2014 PhysicsBowl Solutions

#	Ans	#	Ans	#	Ans	#	Ans	#	Ans
1	B	11	C	21	C	31	C	41	D
2	D	12	C	22	D	32	E	42	C
3	D	13	B	23	B	33	B	43	D
4	A	14	E	24	D	34	A	44	E
5	C	15	D	25	E	35	E	45	B
6	E	16	D	26	B	36	A	46	A
7	C	17	A	27	A	37	D	47	D
8	E	18	B	28	B	38	A	48	E
9	A	19	E	29	A	39	E	49	C
10	D	20	C	30	C	40	B	50	B

1. **B…** $\times 10^6$ is represented by Mega which has a symbol of M
2. **D…** Averaging the values gives $5.06666\ cm$. Using the rules of significant digits, we can only record the answer out to the hundredths place. Hence, we round to obtain $5.07\ cm$.
3. **D…** Using constant acceleration kinematics, we have $\Delta x = v_0 t + \frac{1}{2}at^2 \Rightarrow \Delta x = (0) + \frac{1}{2}(-10)(3)^2 = -45\ m$. The object falls 45 meters.
4. **A…** By considering the free body diagram of the bottom mass, the tension in the string would be Mg. Using the same logic for everything beneath string B, we have a tension of $2Mg$ for B. Finally, string A is effectively supporting all three masses and the tension is $3Mg$. This is ignoring the assumed small mass of the strings.
5. **C…** The period of a pendulum is given by $T = 2\pi\sqrt{\frac{L}{g}}$. Hence, to increase the period of the pendulum, one should lengthen the string.
6. **E…** MRI stands for Magnetic Resonance Imaging
7. **C… METHOD #1:** using constant acceleration kinematics, we can find the acceleration from $v = v_0 + at \Rightarrow a = \frac{v-v_0}{t} = \frac{7.60-0}{3.00} = 2.53\frac{m}{s^2}$. Continuing with this process, we write $\Delta x = v_0 t + \frac{1}{2}at^2 \Rightarrow \Delta x = (0) + \frac{1}{2}(2.53)(3)^2 = 11.4\ m$

   Alternatively: The average velocity can be found for constant acceleration as $\langle v \rangle = \frac{1}{2}(v + v_0) = \frac{1}{2}(7.6 + 0) = 3.80\frac{m}{s}$. By definition, $\langle v \rangle = \frac{\Delta x}{t} \Rightarrow \Delta x = \langle v \rangle t = (3.80)(3) = 11.4\ m$
8. **E…** The amplitude of motion for a simple harmonic oscillator is found from the coefficient in front of the sinusoidal function. This means that the value 0.20 gives the amplitude of the motion.
9. **A…** Vectors have direction. Only force has a direction associated with it.
10. **D…** Electric field lines point away from positive charge and toward negative charge. From the symmetry of the situation, the fields at P are "up and to the left" and "down and to the left" with the up and down pieces cancelling. This means that the total field is directed to the left.
11. **C…** By writing $\Delta KE = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2 \Rightarrow 36 = \frac{1}{2}m(4)^2 - \frac{1}{2}m(2)^2 = 6m \Rightarrow m = \frac{36}{6} = 6\ kg$
12. **C…** From Newton's Second Law, $F_{net} = ma = (5)(3) = 15.0\ N$
13. **B…** The verbal description given is known as Hooke's Law which is useful for springs.
14. **E…** By Newton's Third Law, The gravitational force acting on the box by the Earth has as its pair force the gravitational force acting on the Earth by the box.
15. **D…** To solve this problem, we use linear momentum conservation which gives $\Delta p_A + \Delta p_B = 0$. Using $p = mv$ and choosing "to the right" as positive, we write $m_A(v_{fA} - v_{0A}) + m_B(v_{fB} - v_{0B}) = 0 \Rightarrow (5)(-25 - 25) + (10)\left(v_{fB} - (-20)\right) = 0$.

Solving gives $-250 + 10(v_{fB} + 20) = 0 \Rightarrow v_{fB} + 20 = 25 \Rightarrow v_{fB} = 5.0\frac{m}{s}$. Note that the sign is positive indicating motion to the right.

