AAPT UNITED STATES PHYSICS TEAM
AIP 2021

2021 $F = ma$ Exam

25 QUESTIONS - 75 MINUTES

INSTRUCTIONS

DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

- Use $g = 10$ N/kg throughout this contest.
- You may write in this booklet of questions. However, you will not receive any credit for anything written in this booklet. You may only use the scratch paper provided by the proctor.
- This test contains 25 multiple choice questions. Select the answer that provides the best response to each question. Please be sure to use a No.2 pencil and completely fill the box corresponding to your choice. If you change an answer, the previous mark must be completely erased. Only the boxes preceded by numbers 1 through 25 are to be used on the answer sheet.
- All questions are equally weighted, but are not necessarily of the same level of difficulty.
- Correct answers will be awarded one point; incorrect answers or leaving an answer blank will be awarded zero points. There is no additional penalty for incorrect answers.
- A hand-held calculator may be used. Its memory must be cleared of data and programs. You may use only the basic functions found on a simple scientific calculator. Calculators may not be shared. Cell phones may not be used during the exam or while the exam papers are present. You may not use any tables, books, or collections of formulas.
- The question booklet, the answer sheet and the scratch paper will be collected at the end of this exam.
- **In order to maintain exam security, do not communicate any information about the questions (or their answers or solutions) on this contest until after February 1, 2021.**

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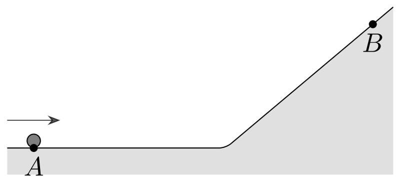
We acknowledge the following people for their contributions to this year's exams (in alphabetical order):

JiaJia Dong, Mark Eichenlaub, Abi Krishnan, Kye Shi, Brian Skinner, Mike Winer and Kevin Zhou

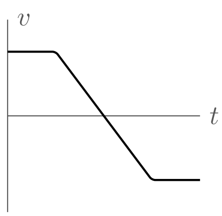
AAPT | UNITED STATES PHYSICS TEAM
AIP | **2021**

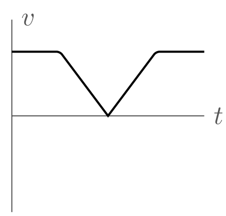
2021 $F = ma$ Competition

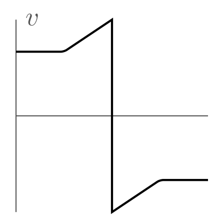
The following information applies to problems 1 and 2.
At time $t = 0$, a small ball is released on the track shown, with an initial rightward velocity. Assume the ball always rolls along the track without slipping.

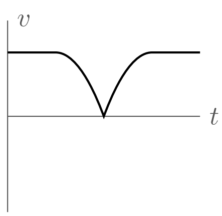


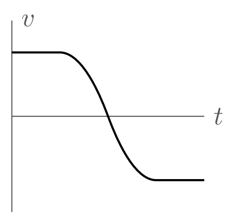
1. The ball starts at point A, turns around at point B, and returns to point A. Which of the following shows the **speed** of the ball as a function of time?

(A) 

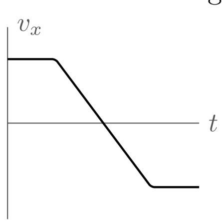
(C) 

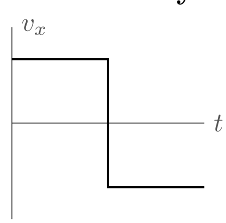
(E) 

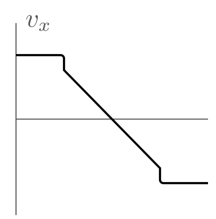
(B) 

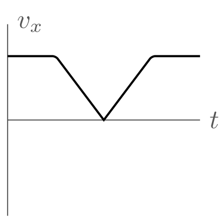
(D) 

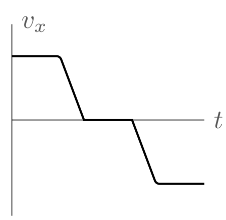
2. Which of the following shows the horizontal **velocity** of the ball as a function of time?

