



## 2014 $F = ma$ Contest

25 QUESTIONS - 75 MINUTES

### INSTRUCTIONS

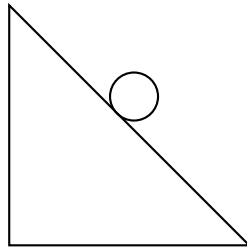
**DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN**

- Use  $g = 10 \text{ N/kg}$  throughout this contest.
- You may write in this booklet of questions. However, you will not receive any credit for anything written in this booklet.
- Your answer to each question must be marked on the optical mark answer sheet.
- Select the single answer that provides the best response to each question. Please be sure to use a No. 2 pencil and completely fill the box corresponding to your choice. If you change an answer, the previous mark must be completely erased.
- Correct answers will be awarded one point; incorrect answers will result in a deduction of  $\frac{1}{4}$  point. There is no penalty for leaving an answer blank.
- A hand-held calculator may be used. Its memory must be cleared of data and programs. You may use only the basic functions found on a simple scientific calculator. Calculators may not be shared. Cell phones may not be used during the exam or while the exam papers are present. You may not use any tables, books, or collections of formulas.
- This test contains 25 multiple choice questions. Your answer to each question must be marked on the optical mark answer sheet that accompanies the test. Only the boxes preceded by numbers 1 through 25 are to be used on the answer sheet.
- All questions are equally weighted, but are not necessarily the same level of difficulty.
- **In order to maintain exam security, do not communicate any information about the questions (or their answers or solutions) on this contest until after February 20, 2014.**
- The question booklet and answer sheet will be collected at the end of this exam. You may not use scratch paper.






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Contributors to this year's exam include David Fallest, David Jones, Jiajia Dong, Paul Stanley, Warren Turner, Qiuzi Li, and former US Team members Andrew Lin, Matthew Huang, Samuel Zbarsky.

1. A car turning to the right is traveling at constant speed in a circle. From the driver's perspective, the angular momentum vector about the center of the circle points  $X$  and the acceleration vector of the car points  $Y$  where
- (A)  $X$  is left,  $Y$  is left.
  - (B)  $X$  is forward,  $Y$  is right.
  - (C)  $X$  is down,  $Y$  is forward.
  - (D)  $X$  is left,  $Y$  is right.
  - (E)  $X$  is down,  $Y$  is right.
2. A ball rolls without slipping down an inclined plane as shown in the diagram.

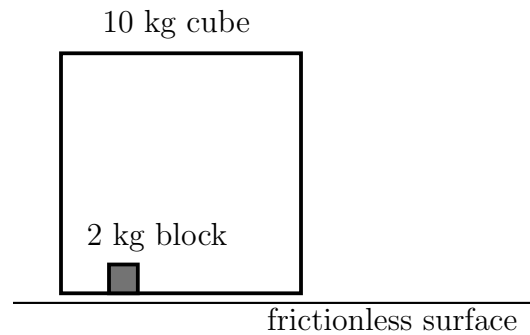


Which of the following vectors best represents the direction of the total force that the ball exerts on the plane?

- (A) 
- (B) 
- (C) 
- (D) 
- (E) 

3. An object of uniform density floats partially submerged so that 20% of the object is above the water. A 3 N force presses down on the top of the object so that the object becomes fully submerged. What is the volume of the object? The density of water is  $\rho_{\text{H}_2\text{O}} = 1000 \text{ kg/m}^3$ .
- (A)  $V_{\text{object}} = 0.3 \text{ L}$   
(B)  $V_{\text{object}} = 0.67 \text{ L}$   
(C)  $V_{\text{object}} = 1.2 \text{ L}$   
(D)  $V_{\text{object}} = 1.5 \text{ L}$   
(E)  $V_{\text{object}} = 3.0 \text{ L}$
4. What are the correct values of the numbers in the following statements? Assume there are no external forces, and take  $N = 1$  to mean that the statement cannot be made for any meaningful number of particles.
- If a particle at rest explodes into  $N_1$  or fewer particles with known masses, and the total kinetic energy of the new particles is known, the kinetic energy of each of the new particles is completely determined.
  - If a particle at rest explodes into  $N_2$  or fewer particles, the velocities of the new particles must lie in a line.
  - If a particle at rest explodes into  $N_3$  or fewer particles, the velocities of the new particles must lie in a plane.
- (A)  $N_1 = 2, N_2 = 1, N_3 = 1$   
(B)  $N_1 = 1, N_2 = 2, N_3 = 3$   
(C)  $N_1 = 2, N_2 = 2, N_3 = 3$   
(D)  $N_1 = 3, N_2 = 2, N_3 = 3$   
(E)  $N_1 = 2, N_2 = 3, N_3 = 4$
5. A unicyclist goes around a circular track of radius 30 m at a (*amazingly fast!*) constant speed of 10 m/s. At what angle to the left (or right) of vertical must the unicyclist lean to avoid falling? Assume that the height of the unicyclist is much smaller than the radius of the track.
- (A)  $9.46^\circ$   
(B)  $9.59^\circ$   
(C)  $18.4^\circ$   
(D)  $19.5^\circ$   
(E)  $70.5^\circ$

