AAPT AIP UNITED STATES PHYSICS TEAM 2018

USA Physics Olympiad Exam

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Important Instructions for the Exam Supervisor

- This examination consists of two parts: Part A has three questions and is allowed 90 minutes; Part B also has three questions and is allowed 90 minutes.
- The first page that follows is a cover sheet. Examinees may keep the cover sheet for both parts of the exam.
- The parts are then identified by the center header on each page. Examinees are only allowed to do one part at a time, and may not work on other parts, even if they have time remaining.
- Allow 90 minutes to complete Part A. Do not let students look at Part B. Collect the answers to Part A before allowing the examinee to begin Part B. Examinees are allowed a 10 to 15 minutes break between parts A and B.
- Allow 90 minutes to complete Part B. Do not let students go back to Part A.
- Ideally the test supervisor will divide the question paper into 3 parts: the cover sheet (page 2), Part A (pages 3-5), Part B (pages 7-9), and several answer sheets for one of the questions in Part A (pages 11-13). Examinees should be provided parts A and B individually, although they may keep the cover sheet. The answer sheets should be printed single sided!
- The supervisor *must* collect all examination questions, including the cover sheet, at the end of the exam, as well as any scratch paper used by the examinees. Examinees may *not* take the exam questions. The examination questions may be returned to the students after April 21, 2018.
- Examinees are allowed calculators, but they may not use symbolic math, programming, or graphic features of these calculators. Calculators may not be shared and their memory must be cleared of data and programs. Cell phones, PDA's or cameras may not be used during the exam or while the exam papers are present. Examinees may not use any tables, books, or collections of formulas.

AAPT AIP UNITED STATES PHYSICS TEAM 2018

USA Physics Olympiad Exam

INSTRUCTIONS

DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

- Work Part A first. You have 90 minutes to complete all three problems. Each question is worth 25 points. Do not look at Part B during this time.
- After you have completed Part A you may take a break.
- Then work Part B. You have 90 minutes to complete three problems. Each question is worth 25 points. Do not look at Part A during this time.
- Show all your work. Partial credit will be given. Do not write on the back of any page. Do not write anything that you wish graded on the question sheets.
- Start each question on a new sheet of paper. Put your AAPT ID number, your proctor's AAPT ID number, the question number and the page number/total pages for this problem, in the upper right hand corner of each page. For example,

  Student AAPT ID #

  Proctor AAPT ID #

  A1 - 1/3

- A hand-held calculator may be used. Its memory must be cleared of data and programs. You may use only the basic functions found on a simple scientific calculator. Calculators may not be shared. Cell phones, PDA's or cameras may not be used during the exam or while the exam papers are present. You may not use any tables, books, or collections of formulas.
- Questions with the same point value are not necessarily of the same difficulty.
- **In order to maintain exam security, do not communicate any information about the questions (or their answers/solutions) on this contest until after April 13, 2018.**

Possibly Useful Information. You may use this sheet for both parts of the exam.

$g = 9.8$ N/kg	$G = 6.67 \times 10^{-11}$ N $\cdot$ m^2/kg^2
$k = 1/4\pi\epsilon_0 = 8.99 \times 10^9$ N $\cdot$ m^2/C^2	$k_\mathrm{m} = \mu_0/4\pi = 10^{-7}$ T $\cdot$ m/A
$c = 3.00 \times 10^8$ m/s	$k_\mathrm{B} = 1.38 \times 10^{-23}$ J/K
$N_\mathrm{A} = 6.02 \times 10^{23}$ (mol)$^{-1}$	$R = N_\mathrm{A} k_\mathrm{B} = 8.31$ J/(mol $\cdot$ K)
$\sigma = 5.67 \times 10^{-8}$ J/(s $\cdot$ m^2 $\cdot$ K^4)	$e = 1.602 \times 10^{-19}$ C
1 eV $= 1.602 \times 10^{-19}$ J	$h = 6.63 \times 10^{-34}$ J $\cdot$ s $= 4.14 \times 10^{-15}$ eV $\cdot$ s
$m_e = 9.109 \times 10^{-31}$ kg $= 0.511$ MeV/c^2	$(1+x)^n \approx 1 + nx$ for $\lvert x\rvert \ll 1$
$\sin\theta \approx \theta - \frac{1}{6}\theta^3$ for $\lvert\theta\rvert \ll 1$	$\cos\theta \approx 1 - \frac{1}{2}\theta^2$ for $\lvert\theta\rvert \ll 1$