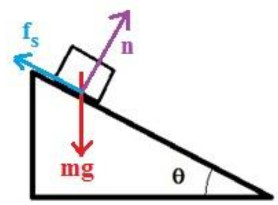
16. D… By definition, average velocity is displacement divided by time. That means, $\langle v \rangle = \frac{\Delta x}{\Delta t} = \frac{-10}{5} = -2\frac{cm}{s}$. The magnitude of this quantity is $2.0\frac{cm}{s}$.

17. A… By definition, average acceleration is change in velocity divided by time. That means, $\langle a \rangle = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t} = \frac{-40-60}{5} = -20\frac{cm}{s^2}$. The magnitude of this quantity is $20.0\frac{cm}{s^2}$

18. B… Density is mass divided by volume. By writing this, we have $\rho = \frac{m}{V} = \frac{17.8\ g}{(4.00\ cm)(3.00\ cm)(2.00\ cm)} = 0.742\frac{g}{cm^3}$. Converting units, $0.742\frac{g}{cm^3} \times \frac{1\ kg}{1000\ g} \times \left(\frac{100\ cm}{1\ m}\right)^3 = 742\frac{kg}{m^3} = 7.42 \times 10^2\frac{kg}{m^3}$

19. E… The Nobel Prize was awarded for the experimental verification of the Higgs Boson, named after Peter Higgs.

20. C… METHOD #1: The free body diagram will have a gravitational force, a normal force, and a friction force acting. By writing those forces into components and noting that there is no acceleration, we have $F_{net_x} = ma_x \Rightarrow mg\sin\theta - f_s = 0$ and $F_{net_y} = ma_y \Rightarrow n - mg\cos\theta = 0$. By solving for the normal force and the friction force, we take the ratio and obtain $\frac{f_s}{n} = \frac{mg\sin\theta}{mg\cos\theta} = \tan\theta \Rightarrow \theta = \tan^{-1}\left(\frac{38}{62}\right) = 31.5^\circ$. Finally, using either expression from Newton's Law, we find the mass as $m = \frac{f_s}{g\sin\theta} = \frac{38}{10\sin 31.5^\circ} = 7.27\ kg$.



METHOD #2: Noting that the object is in static equilibrium, the sum of the vector forces is equal to zero. This means that the sum of the normal and friction forces must be equal and opposite to the gravitational force. $\vec{n} + \vec{f_s} = -m\vec{g}$. The size of the force can be found from the Pythagorean Theorem, since the normal and friction forces are at right angles to each other, to give $mg = \sqrt{(38)^2 + (62)^2} = 72.7\ N$. Consequently, the mass is $7.27\ kg$.

21. C… For a tube closed at the bottom, only odd harmonics exist. The expression for the frequencies of standing wave modes is $f_n = nf_1 = n\left(\frac{v}{4L}\right) \Rightarrow f_n = \frac{(3)(342)}{4(0.57)} = 450\ Hz$.

22. D… For the way that the circuit is connected, all of the current is through bulb #3. By following electron flow from the negative end of the battery through bulb #3, we find a junction resulting in ½ of the current through each of bulbs #1 and #2. Since the resistances of the bulbs are all the same, we use $P = I^2R$ to determine the bulb brightness, leading to $P_{bulb\ \#1} = P_{bulb\ \#2} < P_{bulb\ \#3}$.

23. B… By writing Newton's Second Law, there is only one inward directed force (friction) leading to $F_{net} = ma \Rightarrow f_f = ma$. Noting that $a = \frac{v_t^2}{r}$, we will write it as $a = \frac{(r\omega)^2}{r} = r\omega^2$ leading to $f_f = mr\omega^2$. Both the mass of the coins as well as the angular speed, ω, for all points on the disk is the same, hence, from $f_f = mr\omega^2$, we see that the friction force is proportional to the distance from the center of the disk. This means that because $d_x = \frac{1}{2}d_Y$, then $f_x = \frac{1}{2}f_Y$

24. D… Magnetic field lines point from North to South poles outside the magnet. As a result, the field points directly to the left above the middle of the bar magnet shown.