(A) 

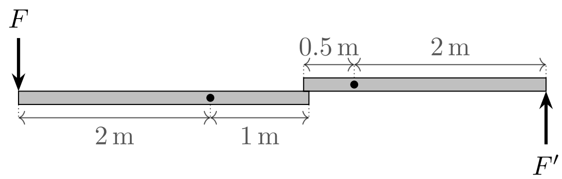
(C) 

(E) 

(B) 

(D) 

3. Two massless rods are attached to frictionless pivots, with their ends touching. The distances between the pivot points and the endpoints of the rods are shown below.



Neglecting friction between the rods, if a force F is applied at the left end of the left rod, what force F' must be applied at the right end of the right rod to keep the system in equilibrium?

(A) $F/8$
(B) $F/2$
(C) $4F/7$
(D) $6F/5$
(E) $2F$

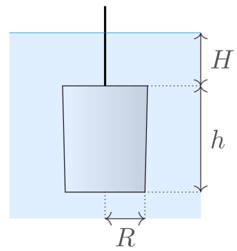
4. Alice and Bethany stand side by side on the Earth's equator. If Alice jumps directly upward, in her frame of reference, to a small height h much less than the radius of the Earth, she will land a distance D to the west of Bethany. If Alice had instead jumped to a height $2h$, how far to the west of Bethany would she land? Neglect air resistance.

(A) $D/\sqrt{2}$
(B) D
(C) $\sqrt{2}\,D$
(D) $2D$
(E) $2^{3/2}D$

5. A train starts from city A and stops in city B. The distance between the cities is s. The train's maximal acceleration is a_1 and its maximal deceleration is a_2 (in absolute value). What is the shortest time in which the train can travel between A and B?

(A) $2\sqrt{\dfrac{s}{a_1 + a_2}}$
(B) $2\sqrt{\dfrac{s}{\sqrt{a_1 a_2}}}$
(C) $\sqrt{2\dfrac{s(a_1 + a_2)}{a_1 a_2}}$
(D) $\sqrt{\dfrac{2sa_2}{a_1(a_1 + a_2)}}$
(E) $\sqrt{\dfrac{s\sqrt{a_1 a_2}}{(a_1 + a_2)^2}}$

6. A cylindrical bucket of negligible mass has radius R and height h, and is open at the top. It is submerged in water of density ρ, with its top a distance H below the surface. How much work is needed to pull the bucket slowly up so that its bottom is just above the lake surface?



(A) $\rho g \pi R^2 h(H - h)$

(B) $\rho g \pi R^2 h^2$

(C) $\rho g \pi R^2 (H - h)^2$

(D) $\rho g \pi R^2 h^2/2$

(E) $\rho g \pi R^2 h(H - h)/2$

7. A point mass slides with speed v on a frictionless horizontal surface between two fixed parallel walls, initially a distance L apart. It bounces between the walls perfectly elastically. You move one of the walls towards the other by a distance $0.01L$, with speed $0.0001v$. What is the final speed of the point mass?

(A) $1.001v$

(B) $1.002v$

(C) $1.005v$

(D) $1.01v$

(E) $1.02v$

8. A mint produces 100,000 coins. Upon weighing some of them with a precise scale, officials find that the coins vary slightly in weight, with an independent uncertainty of 1%. About how many coins must be randomly sampled and weighed in order to determine the total weight of the coins to within 0.1% uncertainty? Assume there are no sources of systematic uncertainty.

(A) Almost all of the coins must be weighed.

(B) 10,000

(C) 1,000

(D) 100

(E) 10

9. NASA trains astronauts to experience weightlessness with an airplane which flies in a parabolic arc with constant acceleration g toward the ground. The plane can remain on this trajectory for at most 25 seconds, due to the large change in altitude required. If instead of simulating weightlessness, NASA wanted to fly a trajectory that would simulate the gravitational acceleration of Mars 3.7 m/s^2, for what length of time can the plane simulate Mars gravity? Assume that the maximum change in altitude is the same for both trajectories.

(A) 25 s

(B) 31 s

(C) 41 s

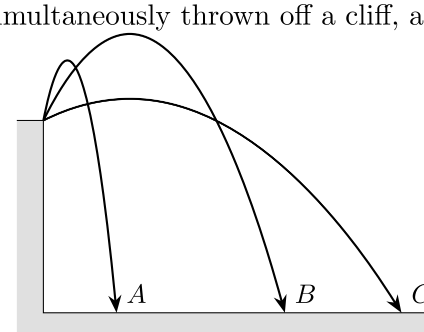
(D) 68 s

(E) 183 s

10. A uniform solid circular disk of mass m is on a flat, frictionless horizontal table. The center of mass of the disk is at rest and the disk is spinning with angular frequency ω_0. A stone, modeled as a point object also of mass m, is placed on the edge of the disk, with zero initial velocity relative to the table. A rim built into the disk constrains the stone to slide, with friction, along the disk's edge. After the stone stops sliding with respect to the disk, what is the angular frequency of rotation of the disk and stone together?

