AAPT UNITED STATES PHYSICS TEAM
AIP 2007

2007 *F=ma Contest SOLUTIONS*

Multiple Choice Answers

1. **e**
2. **b**
3. **b**
4. **c**
5. **a**
6. **b**
7. **d**
8. **b**
9. **d**
10. **d**
11. **e**
12. **c**
13. **d**
14. **a**
15. **b**
16. **b**
17. **a**
18. **b**
19. **a**
20. **c**
21. **e**
22. **d**
23. **c**
24. **b**
25. **d**
26. **b**
27. **e**
28. **a**
29. **b**
30. **e**
31. **d**
32. **e**
33. **c**
34. **b**
35. **a**
36. **d**
37. **c**
38. **d**

Solutions to Free Response

26. Since the acceleration is constant and the sled starts from rest,

$$\Delta x = \frac{1}{2} a t^2 \qquad (26\text{-}1)$$

so $a = 0.588$ m/s^2.

With the y-axis perpendicular to the incline, $a_y = 0$, so the normal force is

$$N = mg\cos\theta \qquad (26\text{-}2)$$

Applying Newton's second law parallel to the incline with f = force of kinetic friction

$$mg\sin\theta - f = ma \qquad (26\text{-}3)$$

Using $f = \mu N$ along with equations (26-2) and (26-3) we find that

$$\mu = \tan\theta - \frac{a}{g\cos\theta}$$

$$\mu = \tan 25 - \frac{0.588 m/s^2}{(10 m/s^2)\cos 25} = 0.40$$

27. An accelerated reference frame is equivalent to a gravitational field. We will denote all quantities that change when the astronauts move with a primed superscript after the move. Due to circular motion and the fact that the radius does not change and that $v = r\omega$, we find that

$$\frac{g'}{g} = \frac{v'^2}{v^2} = \frac{\omega'^2}{\omega^2} \qquad (27\text{-}1)$$

Angular momentum is conserved since there is no external torque acting on the system. Therefore,

$$I\omega = I'\omega' \qquad (27\text{-}2)$$

Since the corridors are long, we can consider the astronauts to be point masses. So, with r = the distance from the central hub to the living modules, m = the mass of one astronaut, and with two living modules each with N astronauts originally, we find that the rotational inertia before the astronauts move is

$$I = 2Nmr^2 \qquad (27\text{-}3)$$

After the two astronauts climb into the central hub,

$$I' = 2(N-1)mr^2 \qquad (27\text{-}4)$$

When we substitute (27-3) and (27-4) into (27-2) we obtain

$$2Nmr^2\omega = 2(N-1)mr^2\omega' \qquad (27\text{-}5)$$

$$\frac{\omega'}{\omega} = \frac{N}{N-1} \qquad (27\text{-}6)$$

Finally, substituting (27-6) into (27-1), we find

$$\frac{g'}{g} = \left(\frac{\omega'}{\omega}\right)^2 = \left(\frac{N}{N-1}\right)^2 \qquad (27\text{-}7)$$

28. For static equilibrium in an accelerated reference frame, we need to calculate torques about the center of mass. Let N_1 be the normal force on the front tire and N_2 the normal force on the rear tire. Let f_1 be the force of friction on the front tire and f_2 the force of friction on the rear tire. If the front tire just barely remains in contact with the ground then $N_1 = f_1 = 0$.

Then setting the counter-clockwise torque due to friction on the rear tire = the clockwise torque due to the normal force on the rear tire, we have

$$f_2 h = N_2 \frac{w}{2} \qquad (28\text{-}1)$$

Substituting $f_2 = \mu N_2$ into (28-1),

$$\mu N_2 h = N_2 \frac{w}{2} \qquad (28\text{-}2)$$

$$\mu = \frac{w}{2h} \qquad (28\text{-}3)$$

29 – 30. Applying Newton's Second Law to the horizontal direction,

$$f_1 + f_2 = Ma \qquad (29\text{-}1)$$

Setting clockwise torque = counterclockwise torques:

$$\frac{N_2 w}{2} = \frac{N_1 w}{2} + (f_1 + f_2)h \qquad (29\text{-}2)$$

Substituting (28-4) into (28-5) and solving for a,

$$Mah = \frac{w}{2}(N_2 - N_1) \qquad (29\text{-}3)$$

$$a = \frac{w}{2}\frac{(N_2 - N_1)}{Mh} \qquad (29\text{-}4)$$

The maximum acceleration will clearly occur when N_1=0. In that case, $N_2 = Mg$, and

$$a = \frac{w}{2}\frac{g}{h} \qquad (29\text{-}5)$$

(This is the answer to question 29. Note that this answer did not depend at all upon whether the coefficient of sliding friction for each tire and the ground is the same or different. Therefore, this is the answer to question 30 also.)

31. The rotational inertia of a thin, uniform rod about its center is

$$I_{cm} = \frac{1}{12}mL^2 \qquad (31\text{-}1)$$

We are given that the rotational inertia of the rod about its center is md^2. Setting this expression equal to (31-1), we obtain

$$md^2 = \frac{1}{12}mL^2 \qquad (31\text{-}2)$$

Therefore,

$$L^2 = 12d^2 \tag{31-3}$$

and

$$\frac{L}{d} = \sqrt{12} = 2\sqrt{3} \tag{31-4}$$

32. The torque due to gravity is the same as if the entire mass were located at the center of mass. Therefore, the gravitational torque on the rod about an axis through the suspension point a distance *kd* from the center when the rod making an angle θ to the vertical is

$$\tau_p = -mgkd\sin\theta \tag{32-1}$$

where the subscript *p* denotes the pivot point. We now need the parallel axis theorem to find the rotational inertia about the pivot point.

$$I_p = I_{cm} + mh^2 = md^2 + m(kd)^2 \tag{32-2}$$

$$I_p = md^2(1+k^2) \tag{32-3}$$

Now, apply the rotational analogue of Newton's Second Law to the axis through the pivot point. Noting that the force of the pivot does not exert a torque about an axis through the pivot and using equations (32-1) and (32-3), we find

$$\tau_p = I_p\alpha \tag{32-4}$$

$$-mgkd\sin\theta = md^2(1+k^2)\frac{d^2\theta}{dt^2} \tag{32-5}$$

For small oscillations, $\sin\theta \approx \theta$. Therefore,

$$\frac{d^2\theta}{dt^2} = -\frac{gk}{d(1+k^2)}\theta \tag{32-6}$$

Since an object oscillates with angular frequency ω when the object's motion is governed by the differential equation

$$\frac{d^2\theta}{dt^2} = -\omega^2\theta \qquad (32\text{-}7)$$

we find that

$$\omega = \sqrt{\frac{gk}{d(1+k^2)}} = \sqrt{\frac{k}{1+k^2}}\sqrt{\frac{g}{d}} \qquad (32\text{-}8)$$

$$\omega = \beta\sqrt{\frac{g}{d}} \qquad (32\text{-}9)$$

where

$$\beta = \sqrt{\frac{k}{1+k^2}} \qquad (32\text{-}10)$$

33. To find the value of *k* that gives the maximum value of β, square equation (32-10) and then differentiate both sides with respect to *k*.

$$2\beta\frac{d\beta}{dk} = \frac{(1+k^2)-k(2k)}{(1+k^2)^2} = \frac{1-k^2}{(1+k^2)^2} \qquad (33\text{-}1)$$

$$\frac{d\beta}{dk} = 0 \quad \text{when } k = 1 \qquad (33\text{-}2)$$

Substituting $k = 1$ into (32-10), we find that

$$\beta = \sqrt{\frac{1}{2}} \qquad (33\text{-}3)$$

34 – 36. Since the velocity is perpendicular to the rope, the rope does not do any work on the object. Since the object is moving on a horizontal frictionless surface, the net work done on the object is zero and therefore the change in kinetic energy of the object is zero.

Thus, the kinetic energy of the object at the instant that the rope breaks is the same as the initial kinetic energy of the object:

$$K = \frac{mv_0^2}{2} \qquad (34\text{-}1)$$

(This is the answer to #35.)