25. E… The bending of light occurs because of a change in the index of refraction… the phenomenon is refraction.

26. B… The masses are equal, but the density of iron is much greater than the density of aluminum. As a result, the volume of aluminum is much greater than the volume of the iron. The buoyant force on an object in a fluid is computed as $B = \rho_{fluid} g V_{displaced}$. The density is the same for each object since it is the density of water that is important in buoyancy. Hence, the aluminum cube has the larger buoyant force as it displaces more water than the iron cube. Note that the force from the bottom surface plays no role in determining the buoyant force and that the force from the bottom is different for each cube.

27. **A…** The acceleration due to gravity at the surface of a planet can be found using Newton's General Law of Gravitation. That is, $\frac{GmM}{r^2} = ma$ which means that the acceleration near a Planet's surface is computed as $a = \frac{GM}{r^2}$. Taking the ratio of these quantities for the planets involved gives $\frac{g_x}{g_y} = \frac{GM_x/r_x^2}{GM_y/r_y^2} = \frac{M_x r_y^2}{M_y r_x^2}$. The mass is found using the density expression and the volume of the planet which gives $\frac{g_x}{g_y} = \frac{M_x r_y^2}{M_y r_x^2} = \frac{\rho\left(\frac{4}{3}\pi r_x^3\right) r_y^2}{\rho\left(\frac{4}{3}\pi r_y^3\right) r_x^2} = \frac{r_x}{r_y} = 2$

28. **B…** When a wave travels from one medium to the next, the one quantity that is unaffected is the frequency. Since the speed of sound in water is greater than the speed of sound in air, and $v = f\lambda$, with an increased speed and unchanged frequency, the wavelength will be larger in the water than the air.

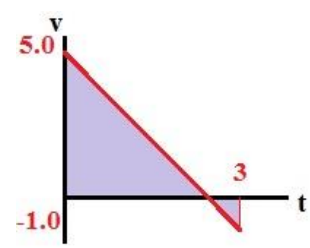
29. **A…** The magnetic force deflects charged particles and does NO work on them.

30. **C… METHOD #1:** It takes about 1 second for light from the Moon to reach the earth, which means that the Earth-Moon distance is approximately $3.0 \times 10^8\ m$. Using a free body diagram analysis, we write $F_{net} = ma \Rightarrow \frac{GM_E M_M}{r^2} = M_M a$. So, upon solving for the acceleration, we find $a = \frac{GM_E}{r^2} = \frac{(6.7\times10^{-11})(6\times10^{24})}{(3\times10^8)^2} = 4.47 \times 10^{-3}\ \frac{m}{s^2}$. The closest answer is $3 \times 10^{-3}\ \frac{m}{s^2}$.
METHOD #2: To find the actual distance from the Moon to the Earth, we can employ a free body diagram analysis (or go directly to Kepler's Third Law) showing $F_{net} = ma \Rightarrow \frac{GM_E M_M}{r^2} = M_M \frac{v^2}{r}$. Using that the Moon moves in an approximate circle around the Earth, we have $v = \frac{2\pi r}{T}$ leading to $\frac{GM_E}{r^2} = \frac{v^2}{r} \Rightarrow \frac{GM_E}{r^2} = \frac{\left(2\pi r/T\right)^2}{r} = \frac{4\pi^2 r}{T^2}$ or $r = \left(\left(\frac{GM_E}{4\pi^2}\right) T^2\right)^{\frac{1}{3}}$. The period of the Moon around the Earth is approximately 28 days and converting that to seconds leads to $28\ dys \times \frac{24\ hr}{1\ dy} \times \frac{3600\ s}{1\ hr} = 2.42 \times 10^6\ s$. Substitution into the expression for the Earth-Moon distance gives $r = \left(\frac{(6.7\times10^{-11})(6\times10^{24})(2.42\times10^6)^2}{4\pi^2}\right)^{1/3} = 3.9 \times 10^8\ m$
Substituting this back into $a = \frac{GM_E}{r^2}$ yields $a = 2.6 \times 10^{-3}\ \frac{m}{s^2}$

31. **C… METHOD #1:** Since we know the useful output going in to lifting the mass, we employ $P = Fv\cos\theta = Mgv$. By substitution, we obtain $v = \frac{P}{Mg} = \frac{0.10\ W}{(0.20\ kg)\left(10\frac{N}{kg}\right)} = 0.05\ \frac{m}{s}$
METHOD #2: Since power is work per time, we write $P = \frac{Mg\Delta y}{\Delta t} = Mgv$ where the gravitational potential energy change is where the useful power went.
Alternatively: In a variation on METHOD #2… assume a time of $t = 1\ s$ and find the height that the object would be raised. The speed would then be the height/ 1 s.