(A) ω_0
(B) $2\omega_0/3$
(C) $\omega_0/2$
(D) $\omega_0/3$
(E) $\omega_0/4$

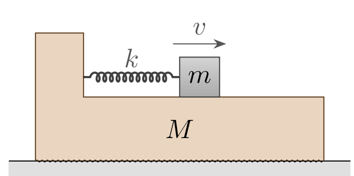
11. Projectiles A, B, and C are simultaneously thrown off a cliff, and take the trajectories shown.



Neglecting air resistance, rank the times t_A, t_B, and t_C they take to hit the ground.

(A) $t_A < t_B < t_C$
(B) $t_A < t_C < t_B$
(C) $t_C < t_B < t_A$
(D) $t_C < t_A < t_B$
(E) There is not enough information to decide.

12. An object of mass $m = 1\,\mathrm{kg}$ is attached to a platform of mass $M = 4\,\mathrm{kg}$ with a spring of spring constant $k = 400\,\mathrm{N/m}$. There is no friction between the object and the platform, and the coefficient of static friction between the platform and the ground is $\mu = 0.1$. The object is placed at its equilibrium position, and then given a horizontal velocity v.



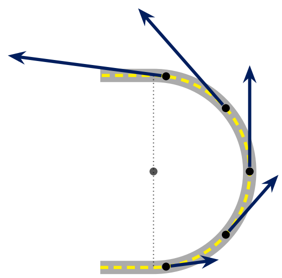
For what v will the platform never slip on the ground?

(A) $v \le 0.1\,\mathrm{m/s}$
(B) $v \le 0.2\,\mathrm{m/s}$
(C) $v \le 0.25\,\mathrm{m/s}$

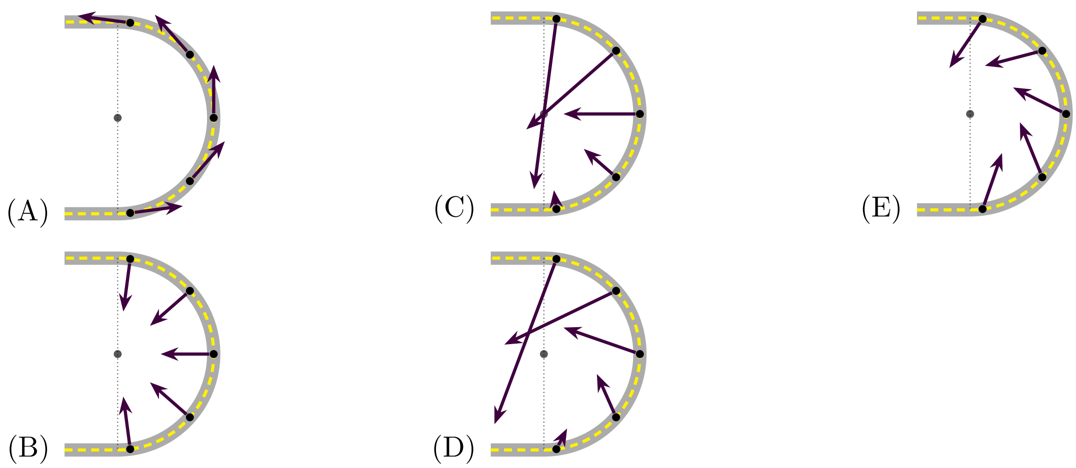
(D) $v \le 0.4\,\text{m/s}$

(E) $v \le 0.5\,\text{m/s}$

13. A car is driving on a semicircular racetrack. Its velocity at several points along the track is shown below.



Which of the following could depict its acceleration at the corresponding points?