Part A

Question A1

a. Suppose you drop a block of mass m vertically onto a fixed ramp with angle θ with coefficient of static and kinetic friction μ. The block is dropped in such a way that it does not rotate after colliding with the ramp. Throughout this problem, assume the time of the collision is negligible.

   i. Suppose the block's speed just before it hits the ramp is v and the block slides down the ramp immediately after impact. What is the speed of the block right after the collision?

   ii. What is the minimum μ such that the speed of the block right after the collision is 0?

b. Now suppose you drop a sphere with mass m, radius R and moment of inertia $\beta m R^2$ vertically onto the same fixed ramp such that it reaches the ramp with speed v.

   i. Suppose the sphere immediately begins to roll without slipping. What is the new speed of the sphere in this case?

   ii. What is the minimum coefficient of friction such that the sphere rolls without slipping immediately after the collision?

Question A2

For this problem, graphical answers should be drawn on the answer sheets graphs provided. Supporting work is to be written on blank answer sheets. Incorrect graphs without supporting work will receive no partial credit.

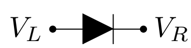
The current I as a function of voltage V for a certain electrical device is

$$I = I_0 e^{-qV_0/k_B T}\left(e^{qV/k_B T} - 1\right)$$

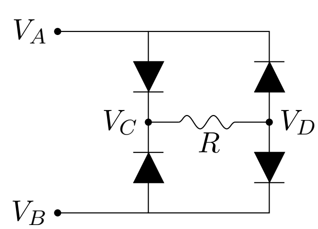
where q is the magnitude of the charge on an electron, k_B is Boltzmann's constant, and T is the absolute temperature. I_0 and V_0 are non-zero positive constants. Throughout this problem assume low temperature values $k_B T \ll qV_0$.

a. On the answer sheets, sketch a graph of the current versus voltage for low temperature values $k_B T \ll qV_0$, clearly indicating any asymptotic behavior.

Shown is a schematic for the device. Positive voltage means that the electric potential of the left hand side of the device is higher than the right hand side. For this device, $I_0 = 25\,\mu\text{A}$ and $V_0 = 1.0\,\text{V}$.

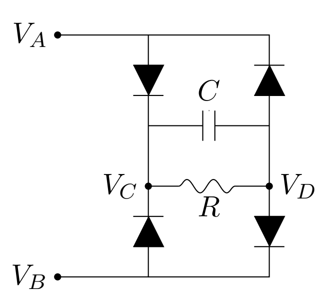


Below is a circuit made up of these elements. The voltage supplied the circuit is sinusoidal, $V_{AB} = V_A - V_B = V_s \sin\omega t$, and is also shown on answer sheets. The resistance is $R = 5.0\,\Omega$ and $V_s = 5.0\,\text{V}$.



b. Sketch the potential difference $V_{CD} = V_C - V_D$ as a function of time on the answer sheet. For your convenience, V_{AB} is shown in light gray. Assume that V_{AB} has been running for a long time.