32. **E…** From units: $\frac{(Js)\left(\frac{m}{s}\right)}{m} = J$ which is a unit of energy. Planck's constant is on the constants sheet provided with the contest.

33. **B… METHOD #1:** If one makes a graph of the velocity vs. time for the object, a crossover from positive to negative velocity occurs. The total distance traveled (needed for finding average speed) is computed as the magnitude of the area under the curve. From the triangles we have in the graph, the total distance traveled is $\frac{1}{2}\left(5\frac{m}{s}\right)(2.5\ s) + \frac{1}{2}\left(1\frac{m}{s}\right)(0.5\ s) =$



$6.25 + 0.25\ m = 6.5\ m$. The average speed would be computed as $speed = \frac{dist}{time} = \frac{6.5m}{3.0s} = 2.17\frac{m}{s}$.
METHOD #2: Using algebra, we need to find when the velocity becomes zero. The acceleration of the object would be found as $a = \frac{v_f - v_0}{t} = \frac{-1-5}{3} = -2.0\frac{m}{s^2}$. This means that the velocity is $0\frac{m}{s}$ when $v = v_0 + at \ \Rightarrow 0 = 5 + (-2)t \ \Rightarrow \ t = 2.5\ s$. The change in position going forward would be $\Delta x = v_0 t + \frac{1}{2}at^2 = (5)(2.5) + \frac{1}{2}(-2)(2.5)^2 = 6.25\ m$. Finally, the same process can be used for the last 0.5 seconds giving an extra 0.25 m and then $speed = \frac{dist}{time} = \frac{6.5m}{3.0s} = 2.17\frac{m}{s}$

34. A… By a right-hand rule, the fingers curl in the direction of the conventional current (clockwise) and the right thumb points in the direction of the magnetic field at the center of the loop (into the plane). Because magnetic field lines form closed loops, the field is directed out of the plane of the page inside the loop on the right. Because the resistance of the left-hand circuit is increasing, the current in that loop is decreasing, thereby decreasing the magnetic field strength. In the right-hand loop, since the field through the loop is decreasing in magnitude, induction occurs to reinforce the lost field. This means from the right-hand rule that the fingers will curl counterclockwise around the loop to produce a magnetic field out of the plane, trying to replace the field lines that are being lost.

35. E… The last statement is a property of conductors in electrostatic equilibrium. The coefficient of friction can exceed the value of 1! Tires on the ground can have values that exceed unity. An object in an elevator that is accelerating would have the gravitational and normal forces be different values. An ideal gas undergoing an isothermal process has both work done and an exchange through heat. Answer (D) is only incorrect in that increasing the spacing between the slits decreases the spacing between dark fringes.

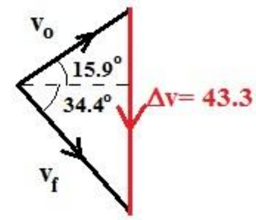
36. A… By placing the object in Region I, the image formed will be real and located in Region I. Once inside the focal length of the mirror, the image formed will become virtual and appear on the right-hand side of the mirror. Using the mirror equation, $1/f = 1/p + 1/q$, we can compute the image location using the object location as nf where n is a number. We have then $\frac{1}{f} = \frac{1}{nf} + \frac{1}{q} \ \Rightarrow \ q = \frac{n}{n-1}f$. Letting $n = \frac{3}{4}$, e.g., then $q = -3f$ which is in Region IV while if $n = \frac{1}{4}$, then $q = -\frac{1}{3}f$ which is in Region III. It is impossible to form an image anywhere in Region II,

37. D… The root-mean-square speed is computed as $v_{rms} = \sqrt{\frac{3RT}{M}}$. From the periodic table entry, we have $M = 458.7\frac{g}{mol}$. As a result, we substitute our values to obtain $v_{rms} = \sqrt{\frac{3(8.31)(293)}{0.4587}} = 126\frac{m}{s}$ as we have converted the temperature into Kelvin and the molar mass into $\frac{kg}{mol}$.