6. A cubical box of mass 10 kg with edge length 5 m is free to move on a frictionless horizontal surface. Inside is a small block of mass 2 kg, which moves without friction inside the box. At time  $t = 0$ , the block is moving with velocity 5 m/s directly towards one of the faces of the box, while the box is initially at rest. The coefficient of restitution for any collision between the block and box is 90%, meaning that the relative speed between the box and block immediately after a collision is 90% of the relative speed between the box and block immediately before the collision.

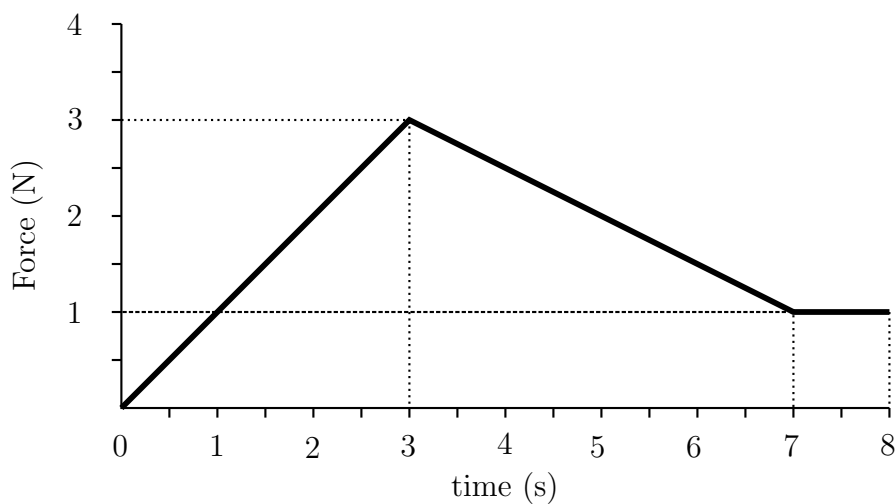


- After 1 minute, the block is a displacement  $x$  from the original position. Which of the following is closest to  $x$ ?
- (A) 0 m
  - (B) 50 m
  - (C) 100 m
  - (D) 200 m
  - (E) 300 m
7. A 1.00 m long stick with uniform density is allowed to rotate about a point 30.0 cm from its end. The stick is perfectly balanced when a 50.0 g mass is placed on the stick 20.0 cm from the same end. What is the mass of the stick?
- (A) 35.7 g
  - (B) 33.3 g
  - (C) 25.0 g
  - (D) 17.5 g
  - (E) 14.3 g

8. An object of mass  $M$  is hung on a vertical spring of spring constant  $k$  and is set into vertical oscillations. The period of this oscillation is  $T_0$ . The spring is then cut in half and the same mass is attached and the system is set up to oscillate on a frictionless inclined plane making an angle  $\theta$  to the horizontal. Determine the period of the oscillations on the inclined plane in terms of  $T_0$ .

- (A)  $T_0$
- (B)  $T_0/2$
- (C)  $2T_0 \sin \theta$
- (D)  $T_0/\sqrt{2}$
- (E)  $T_0 \sin \theta/\sqrt{2}$

9. A 5.0 kg object undergoes a time-varying force as shown in the graph below. If the velocity at  $t = 0.0$  s is +1.0 m/s, what is the velocity of the object at  $t = 7$  s?