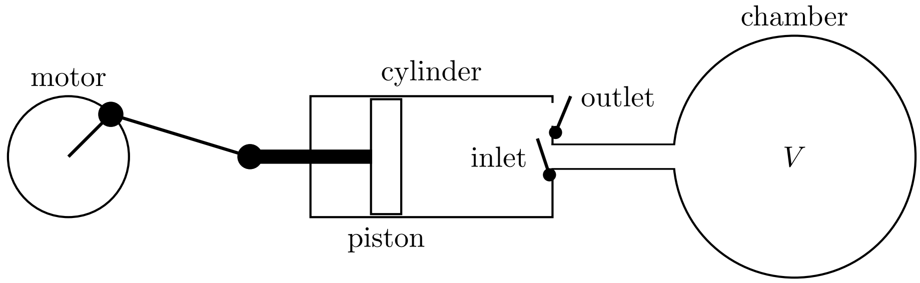
A capacitor is connected to the circuit as shown below. The capacitance is $C = 50\,\text{mF}$.



c. Sketch the new potential difference $V_{CD} = V_C - V_D$ as a function of time on the answer sheet. For your convenience, V_{AB} is shown in light gray. Assume that V_{AB} has been running for a long time.

Question A3

A vacuum system consists of a chamber of volume V connected to a vacuum pump that is a cylinder with a piston that moves left and right. The minimum volume in the pump cylinder is V_0, and the maximum volume in the cylinder is $V_0 + \Delta V$. You should assume that $\Delta V \ll V$.



The cylinder has two valves. The inlet valve opens when the pressure inside the cylinder is lower than the pressure in the chamber, but closes when the piston moves to the right. The outlet valve opens when the pressure inside the cylinder is greater than atmospheric pressure P_a, and closes when the piston moves to the left. A motor drives the piston to move back and forth. The piston moves at such a rate that heat is not conducted in or out of the gas contained in the cylinder during the pumping cycle. One complete cycle takes a time Δt. You should assume that Δt is a very small quantity, but $\Delta V/\Delta t = R$ is finite. The gas in the chamber is ideal monatomic and remains at a fixed temperature of T_a.

Start with assumption that $V_0 = 0$ and there are no leaks in the system.

a. At $t = 0$ the pressure inside the chamber is P_a. Find an equation for the pressure at a later time t.

b. Find an expression for the temperature of the gas as it is emitted from the pump cylinder into the atmosphere. Your answer may depend on time.

For the remainder of this problem $0 < V_0 < \Delta V \ll V$.

c. Find an expression for the minimum possible pressure in the chamber, P_{min}.

STOP: Do Not Continue to Part B

If there is still time remaining for Part A, you should review your work for Part A, but do not continue to Part B until instructed by your exam supervisor.

Part B

Question B1

The electric potential at the center of a cube with uniform charge density ρ and side length a is

$$\Phi \approx \frac{0.1894\rho a^2}{\epsilon_0}.$$

You do not need to derive this.[1]

For the entirety of this problem, any computed numerical constants should be written to three significant figures.

a. What is the electric potential at a corner of the same cube? Write your answer in terms of ρ, a, ϵ_0, and any necessary numerical constants.

b. What is the electric potential at the tip of a pyramid with a square base of side length a, height $a/2$, and uniform charge density ρ? Write your answer in terms of ρ, a, ϵ_0, and any necessary numerical constants.

c. What is the electric potential due to a square plate with side length a of uniform charge density σ at a height $a/2$ above its center? Write your answer in terms of σ, a, ϵ_0, and any necessary numerical constants.

d. Let $E(z)$ be the electric field at a height z above the center of a square with charge density σ and side length a. If the electric potential at the center of the square is approximately $\frac{0.281a\sigma}{\epsilon_0}$, estimate $E(a/2)$ by assuming that $E(z)$ is linear in z for $0 < z < a/2$. Write your answer in terms of σ, a, ϵ_0, and any necessary numerical constants.

[1] See https://arxiv.org/pdf/chem-ph/9508002.pdf for more details if you are interested.

Question B2

In this problem, use a particle-like model of photons: they propagate in straight lines and obey the law of reflection, but are subject to the quantum uncertainty principle. You may use small-angle approximations throughout the problem.