38. A… After a long time has passed, the current through the circuit is zero Amps. Using a Kirchhoff Loop Rule (KLR) around the outside of the circuit reveals that the potential difference across the capacitor is $9\ V$ since there is no current after a long time. Once the switch is closed, there is a loop that encloses the battery and $3\ k\Omega$ resistor resulting in a total of $9 = 3000\ I \ \Rightarrow I = 3\ mA$ through the resistor. For the other branch on the right, there is a capacitor initially charged and a $1\ k\Omega$ resistor. Writing the KLR for this loops results in 9 volts from the capacitor and 9 volts across the resistor. Hence, we find $9 - 1000I = 0 \ \Rightarrow I = 9\ mA$. So, when the switch is initially closed, from Kirchhoff's Junction Rule, we have $3\ mA$ from the left and $9\ mA$ from the right resulting in $12\ mA$ through the switch. Eventually, the capacitor is completely discharged and there is no more current in the right half of the circuit, meaning that only the $3\ mA$ through the $3\ k\Omega$ resistor exists in the circuit.

39. E… With equal initial kinetic energy, one can write for these rotating disks that $KE = \frac{1}{2}I\omega^2 = \frac{L^2}{2I}$. As a result, $L = \sqrt{2I(KE)}$ which means that $L_X < L_Y$ as object Y has more mass. By applying the same force on the outside of each disk, the torque from the center of each disk is the same. From the angular impulse momentum theorem, $\langle\tau\rangle\Delta t = \Delta L$ and since the torques and times are equal, the change in angular momentum is the same for each. Consequently, $L_X < L_Y$ after the push as well. The kinetic energy depends on the total "distance" through which each force acted. Since disk X is rotating at a higher rate than disk Y (compensating for the difference in mass to have the same initial kinetic energy), and the applied force will increase the angular speed of each disk, there will be a greater angle swept out by disk X compared to disk Y (not only is its angular speed greater, but so is its angular acceleration). As a result, the kinetic energy change for disk X is greater than for disk Y meaning that after the push, , $K_X > K_Y$

40. B… METHOD #1: By looking at a vector picture with velocities, we have the picture as shown since $\vec{v}_f = \vec{v}_0 + \vec{a}t$ with $\Delta\vec{v} = -gt\,\hat{y} = -43.3\frac{m}{s}$. We can now use the Law of Sines from the triangle construction to obtain $\frac{v_f}{\sin 74.1^\circ} = \frac{43.3}{\sin 50.3^\circ} \Rightarrow$ $v_f = \frac{43.3\sin 74.1^\circ}{\sin 50.3^\circ} = 54.1\frac{m}{s}$.



METHOD #2: From constant acceleration kinematics, we have

$$v_{fy} = v_{0y} + at \ \Rightarrow v_{fy} = v_{0y} - 43.3$$

Also, since the x-component of the velocity is unchanged, we can find the ratio of the y-components of the velocity from $-\frac{\tan 34.4^\circ}{\tan 15.9^\circ} = \frac{v_{fy}/v_x}{v_{0y}/v_x} = \frac{v_{fy}}{v_{0y}} \ \Rightarrow \ v_{0y} = -\frac{v_{fy}}{2.40}$. Substituting this gives $v_{fy} = -\frac{v_{fy}}{2.40} - 43.3$ which is evaluated as $v_{fy}\left(1 + \frac{1}{2.40}\right) = -43.3 \ \Rightarrow \ v_{fy} = -30.56\frac{m}{s}$. Finally, we have $|v_{fy}| = v_f \sin 34.4^\circ$ which gives $v_f = \frac{30.56}{\sin 34.4^\circ} = 54.1\frac{m}{s}$

METHOD #3: One could solve the problem by starting with the answer choices and finding components. Let's assume we choose answer C since it is in the middle. Using that as the final speed, we have components of velocity given as $v_x = 46.4\cos 34.4^\circ = 38.3\frac{m}{s}$ and $v_y = -46.4\sin 34.4^\circ = -26.2\frac{m}{s}$. From constant acceleration kinematics, we now write $v_y = v_{0y} + a_y t \ \Rightarrow$ $-26.2 = v_{0y} + (-10)(4.33) \ \Rightarrow v_{0y} = 17.1\frac{m}{s}$. Using the components of the initial velocity now, we see that the angle of launch would have been $\tan\theta = \frac{v_{0y}}{v_{0x}} = \frac{17.1}{38.3} \ \Rightarrow \theta = 24.1^\circ$. Since this angle is too high, we would need a higher final speed leading to a smaller initial y-component of velocity meaning that the answer is either (A) or (B). using the same process leads to (B) being the correct choice.