14. A bacterium cell swims by rotating its bundle of flagella to counter a viscous drag force in the medium. The drag force $F(R, v, \eta)$ only depends on the typical length scale of the cell R, its speed v, and the viscosity of the fluid η, which has units of $\text{kg}/(\text{m}\cdot\text{s})$. It is observed under a microscope that a cell of length 1 μm swims at about 20 μm/s. Estimate the speed of a cell of length 0.5 μm, assuming cells of all sizes generate the same amount of force from their flagella.

(A) 5 μm/s

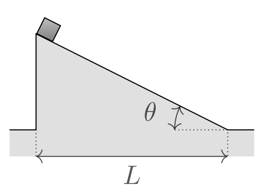
(B) 10 μm/s

(C) 40 μm/s

(D) 80 μm/s

(E) There is not enough information to decide.

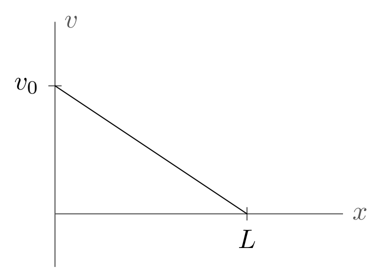
15. A block is released from rest at the top of a fixed, frictionless ramp with horizontal length L and inclination θ.



For a fixed value of L, which value of θ minimizes the time needed for the block to reach the bottom of the ramp?

(A) 30°
(B) 45°
(C) 60°
(D) 75°
(E) 80°

16. A particle begins at the point $x = 0$ and moves along the x-axis with initial rightward velocity v_0. Later, the particle reaches the point $x = L$. The velocity as a function of position during this time interval is shown below.



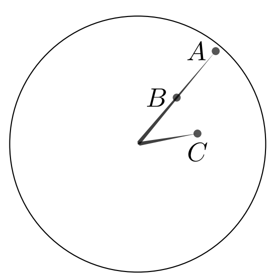
Consider the following three statements.

I. The particle instantaneously stops at $x = L$.
II. The particle is uniformly accelerated.
III. The particle could be performing simple harmonic motion.

Which of these statements are true?

(A) Only I.
(B) Only II.
(C) Both I and II.
(D) Both I and III.
(E) None of the above.

17. In a certain country, the short hand of a clock is exactly half as long as the long hand, and rotates twice for each rotation of the long hand.



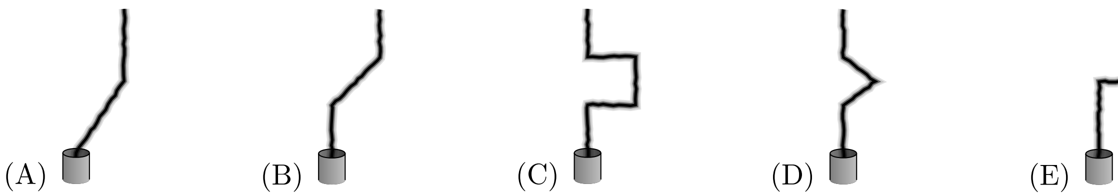
The three points shown on the clock hands have accelerations of magnitude a_A, a_B, and a_C. The point B is at the midpoint of the long hand. Which of the following is true?

(A) $a_A < a_B = a_C$
(B) $a_A = a_B < a_C$
(C) $a_B < a_A < a_C$

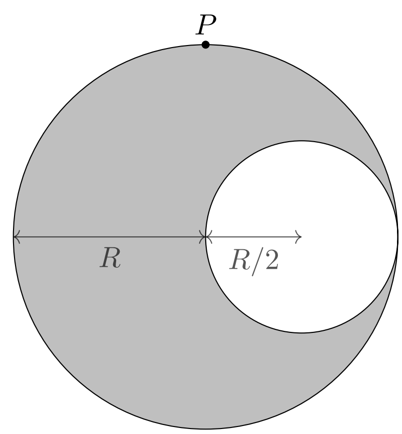
(D) $a_B < a_A = a_C$

(E) $a_B < a_C < a_A$

18. A factory's smokestack continually produces smoke. The smoke always rises with a constant speed relative to the air around it. Suppose that the air was initially still, then abruptly started blowing to the right, then abruptly became still again for some time. Which of the following could show the resulting shape of the smoke plume?



19. A cavity of radius $R/2$ is dug out of a spherical planet with uniform mass density of mass M and radius R. What is the magnitude of the gravitational field at point P in the diagram below?