A photon with wavelength λ has traveled from a distant star to a telescope mirror, which has a circular cross-section with radius R and a focal length $f \gg R$. The path of the photon is nearly aligned to the axis of the mirror, but has some slight uncertainty $\Delta\theta$. The photon reflects off the mirror and travels to a detector, where it is absorbed by a particular pixel on a charge-coupled device (CCD).

Suppose the telescope mirror is manufactured so that photons coming in parallel to each other are focused to the same pixel on the CCD, regardless of where they hit the mirror. Then all small cross-sectional areas of the mirror are equally likely to include the point of reflection for a photon.

a. Find the standard deviation Δr of the distribution for r, the distance from the center of the telescope mirror to the point of reflection of the photon.

b. Use the uncertainty principle, $\Delta r \Delta p_r \geq \hbar/2$, to place a bound on how accurately we can know the angle of the photon from the axis of the telescope. Give your answer in terms of R and λ. If you were unable to solve part a, you may also give your answer in terms of Δr.

c. Suppose we want to build a telescope that can tell with high probability whether a photon it detected from Alpha Centauri A came the left half or right half of the star. Approximately how large would a telescope have to be to achieve this? Alpha Centauri A is approximately 4×10^{16} m from Earth and has a radius approximately 7×10^{8} m. Assume visible light with $\lambda = 500$ nm.

Question B3

Radiation pressure from the sun is responsible for cleaning out the inner solar system of small particles.

a. The force of radiation on a spherical particle of radius r is given by

$$F = PQ\pi r^2$$

where P is the radiation pressure and Q is a dimensionless quality factor that depends on the relative size of the particle r and the wavelength of light λ. Throughout this problem assume that the sun emits a single wavelength λ_{max}; unless told otherwise, leave your answers in terms of symbolic variables.

i. Given that the total power radiated from the sun is given by $L_\odot$, find an expression for the radiation pressure a distance R from the sun.

ii. Assuming that the particle has a density ρ, derive an expression for the ratio $\frac{F_{radiation}}{F_{gravity}}$ in terms of $L_\odot$, mass of sun $M_\odot$, ρ, particle radius r, and quality factor Q.

iii. The quality factor is given by one of the following

- If $r \ll \lambda$, $Q \sim (r/\lambda)^2$
- If $r \sim \lambda$, $Q \sim 1$.
- If $r \gg \lambda$, $Q = 1$

Considering the three possible particle sizes, which is most likely to be blown away by the solar radiation pressure?

b. The **Poynting-Robertson** effect acts as another mechanism for cleaning out the solar system.

i. Assume that a particle is in a circular orbit around the sun. Find the speed of the particle v in terms of $M_\odot$, distance from sun R, and any other fundamental constants.

ii. Because the particle is moving, the radiation force is *not* directed directly away from the sun. Find the torque τ on the particle because of radiation pressure. You may assume that $v \ll c$.

iii. Since $\tau = dL/dt$, the angular momentum L of the particle changes with time. As such, develop a differential equation to find dR/dt, the rate of change of the radial location of the particle. You may assume the orbit is always quasi circular.

iv. Develop an expression for the time required to remove particles of size $r \approx 1\,\text{cm}$ and density $\rho \approx 1000\,\text{kg/m}^3$ originally in circular orbits at a distance $R = R_{earth}$, and use the numbers below to simplify your expression.

Some useful constants include

$$\begin{aligned} M_\odot &= 1.989 \times 10^{30}\,\text{kg} \\ L_\odot &= 3.828 \times 10^{26}\,\text{W} \\ R_{earth} &= 1.5 \times 10^{11}\,\text{m} \\ \lambda_{\text{max}} &= 500\,\text{nm} \end{aligned}$$