41. D… The sentence is usually described as the Pauli Exclusion Principle.

42. C… Applying Malus' Law, $I = I_0\cos^2\theta$. Applying this to our problem yields $\frac{1}{4}I = I\cos^2\theta$ **.** By solving for the angle, we now have $\cos\theta = \frac{1}{2} \ \Rightarrow \ \theta = 60^\circ$

43. D… For the rod, the gravitational force acts at the center, so from the pivot, we will write out Newton's Second Law in rotational form to obtain $\tau_{net} = I\alpha$. The moment of inertia of a rod is $\frac{1}{12}ML^2$ about the center. Using the parallel axis theorem, the moment of inertia about one end is computed as $I = \frac{1}{12}ML^2 + M\left(\frac{L}{2}\right)^2 = \frac{1}{3}ML^2$. The torque from the gravitational force about the end is computed as $\tau = (Mg)\left(\frac{L}{2}\right)\sin(90-\theta) = \frac{MgL}{2}\cos\theta$. Equating our results, we obtain $\frac{MgL}{2}\cos\theta = \frac{1}{3}ML^2\alpha \ \Rightarrow$

$$\alpha = {}^{3g}/_{2L}\cos\theta$$

44. E… Light coming in perpendicular to a surface travels undeflected. When it reaches the back surface, we apply Snel's Law to obtain the angle of refraction as $n_g\sin\theta_g = n_a\sin\theta_{air} \ \Rightarrow \ \theta_{air} = \sin^{-1}\left(\frac{n_g\sin\theta_g}{n_a}\right)$ Evaluation of the argument inside the arcsine yields a value of $\frac{2.00\sin 45^\circ}{1.00} = 1.414$. Since the mathematics is impossible, we have total internal reflection of the light, which bounces it straight downward in the prism. The same analysis now will occur when this ray reaches the glass/air interface. Consequently, internal reflection occurs again and the ray now bounces to the left, where it exits the glass. Hence, the correct answer is E.

45. B… For a physical pendulum, $T = 2\pi\sqrt{\frac{I}{mgd}} = 2\pi\sqrt{\frac{\frac{1}{3}ML^2}{Mg\frac{L}{2}}} = 2\pi\sqrt{\frac{2L}{3g}}$. By doubling the mass, that has no effect on the period, whereas by doubling the distance, we have $T = 2\pi\sqrt{\frac{I}{mgd}} = 2\pi\sqrt{\frac{\frac{1}{3}(2M)(2L)^2}{2Mg\frac{2L}{2}}} = 2\pi\sqrt{\frac{4L}{3g}} = \sqrt{2}\,T$

46. A… For an isobaric process, the work done by the gas on the surroundings is computed as $W = P\Delta V = 80\,J$. The heat associated with this kind of process is

$$Q = nc_p\Delta T = n\left(\frac{5}{2}R\right)\Delta T = \frac{5}{2}P\Delta V = \frac{5}{2}(80) = 200\,J$$

since $P\,\Delta V = nR\Delta T$ for an isobaric process on an ideal gas.

47. D… By disconnecting the battery, the charge on the plates of the capacitor is fixed. Because the capacitance would increase to κC_0. Using for a capacitor that $Q = C\Delta V$, since Q is unchanged and the capacitance went up, this means that the voltage is now $V/_{\kappa}$. Alternatively, since the electric field would be decreased by a factor of κ with the introduction of the dielectric, and because $|\Delta V| = Ed$, then the voltage also decreases by a factor of κ

48. E… From the single slit relation, we have $a\sin\theta = m\lambda$. The angle from the slit to the dark region is computed as $\tan\theta = \frac{0.50\,m}{1.25\,m} \Rightarrow \theta = 21.8^\circ$. The $0.50\,m$ is the distance from the *center* of the bright region to the first dark spot. So, we now write $a = \frac{m\lambda}{\sin\theta} = \frac{632.8\times10^{-9}}{\sin 21.8^\circ} = 1.70\times10^{-6}\,m = 1.70\,\mu m$.

