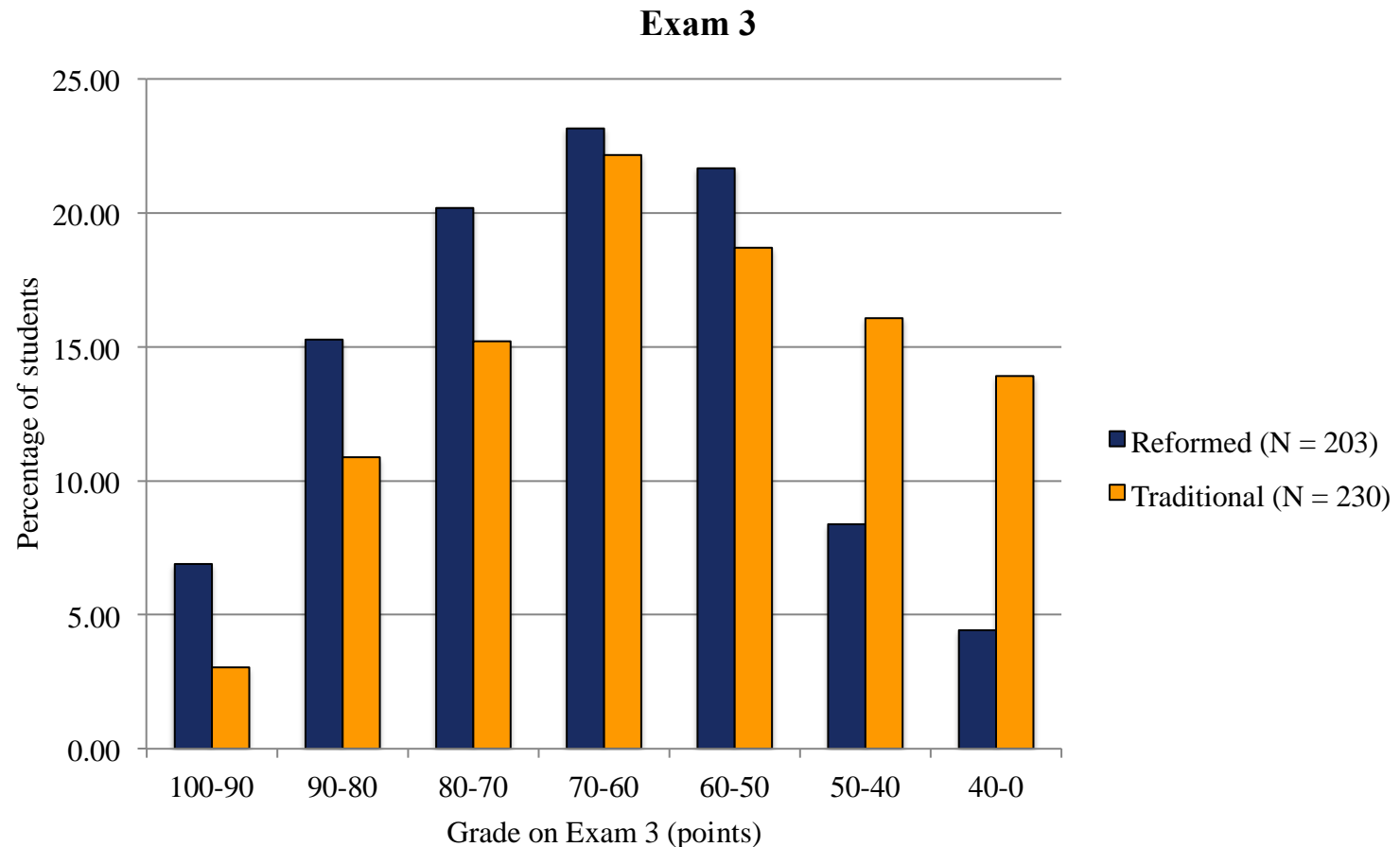
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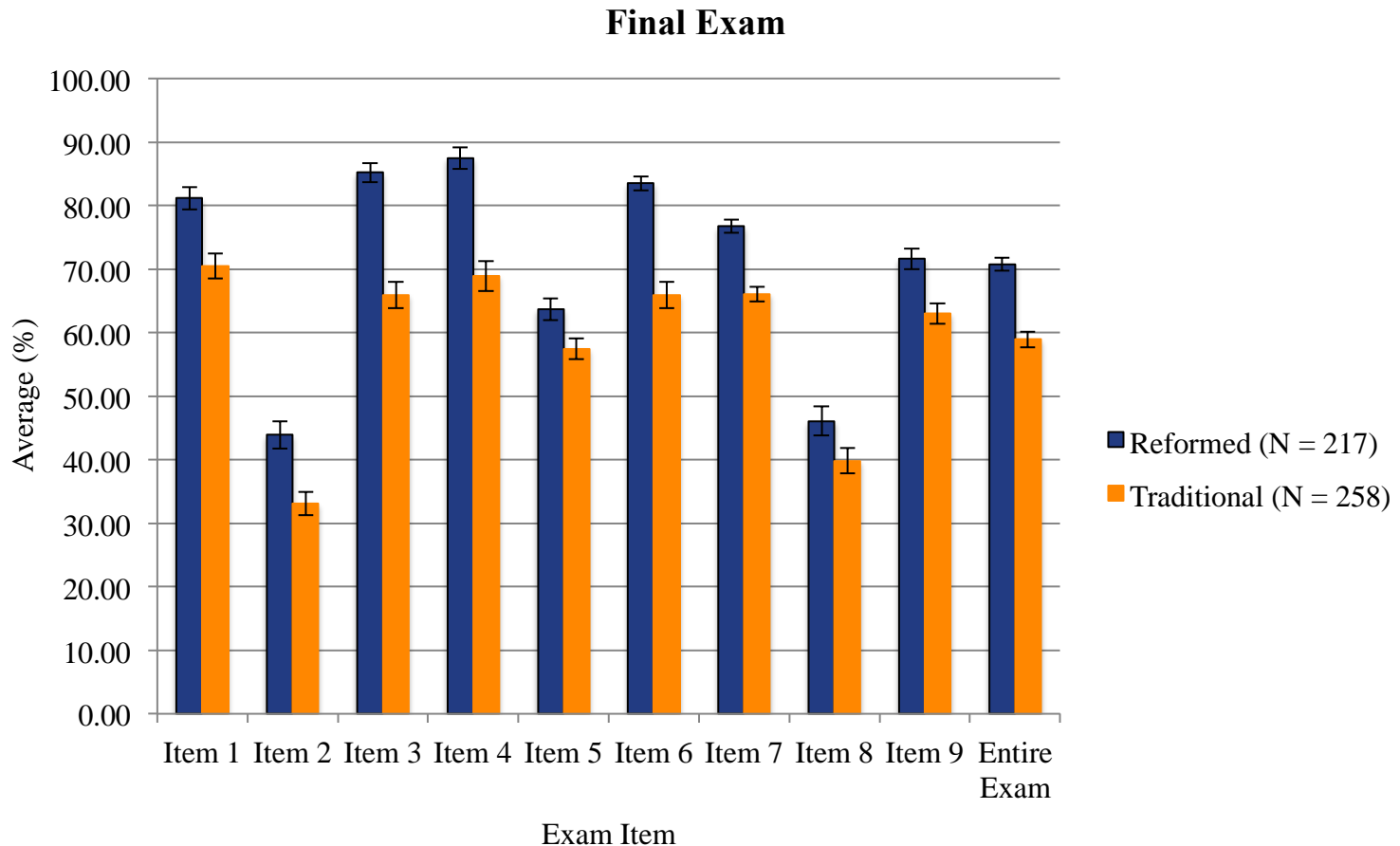


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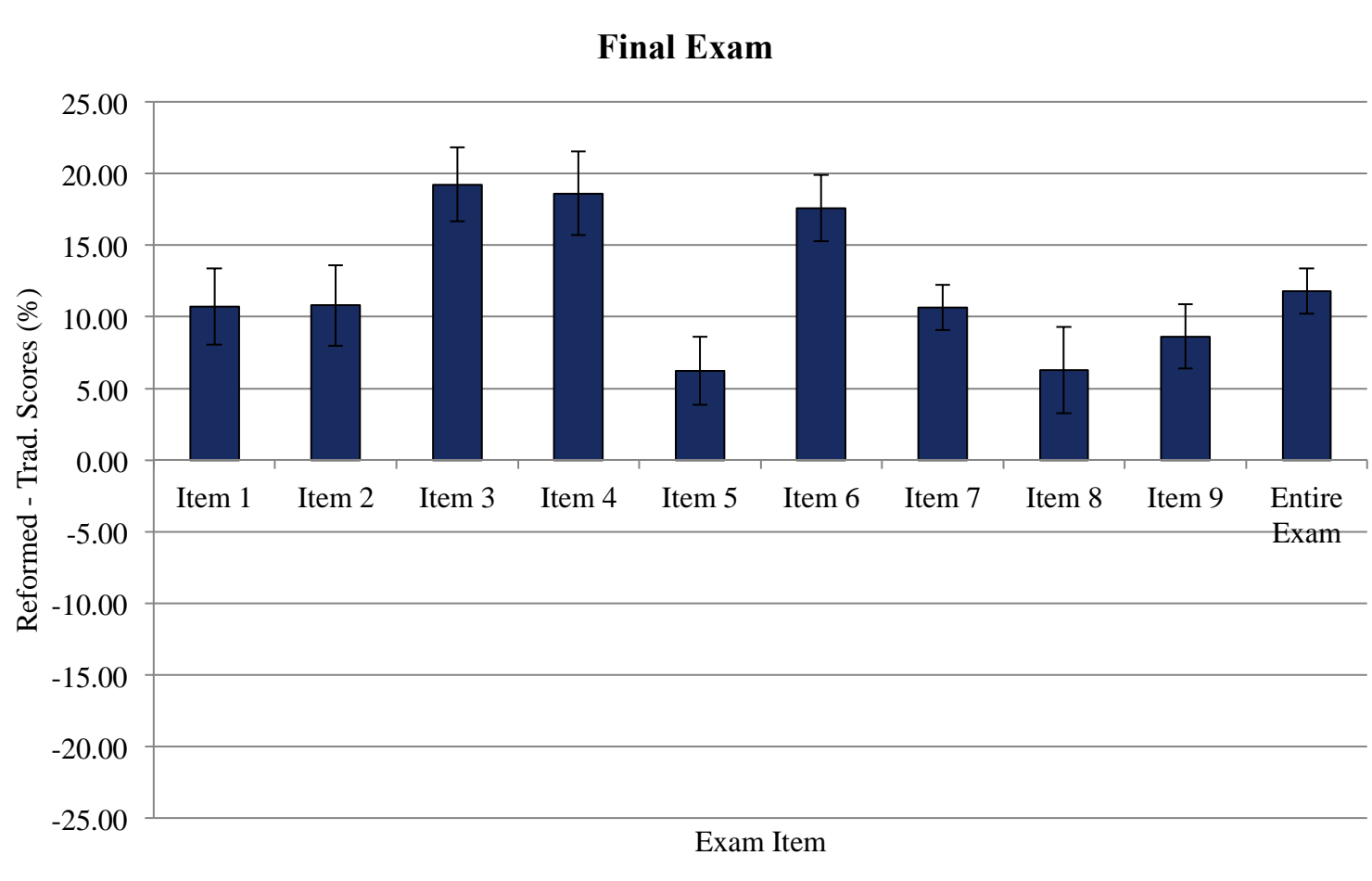




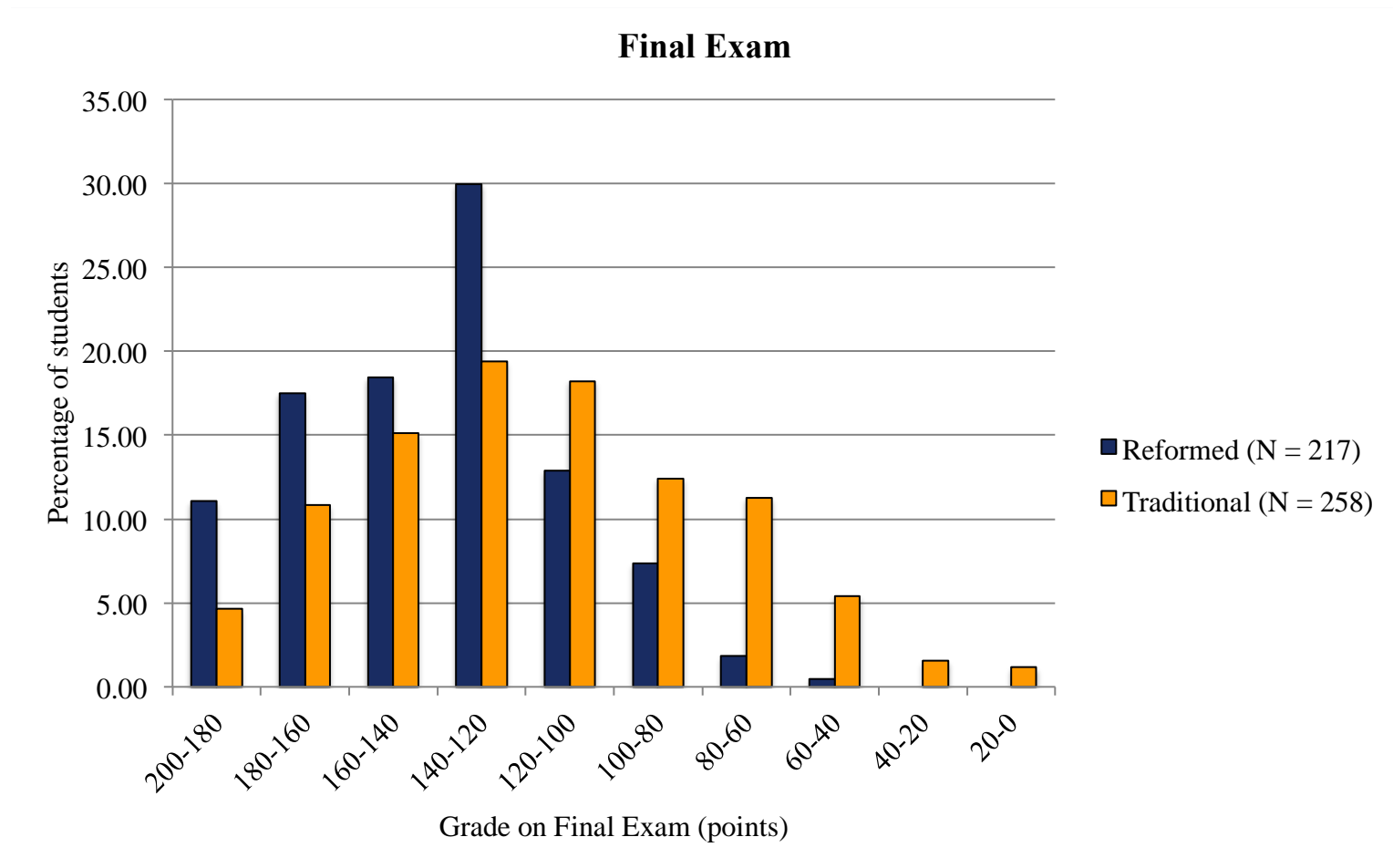
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# Resources

- 1) PowerPoint slide set “Using Think-Pair-Share (TPS) to Promote Quantitative Problem Solving: Sample Questions” on your USB drive
  - Sample quantitative problems with TPS questions for introductory physics
  - Topics include kinematics, Newton’s laws, rotation, static equilibrium, work and energy, collisions, electric fields and forces, electric potential, DC circuits, magnetic fields and forces, induction, and optics
- 2) C. S. Wallace, “Developing Peer Instruction questions for quantitative problems for an upper-division astronomy course,” *American Journal of Physics* (accepted), arXiv: 1909.02394
- 3) Feel free to contact me with any questions:  
[cswphys@email.unc.edu](mailto:cswphys@email.unc.edu)