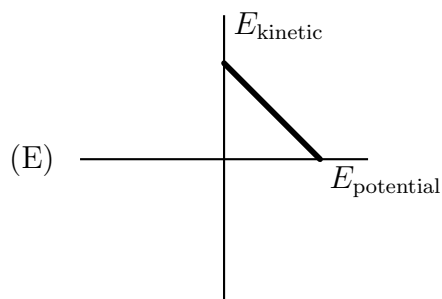
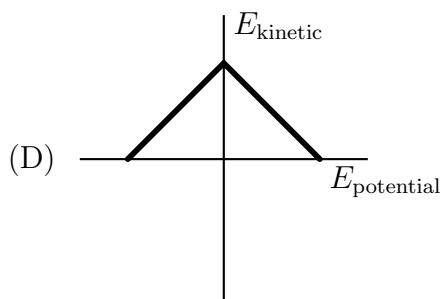
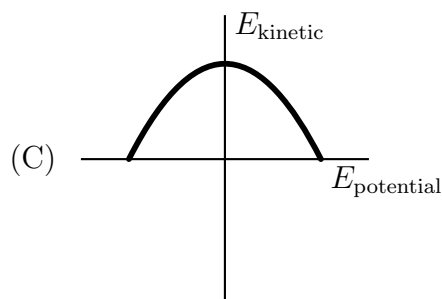
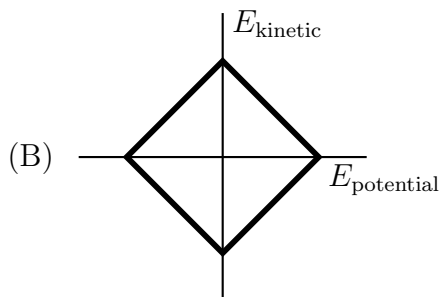
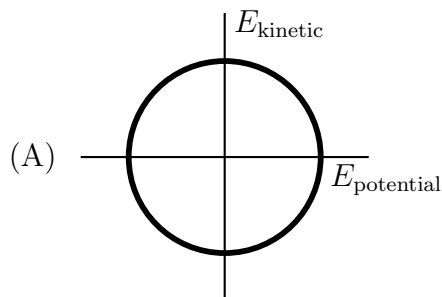
- (A) 2.45 m/s
- (B) 2.50 m/s
- (C) 3.50 m/s
- (D) 12.5 m/s
- (E) 15.0 m/s

10. A radio controlled car is attached to a stake in the ground by a 3.00 m long piece of string, and is forced to move in a circular path. The car has an initial angular velocity of 1.00 rad/s and smoothly accelerates at a rate of 4.00 rad/s<sup>2</sup>. The string will break if the centripetal acceleration exceeds  $2.43 \times 10^2$  m/s<sup>2</sup>. How long can the car accelerate at this rate before the string breaks?

- (A) 0.25 s
- (B) 0.50 s
- (C) 1.00 s
- (D) 1.50 s
- (E) 2.00 s

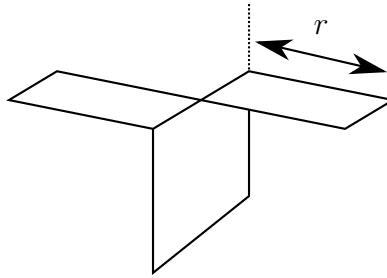
11. A point mass  $m$  is connected to an ideal spring on a horizontal frictionless surface. The mass is pulled a short distance and then released.

Which of the following is the most correct plot of the kinetic energy as a function of potential energy?



**The following information applies to questions 12 and 13**

A paper helicopter with rotor radius  $r$  and weight  $W$  is dropped from a height  $h$  in air with a density of  $\rho$ .



Assuming that the helicopter quickly reaches terminal velocity, a function for the time of flight  $T$  can be found in the form

$$T = kh^\alpha r^\beta \rho^\delta W^\omega.$$

where  $k$  is an unknown dimensionless constant (actually, 1.164).  $\alpha$ ,  $\beta$ ,  $\delta$ , and  $\omega$  are constant exponents to be determined.

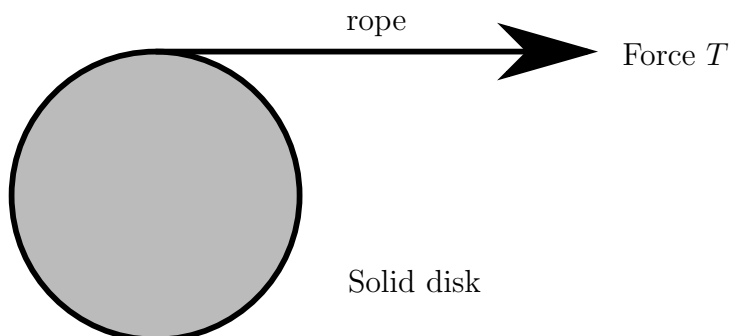
12. Determine  $\alpha$ .

