Apparatus Competition
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AREAL VELOCITY AND THE ORBIT OF MERCURY

Walter L. Triksoko
Department of Physics and Astronomy
Stephen F. Austin State University
Nacogdoches, TX 75962

W 936 468-3001
H 936 560-9046
wtrikosko@sfasu.edu

Abstract

Verifying Kepler's law of areal velocity can be challenging or tedious at best for an introductory physics or astronomy student with limited mathematical skills. What we present here is a scale model of the orbit of the planet Mercury cut from an acrylic sheet. Kepler's second law can be verified by comparing the mass of sections of the model.

Construction of Apparatus:

The eccentric anomaly ψ for the orbit of Mercury is calculated by numerically inverting Kepler's equation $\omega t = \psi - e\sin\psi$ in an Excel spreadsheet (available on request) for every 1/100th of a period. The cosine of the polar angle θ for the orbit is determined by $\cos\theta = \cos\left(\dfrac{\cos\psi - e}{1 - e\cos\psi}\right)$, where e is the eccentricity of Mercury's orbit. The radius is determined using $r = a\dfrac{1-e^2}{1+e\cos\theta}$ where a is the semi-major axis. The polar coordinates for the orbit are converted to Cartesian coordinates and entered into a CAD program from which a DXF file is generated. This file was carried to a local trophy shop where the plastic sheets were laser engraved and cut. The backing was cut from a scrap piece of dry erase board, engraved and the two sheets glued together.

	COST
14"x12"x12" Orange Fluorescent Acrylic Sheet	$13.10
1/16"X12"X24" Laserable Sheet	$10.48
Laser cutting of Acrylic 178"@$0.15/inch	$26.68
	$50.26

Use of Apparatus:

Kepler's second law states: A vector from the sun to a planet sweeps out equal areas in equal times. This is a direct consequence of the principle of conservation of angular momentum and holds for any central force, not just Newton's universal law of gravitation, $F = G\dfrac{Mm}{r^2}$, however, the inverse square law leads to closed elliptical orbits. With this apparatus we will verify Kepler's second law.

The apparatus consists of a base upon which is drawn a scaled ellipse of the orbit of Mercury.   The base is etched with tick marks at every 1/100 of a period. On the major axis the sun is drawn to the same scale as Mercury's orbit and locates one of the foci of Mercury's orbit. Along the bottom are two scales, one marked in multiples of 10^6 km and the other divided into fractions of an astronomical unit. These scales may be used for making measurements or a metric "ruler" can be used for making the necessary measurements depending on how scaling is to be emphasized.

In the cut-out of the base are "puzzle" pieces numbered one through nine each representing the area swept out by the radius vector from the sun to Mercury for different times. Pieces 1 through 5 are for the same fraction of a period, 1/10th period. Pieces 6 through 9 are for 20, 15, 10 and 5 hundredths of a period.

Part one

The first part of this exercise is to establish that for a homogenous medium of uniform thickness, the mass of an irregularly shaped body is proportional to the area. The kit contains a circle, ellipse and a rectangle. The diameter of the circle ($25X10^6$ km, scaled) is equal to the minor axis of the ellipse and the narrow dimension of the rectangle. The major axis of the ellipse ($50X10^6$ km, scaled) is the same as the longer dimension of the rectangle and twice the diameter of the circle.

The areas may be calculated in cm^2, 10^{12} km^2 or AU^2 depending on the concepts you wish to stress. Once this is established we are ready to tackle the law of areal velocity.

Measure and record the length, width and mass of the rectangle.

L=___________cm

W=___________cm

mass=____________gram

Calculate the area of the rectangle $A = L \times W$. *A*=___________ cm^2

Measure and record dimensions and mass of the ellipse

Major axis (L)=___________cm

Minor axis (W)=___________cm

mass=____________gram

Calculate the area of the ellipse $A = \frac{\pi}{4}(L \times W)$. *A*=___________ cm^2

Measure the diameter and mass of circle.

D=___________cm

mass=____________gram

Calculate the area of the circle $A = \frac{\pi}{4} D^2$. *A*=___________ cm^2

	Area (cm^2)	Mass (gm)
Rectangle		
Ellipse		
Circle		

Plot the mass vs. area for the three pieces and draw the best straight line through the data points.

How are the mass and area related for objects made of the same homogeneous material of uniform thickness?

Part two

Weigh each of the nine orbit segments and record the mass and the fraction of an orbital period each represents. Each tic mark around the scale model of Mercury's orbit represents $1/100^{th}$ of a period.