Answer Sheets

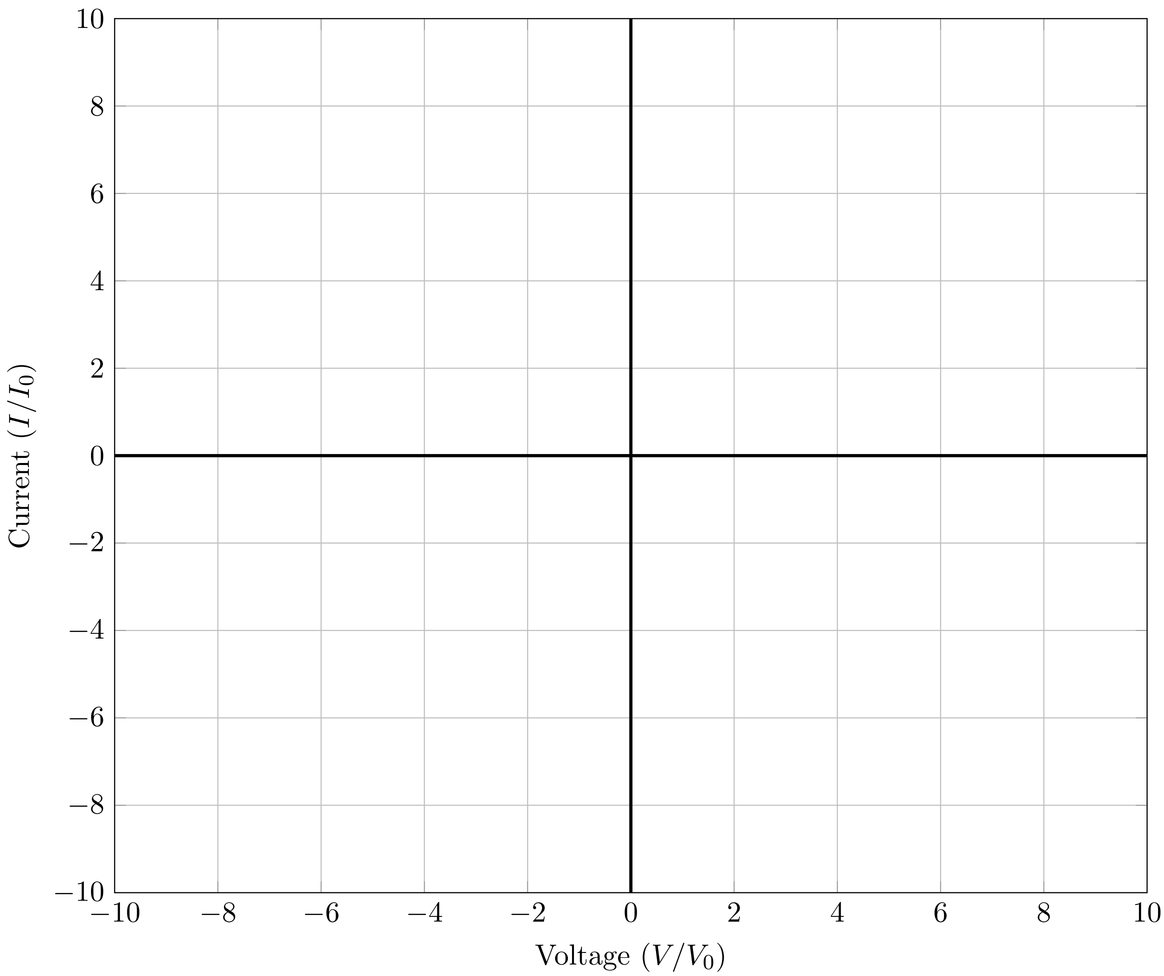
Following are answer sheets for some of the graphical portions of the test.

Student AAPT ID #:

Proctor AAPT ID #:

Question A2

a. Sketch a graph of the current versus voltage for low temperature values $k_B T \ll qV_0$, clearly indicating any asymptotic behavior.