(A) $(0.200)GM/R^2$

(B) $(0.457)GM/R^2$

(C) $(0.829)GM/R^2$

(D) $(0.900)GM/R^2$

(E) $(0.912)GM/R^2$

20. The line between day and night on a planet or moon is called the *terminator*. How fast does the terminator of Earth's moon move across its surface at the equator? The radius of the moon is 1.74×10^6 m.

(A) 0 m/s

(B) 4.5 m/s

(C) 83 m/s

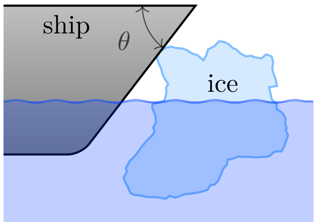
(D) 465 m/s

(E) 2201 m/s

21. A solid sphere sits at the top of a ramp of height h inclined at angle θ to the horizontal. Both the static and kinetic coefficients of friction between the sphere and incline are $\mu_k = \mu_s = 0.2$. The sphere is released from rest at the top of the incline. For which of the following values of θ is the total translational plus rotational kinetic energy of the sphere greatest when it reaches the bottom of the incline?

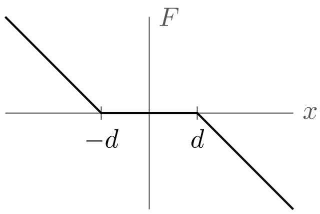
(A) 10°
(B) 45°
(C) 60°
(D) 80°
(E) The mechanical energy is the same for all choices.

22. A ship rams into a hunk of ice floating in the sea. "Icebreakers" are ships designed so that when this happens, the ice is pushed down beneath the ship. If the coefficient of static friction between the ice and the ship is μ_s, what condition applies to the angle θ, as shown in the figure, so that the icebreaker functions as intended?



(A) $\cot\theta > \mu$
(B) $\cos\theta > \mu$
(C) $\cot\theta < \mu$
(D) $\cos\theta < \mu$
(E) It depends on the curvature of the ice.

23. An imperfect spring has a restoring force F that depends on the displacement x from equilibrium as in the graph shown below.



The slope of the curve for $x < -d$ and $x > d$ is a constant $-k$. A mass m is attached to the spring and released from rest at the position $x = A$, where $A > d$. What is the period of the subsequent motion?

(A) $T = \sqrt{\frac{m}{k}}\left(2\pi + \frac{2d}{A}\right)$
(B) $T = \sqrt{\frac{m}{k}}\left(2\pi + \frac{2d}{A-d}\right)$
(C) $T = \sqrt{\frac{m}{k}}\left(2\pi + \frac{4d}{A-d}\right)$

(D) $T = \sqrt{\frac{m}{k}}\left(2\pi + \frac{\pi d}{A-d}\right)$

(E) $T = \sqrt{\frac{m}{k}}\left(2\pi + \frac{2\pi d}{A-d}\right)$

24. Two satellites are in circular orbits around a star with equal radius r, speed v, and period T. The satellites are initially diametrically opposite each other. In order to meet the second satellite in time $T/2$, the first satellite should decrease its speed to approximately

(A) $0.50v$

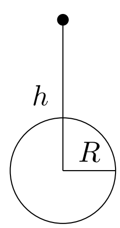
(B) $0.64v$

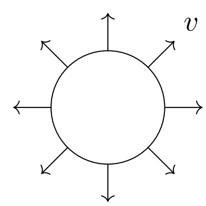
(C) $0.71v$

(D) $0.76v$

(E) $0.82v$

25. Spaceman Fred's trusty pellet sprayer is held at rest a distance h away from the center of Planet Orb, which has radius $R \ll h$. The pellet sprayer ejects pellets radially outward, uniformly in the plane of the page. These pellets are all launched with the same speed v, so that a pellet launched directly away from Orb by the pellet sprayer can just barely escape it. What fraction of the pellets eventually lands on Orb? (Hint: you may use the small angle approximation, $\sin\theta \approx \theta$ for $\theta \ll 1$, where θ is in radians.)





Left: the pellet sprayer relative to Orb Right: a close-up of the pellet sprayer

(A) $\frac{1}{2\pi}\frac{R}{h}$

(B) $\frac{1}{\pi}\frac{R}{h}$

(C) $\frac{2}{\pi}\frac{R}{h}$

(D) $\frac{1}{2\pi}\sqrt{\frac{R}{h}}$

(E) $\frac{1}{\pi}\sqrt{\frac{R}{h}}$