- (A)  $\alpha = -1$
- (B)  $\alpha = -1/2$
- (C)  $\alpha = 0$
- (D)  $\alpha = 1/2$
- (E)  $\alpha = 1$

13. Determine  $\beta$ .

- (A)  $\beta = 1/3$
- (B)  $\beta = 1/2$
- (C)  $\beta = 2/3$
- (D)  $\beta = 1$
- (E)  $\beta$  can not be uniquely determined without more information.

14. A disk of moment of inertia  $I$ , mass  $M$ , and radius  $R$  has a cord wrapped around it tightly as shown in the diagram. The disk is free to slide on its side as shown in the top down view. A constant force of  $T$  is applied to the end of the cord and accelerates the disk along a frictionless surface.



After the disk has accelerated some distance, determine the ratio of the translational KE to total KE of the disk,

$$KE_{\text{translational}}/KE_{\text{total}} =$$

- (A)  $\frac{I}{MR^2}$   
 (B)  $\frac{MR^2}{I}$   
 (C)  $\frac{I}{3MR^2}$   
 (D)  $\frac{I}{MR^2+I}$   
 (E)  $\frac{MR^2}{MR^2+I}$
15. The maximum torque output from the engine of a new experimental car of mass  $m$  is  $\tau$ . The maximum rotational speed of the engine is  $\omega$ . The engine is designed to provide a constant power output  $P$ . The engine is connected to the wheels via a perfect transmission that can smoothly trade torque for speed with no power loss. The wheels have a radius  $R$ , and the coefficient of static friction between the wheels and the road is  $\mu$ .

What is the maximum sustained speed  $v$  the car can drive up a 30 degree incline? Assume no frictional losses and assume  $\mu$  is large enough so that the tires do not slip.

- (A)  $v = 2P/(mg)$   
 (B)  $v = 2P/(\sqrt{3}mg)$   
 (C)  $v = 2P/(\mu mg)$   
 (D)  $v = \tau\omega/(mg)$   
 (E)  $v = \tau\omega/(\mu mg)$



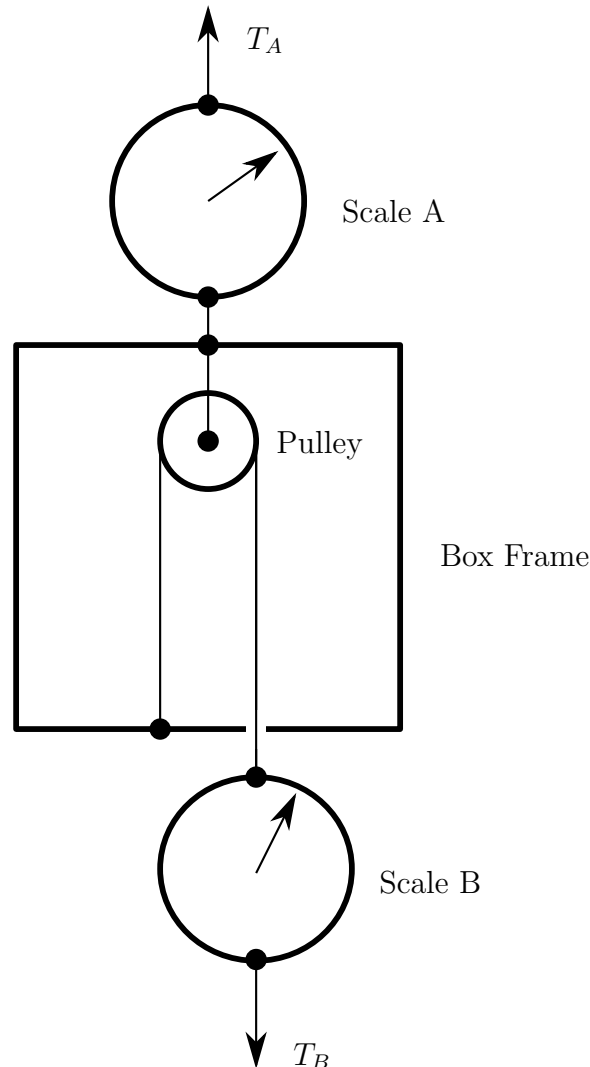
16. An object of mass  $m_1$  initially moving at speed  $v_0$  collides with an object of mass  $m_2 = \alpha m_1$ , where  $\alpha < 1$ , that is initially at rest. The collision could be completely elastic, completely inelastic, or partially inelastic. After the collision the two objects move at speeds  $v_1$  and  $v_2$ . Assume that the collision is one dimensional, and that object one cannot pass through object two.