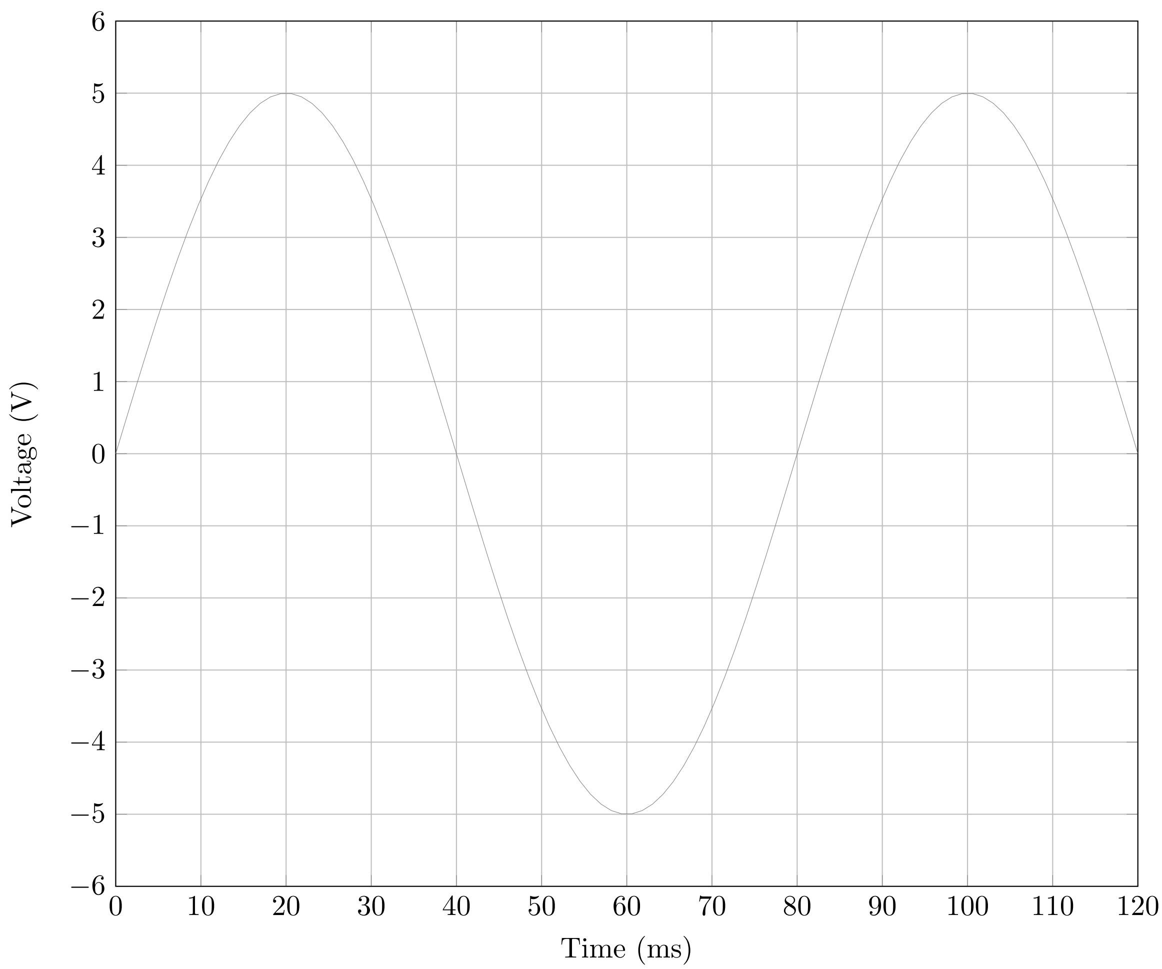


Student AAPT ID #:

Proctor AAPT ID #:

Question A2

b. Sketch the potential difference $V_{CD} = V_C - V_D$ as a function of time. For your convenience, V_{AB} is shown in light gray. Assume that V_{AB} has been running for a long time. There is **no** capacitor in this circuit!



Student AAPT ID #:

Proctor AAPT ID #:

Question A2

c. Sketch the potential difference $V_{CD} = V_C - V_D$ as a function of time. For your convenience, V_{AB} is shown in light gray. Assume that V_{AB} has been running for a long time. There is a capacitor in this circuit!