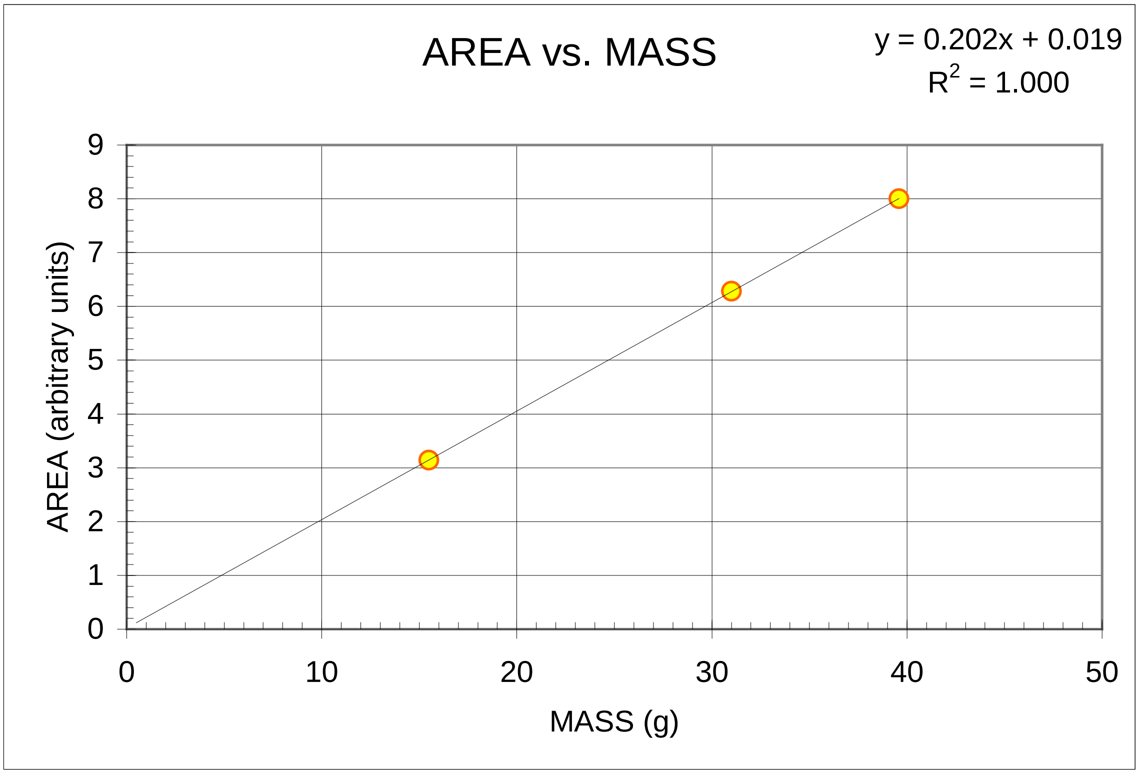
Piece #	Fraction of the orbital period (hundredths of a period)	Mass (gram)
1		
2		
3		
4		
5		
6		
7		
8		
9		

For sections 1 through 5: How are the times for these sections of the orbit related? How are the masses of segments 1 through 5 related and what does this tell you about the areas of segments 1 through 5?

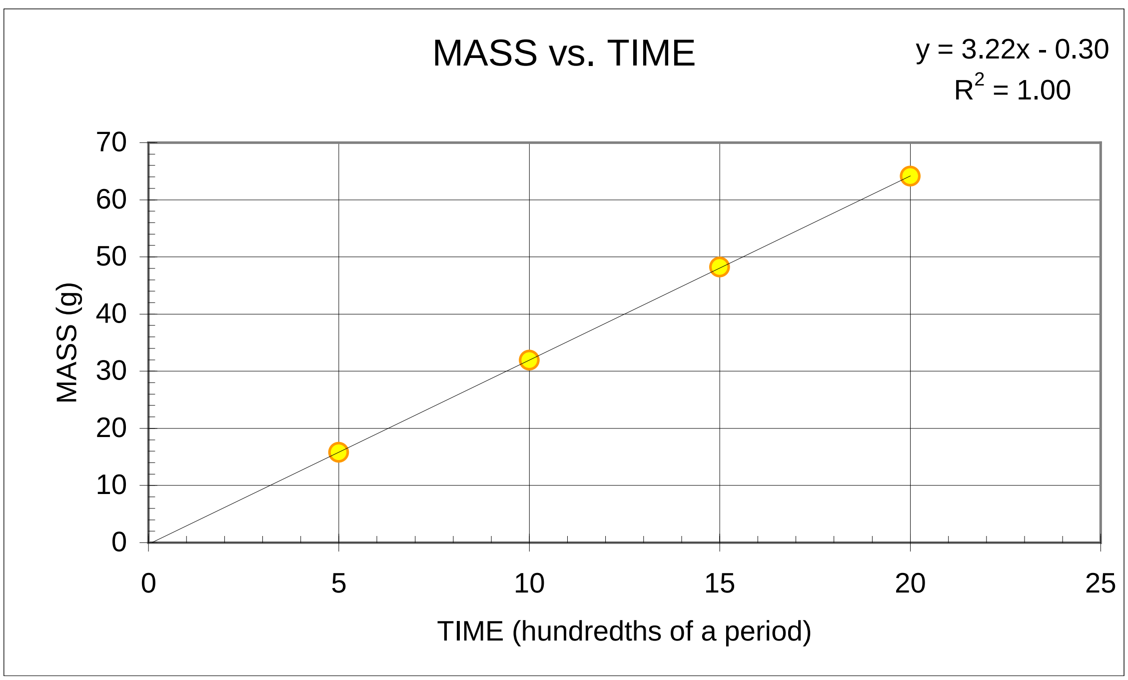
Plot a graph of the "area" (mass) vs. "time" (hundredths of period) for segments 6 through 9?