49. C… METHOD #1: By considering conservation of angular momentum from the point of contact with the floor, before the disk slides, the angular momentum is all from the rotation about the center of the disk which gives $L_i = I\omega_i = \frac{1}{2}MR^2\omega_0$. After allowing it to roll without slipping, the angular momentum can be computed about the instantaneous axis of rotation at the point of contact using the parallel axis theorem to find the moment of inertia as $I = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$. Consequently, by equating the angular momenta, we have $\frac{1}{2}MR^2\omega_0 = \frac{3}{2}MR^2\omega_f \Rightarrow \omega_f = \frac{1}{3}\omega_0$.

Alternatively: The angular momentum after reaching rolling without slipping, there is angular momentum about the center of the disk as well as angular momentum associated with the translation motion of the disk. Hence, we can write $\frac{1}{2}MR^2\omega_0 = Mv_tR + \frac{1}{2}MR^2\omega_f$ with $v_t = R\omega_f$ yielding

$$\frac{1}{2}MR^2\omega_0 = M(R\omega_f)R + \frac{1}{2}MR^2\omega_f = \frac{3}{2}MR^2\omega_f \quad \Rightarrow \omega_f = \frac{1}{3}\omega_0$$

METHOD #2: This problem could be approached using both angular and linear kinematics. Treating "to the right" as positive for translation and "clockwise" as positive for rotation, we can write velocity expressions as $v_f = v_0 + at$ and $\omega_f = \omega_0 + \alpha t$. From the free body diagram, $f_k = ma \Rightarrow \mu mg = ma \Rightarrow a = \mu g$. Likewise, by performing a torque analysis from the center of mass of the disk, we have $\tau_{net} = I\alpha \Rightarrow -Rf = \frac{1}{2}MR^2\alpha \Rightarrow \alpha = -\frac{2\mu g}{R}$. So, we have $v_f = 0 + \mu g t$ and $\omega_f = \omega_0 - \frac{2\mu g}{R}t$. We are looking for the condition at which $v_f = \mu g t = R\omega_f$, so we can write $R\omega_f = R\left(\omega_0 - \frac{2\mu g t}{R}\right) \Rightarrow R\omega_f = R\omega_0 - 2R\omega_f \Rightarrow 3\omega_f = \omega_0 \Rightarrow \omega_f = \frac{1}{3}\omega_0$.

50. B… METHOD #1: The key to this problem is recognizing that what is seen in the telescope is light that has made it all the way back to Earth. This means that for the Earth, the time that passes will be the time it takes the ship to move out some distance X and allow the light to return back to Earth. That is,

$90\ s = t_{out} + t_{back} = \frac{x}{0.8\ c} + \frac{x}{c} = \frac{1.8x}{0.8c} \Rightarrow x = 40\ c = 1.2 \times 10^{10}\ m$. From length contraction, the spaceship claims that a distance of $L_I = \frac{L_P}{\gamma} \Rightarrow \sqrt{1-(0.8)^2}(1.2 \times 10^{10}) = 7.2 \times 10^{9} m$ has been traversed. Hence, at a speed of 0.8c, a time of $t = \frac{L_I}{v} = \frac{7.2\times10^{9}}{0.8\ c} = 30\ s$ would be reading on the clock.

METHOD #2: Another approach would be to use the Doppler Effect equation by imagining that the spaceship clock emits a pulse after every second passes on the ship. The emitting frequency is $1\ Hz$ and the expression is $f_I = f_p\sqrt{\frac{c-v}{c+v}}$ when two objects move directly away from each other. Consequently, the received frequency of pulses on Earth would be $f_I = (1\ Hz)\sqrt{\frac{c-0.8c}{c+0.8c}} = \frac{1}{3} Hz$. Because this occurs for 90 seconds on the Earth, by receiving pulses at a rate of $\frac{1}{3} Hz$, a grand total of 30 pulses would reach Earth, meaning that it appears that only 30 seconds have passed on the spaceship.