After the collision, the speed ratio  $r_1 = v_1/v_0$  of object 1 is bounded by

- (A)  $(1 - \alpha)/(1 + \alpha) \leq r_1 \leq 1$
  - (B)  $(1 - \alpha)/(1 + \alpha) \leq r_1 \leq 1/(1 + \alpha)$
  - (C)  $\alpha/(1 + \alpha) \leq r_1 \leq 1$
  - (D)  $0 \leq r_1 \leq 2\alpha/(1 + \alpha)$
  - (E)  $1/(1 + \alpha) \leq r_1 \leq 2/(1 + \alpha)$
17. A spherical cloud of dust in space has a uniform density  $\rho_0$  and a radius  $R_0$ . The gravitational acceleration of free fall at the surface of the cloud due to the mass of the cloud is  $g_0$ .
- A process occurs (heat expansion) that causes the cloud to suddenly grow to a radius  $2R_0$ , while maintaining a uniform (but not constant) density. The gravitational acceleration of free fall at a point  $R_0$  away from the center of the cloud due to the mass of the cloud is now

- (A)  $g_0/32$
- (B)  $g_0/16$
- (C)  $g_0/8$
- (D)  $g_0/4$
- (E)  $g_0/2$

18. Consider the following diagram of a box and two weight scales. Scale A supports the box via a massless rope. A pulley is attached to the top of the box; a second massless rope passes over the pulley, one end is attached to the box and the other end to scale B. The two scales read indicate the tensions  $T_A$  and  $T_B$  in the ropes. Originally scale A reads 30 Newtons and scale B reads 20 Newtons.



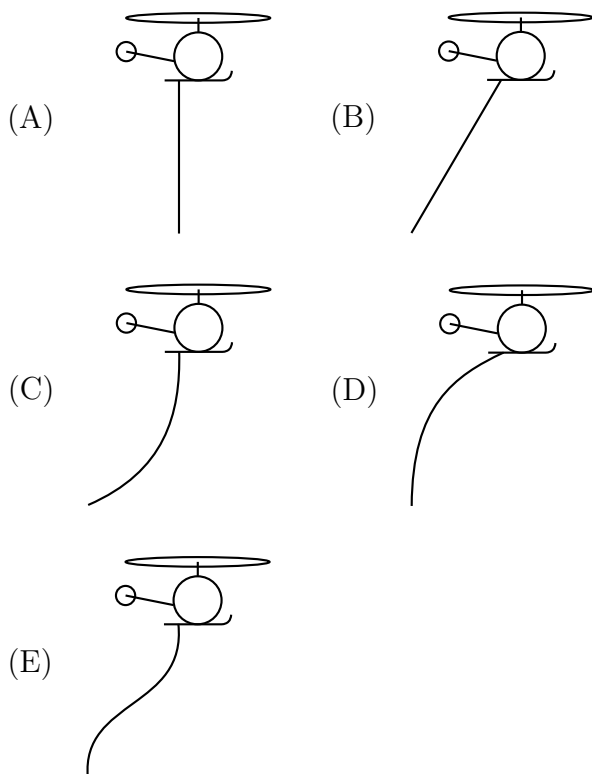
If an additional force pulls down on scale B so that the reading increases to 30 Newtons, what will be the new reading on scale A?

- (A) 35 Newtons
- (B) 40 Newtons
- (C) 45 Newtons
- (D) 50 Newtons
- (E) 60 Newtons

*Adapted from a demonstration by Richard Berg.*

19. A helicopter is flying horizontally at constant speed. A perfectly flexible uniform cable is suspended beneath the helicopter; air friction on the cable is *not* negligible.

Which of the following diagrams best shows the shape of the cable as the helicopter flies through the air to the right?