Therefore, the speed of the object is always v_0. The angular momentum of the object with respect to the axis of the cylinder is

$$L = mv_0 r \qquad (34\text{-}2)$$

where r is the radius of the circular orbit (which is the length of the not yet wound rope.)

At the time that the rope breaks, the tension is

$$T_{\max} = \frac{mv_0^2}{r} \qquad (34\text{-}3)$$

Solve equation (34-3) for r

$$r = \frac{mv_0^2}{T_{\max}} \qquad (34\text{-}4)$$

(This is the answer to #36.)

Substitute (34-4) into (34-2).

$$L = \frac{m^2 v_0^3}{T_{\max}} \qquad (34\text{-}4)$$

(This is the answer to #34.)

37 – 38. The cord breaks when it has exceeded a certain tension, which happens when it exceeds a certain potential energy, U_0. For the inelastic collision, when the cord is slack, we use conservation of momentum

$$mv_0 = 4mv' \tag{37-1}$$

$$v' = \frac{v_0}{4} \tag{37-2}$$

The kinetic energy of the two masses immediately after the collision is

$$K_1 = \frac{mv_0^2}{8} \tag{37-3}$$

All of this kinetic energy gets transferred into potential energy so we know that the cord breaks when

$$U_0 = K_1 = \frac{mv_0^2}{8} \tag{37-4}$$

Now for the elastic collision: First, find the velocities of each mass immediately after the collision while the cord is slack. The easy way to do this is to find that the velocity of the center of mass is

$$v_{cm} = \frac{v_0}{4} \tag{37-5}$$

Then in the center of mass reference frame, before the collision the velocity of m is $\frac{3v_0}{4}$ and the velocity of $3m$ is $\frac{-v_0}{4}$. In a one-dimensional elastic collision, in the center of mass reference frame, each block's velocity after the collision is the same magnitude, but in the opposite direction, of its velocity before the collision. So the velocity of the $3m$ object right after the collision in the center of mass reference frame is $\frac{+v_0}{4}$. Using (37-5) to transform back to the lab reference frame, we find that the velocity of the $3m$ object immediately after the collision is $\frac{v_0}{2}$ and therefore its kinetic energy immediately after the collision (while the cord is still slack) is

$$K_2 = \frac{3mv_0^2}{8} \qquad (37\text{-}6)$$

But since we know that the cord breaks, we know that U_0 of the kinetic energy of the *3m* block will be consumed by the cord. Therefore, the final kinetic energy of the *3m* block, using conservation of energy along with equations (37-4) and (37-6) is

$$K_3 = K_2 - K_1 = \frac{3mv_0^2}{8} - \frac{mv_0^2}{8} = \frac{mv_0^2}{4} \qquad (37\text{-}7)$$

Now we can find that

$$K_3 = \frac{3mv_f^2}{2} = \frac{mv_0^2}{4} \qquad (37\text{-}8)$$

$$\frac{v_f}{v_0} = \sqrt{\frac{1}{6}} \qquad (37\text{-}9)$$

The velocity of the object of mass *m* in the center of mass reference frame immediately after the collision was $\frac{-3v_0}{4}$. Transforming back to the lab reference frame, we find that the mass *m* has a velocity after the collision of $\frac{v_0}{2}$. Therefore, the kinetic energy of *m* after the elastic collision is

$$K_4 = \frac{mv_0^2}{8} \qquad (37\text{-}10)$$

The total kinetic energy of the system after the elastic collision and the cord is broken, using (37-7) and (37-10) is

$$K_3 + K_4 = \frac{mv_0^2}{8} + \frac{mv_0^2}{4} = \frac{3mv_0^2}{8} \qquad (37\text{-}11)$$

So, the ratio of the total kinetic energy of the system after the elastic collision and the cord is broken to the initial kinetic energy of the smaller mass prior to the collision is

$$\frac{\frac{3mv_0^2}{8}}{\frac{mv_0^2}{2}} = \frac{3}{4} \qquad (37\text{-}12)$$