What conclusion can you draw about the relation between the time and the area swept out by the planet Mercury?

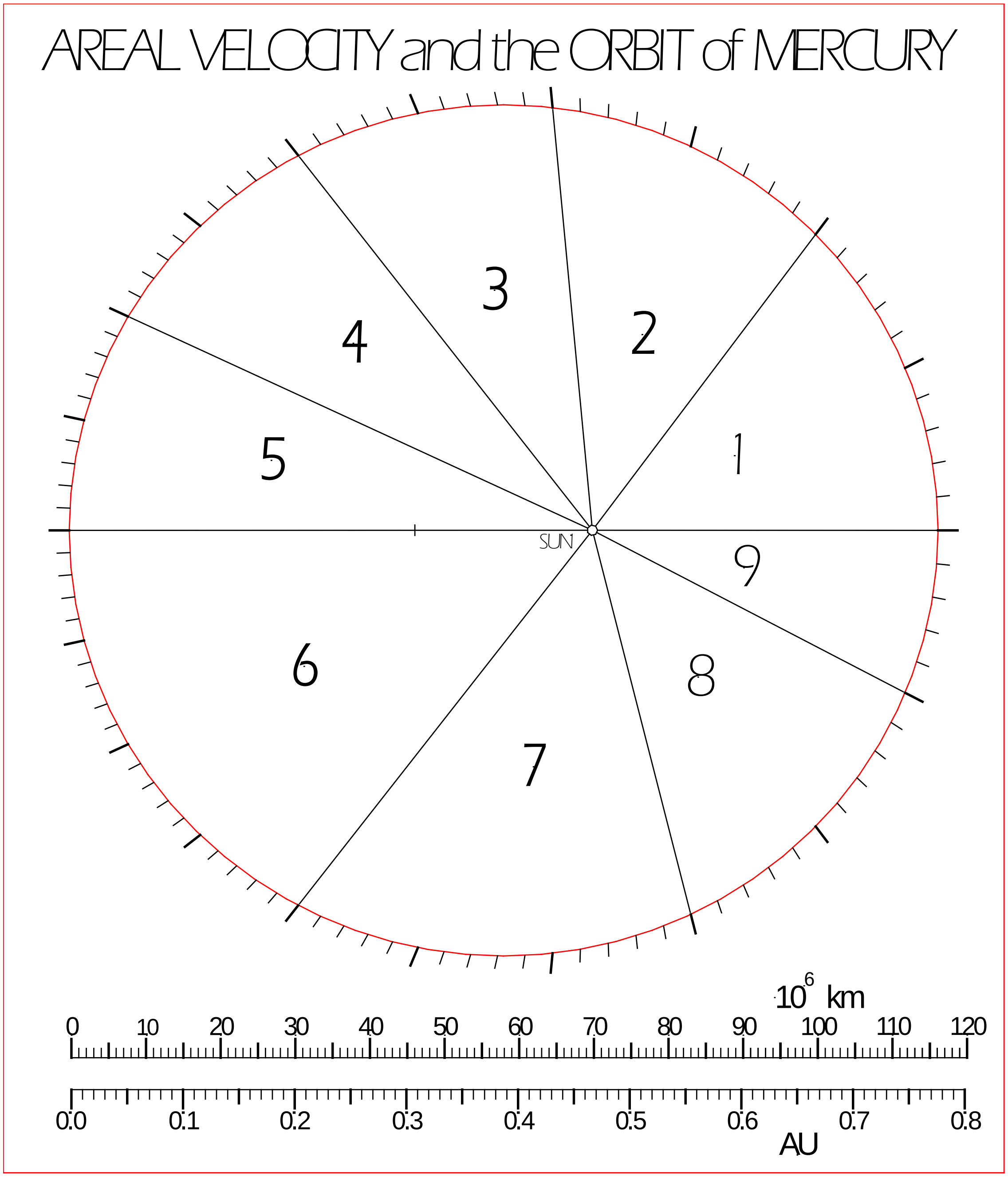
RESULTS FOR PART ONE

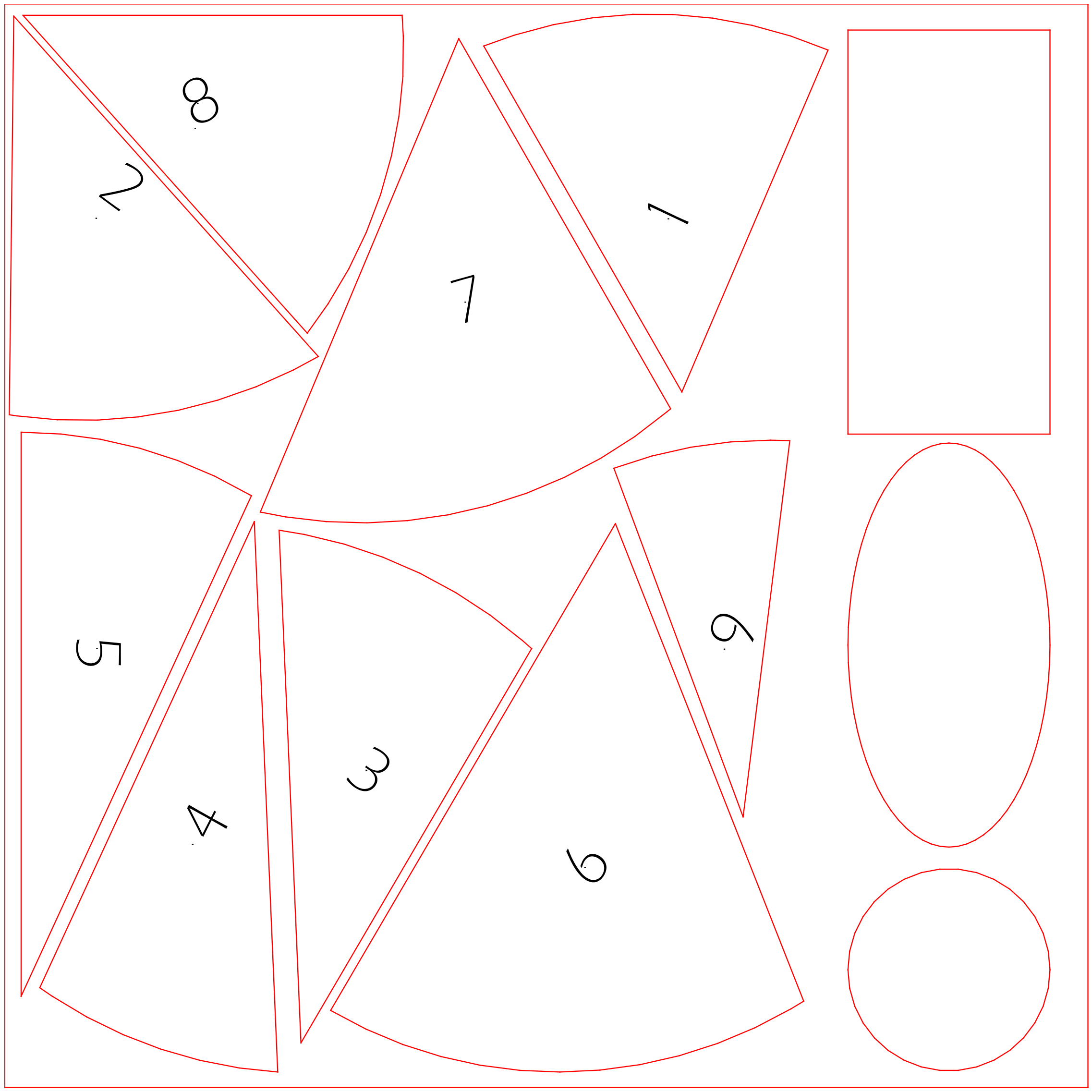


RESULTS FOR PART TWO



TEMPLATES


AREAL VELOCITY and the ORBIT of MERCURY
1
2
3
4
5
6
7
8
9
SUN
10^6 km
0
10
20
30
40
50
60
70
80
90
100
110
120
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
AU


8
2
1
7
5
4
3
6
9