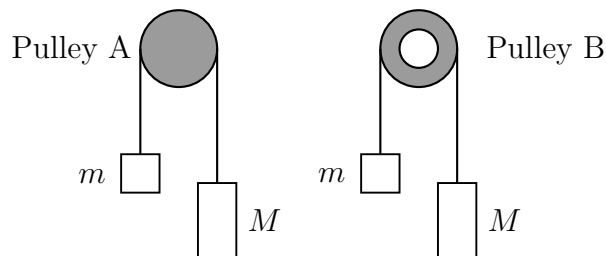


20. A crew of scientists has built a new space station. The space station is shaped like a wheel of radius  $R$ , with essentially all its mass  $M$  at the rim. When the crew arrives, the station will be set rotating at a rate that causes an object at the rim to have radial acceleration  $g$ , thereby simulating Earth's surface gravity. This is accomplished by two small rockets, each with thrust  $T$  newtons, mounted on the station's rim. How long a time  $t$  does one need to fire the rockets to achieve the desired condition?

- (A)  $t = \sqrt{gR^3} M/(2T)$   
 (B)  $t = \sqrt{gR} M/(2T)$   
 (C)  $t = \sqrt{gR} M/T$   
 (D)  $t = \sqrt{gR/\pi} M/T$   
 (E)  $t = \sqrt{gR} M/(\pi T)$

Adapted from a problem in *Physics for Scientists and Engineers* by Richard Wolfson

21. Two pulleys (shown in figure) are made of the same metal with density  $\rho$ . Pulley A is a uniform disk with radius  $R$ . Pulley B is identical except a circle of  $R/2$  is removed from the center. When two boxes  $M = \alpha m$  ( $\alpha > 1$ ) are connected over the pulleys through a massless rope and move without slipping, what is the ratio between the accelerations in system A and B? The mass of pulley A is  $M + m$ .



- (A)  $a_A/a_B = 47/48$   
 (B)  $a_A/a_B = 31/32$   
 (C)  $a_A/a_B = 15/16$   
 (D)  $a_A/a_B = 9/16$   
 (E)  $a_A/a_B = 3/4$
22. A body of mass  $M$  and a body of mass  $m \ll M$  are in circular orbits about their center of mass under the influence of their mutual gravitational attraction to each other. The distance between the bodies is  $R$ , which is much larger than the size of either body. A small amount of matter  $\delta m \ll m$  is removed from the body of mass  $m$  and transferred to the body of mass  $M$ . The transfer is done in such a way so that the orbits of the two bodies remain circular, and remain separated by a distance  $R$ . Which of the following statements is correct?
- (A) The gravitational force between the two bodies increases.  
 (B) The gravitational force between the two bodies remains constant.  
 (C) The total angular momentum of the system increases.  
 (D) The total angular momentum of the system remains constant.  
 (E) The period of the orbit of two bodies remains constant.

**The following information applies to questions 23 and 24**

A 100 kg astronaut carries a launcher loaded with a 10 kg bowling ball; the launcher and the astronaut's spacesuit have negligible mass. The astronaut discovers that firing the launcher results in the ball moving away from her at a relative speed of 50 m/s.

23. What is the impulse delivered to the astronaut when firing the launcher?
- (A) 455 N s
  - (B) 500 N s
  - (C) 550 N s
  - (D) 5000 N s
  - (E) 5500 N s
24. The astronaut in the previous situation is now moving at 10 m/s (as measured in a certain frame of reference). She wishes to fire the launcher so that her velocity turns through as large an angle as possible (in this frame of reference). What is this maximum angle? (*Hint*: a diagram may be useful.)
- (A) 24.4°
  - (B) 26.6°
  - (C) 27.0°
  - (D) 30.0°
  - (E) 180.0°
25. A block with mass  $m$  is released from rest at the top of a frictionless ramp. The block starts at a height  $h_1$  above the base of the ramp, slides down the ramp, and then up a second ramp. The coefficient of kinetic friction between the block and the second ramp is  $\mu_k$ . If both ramps make an angle of  $\theta$  with the horizontal, to what height  $h_2$  above the base of the second ramp will the block rise?
- (A)  $h_2 = (h_1 \sin \theta) / (\mu_k \cos \theta + \sin \theta)$
  - (B)  $h_2 = (h_1 \sin \theta) / (\mu_k + \sin \theta)$
  - (C)  $h_2 = (h_1 \sin \theta) / (\mu_k \cos^2 \theta + \sin \theta)$
  - (D)  $h_2 = (h_1 \sin \theta) / (\mu_k \cos^2 \theta + \sin^2 \theta)$
  - (E)  $h_2 = (h_1 \cos \theta) / (\mu_k \sin \theta + \cos \theta)$