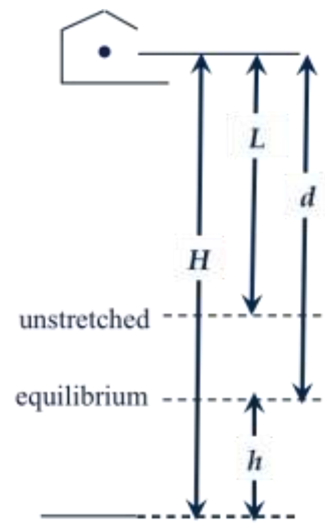
Physics Challenge for Teachers and Students

Solution to the December, 2016 Challenge ***The December Descent.***

First, we make the following assumptions:

1. Neglect dissipative forces such as air resistance and (at least for the first "dip") any internal energy losses in the bungee rope. Therefore we may assume that mechanical energy is conserved.
2. The bungee rope obeys Hooke's law when stretched, with a spring constant k.



If we are going to use conservation of mechanical energy to solve this problem, we need to know how much potential energy is stored in the rope at a given height above ground, for which we would need to know its unstretched length, L. For convenience I also define the equilibrium length of the rope when suspending the student as $d = H\text{-}h$, as shown at right. According to Hooke's law, with the rope's extension $d - L$,

$$mg = k(d-L) \Rightarrow \frac{mg}{k} = d - L. \qquad (1)$$

At the bottom of the first bounce, since the student's speed is instantaneously zero, the gravitational potential energy U_g lost in the fall equals the energy now stored in the rope, i.e.

$$\Delta U_s = -\Delta U_g \Rightarrow \tfrac{1}{2}k(H-L)^2 = mgH \Rightarrow \frac{mg}{k} = \frac{(H-L)^2}{2H} \qquad (2)$$

Equating the terms equal to mg/k in Equations (1) and (2) and solving for L,

$$\frac{(H-L)^2}{2H} = H - h - L \Rightarrow H^2 - 2LH + L^2 = 2H^2 - 2Hh - 2LH$$
$$\Rightarrow L = \sqrt{H(H-2h)} = \sqrt{80(80-40)} = 40\sqrt{2} \approx 56.6 \text{ m}. \qquad (3)$$

Where is the jumper moving fastest? Her speed is a maximum when the acceleration is zero, which is at the equilibrium position at a height h above the water, as used in Equation 1.

Since we now know the extension of the rope at that point is (H-h-L) = d-L, we can use conservation of mechanical energy to find their kinetic energy K as they pass through this equilibrium position. Initially before the jump, the student's kinetic energy = 0 and the rope's potential energy U_s= 0.

After the student has fallen to the equilibrium position, a total height loss $d = H\text{-}h$, we can write:

$$\Delta K + \Delta U_s + \Delta U_g = 0 \Rightarrow \tfrac{1}{2}mv^2 + \tfrac{1}{2}k(d-L)^2 - mgd = 0$$
$$\Rightarrow v^2 = 2gd - g\frac{k}{mg}(d-L)^2 = 2gd - g(d-L) = g(d+L) = g(H-h+L) \tag{4}$$

where we have substituted for mg/k from Equation 1. Finally, taking the square root and using the expression for the unstretched length L from Equation 3,

$$v = \sqrt{g(H-h+L)} = \sqrt{g}\sqrt{H-h+\sqrt{H(H-2h)}} = \sqrt{10}\sqrt{60+40\sqrt{2}} = 10\sqrt{2+\sqrt{2}} \approx 34.14 \text{ m/s.}$$

where g = 10 m/s^2, H = 80 m, and h = 20 m as given. That's about 76 mph!

Alternate method using standard results from simple harmonic motion

The student's motion where the rope is taut is part of a simple harmonic motion cycle around the equilibrium position, with amplitude $A=h$ and angular frequency given by $\omega = \sqrt{k/m}$. Therefore the maximum speed attained is,

$$|v_{\max}| = A\omega = h\sqrt{\frac{k}{m}} = h\sqrt{g}\sqrt{\frac{k}{mg}} = h\frac{\sqrt{g}}{\sqrt{d-L}}$$
$$= 20\frac{\sqrt{10}}{\sqrt{H-h-L}} = 20\frac{\sqrt{10}}{\sqrt{60-40\sqrt{2}}} = 10\left(2+\sqrt{2}\right) \approx 34.14 \text{ m/s.}$$

where we use Equation 1 to substitute for mg/k.

One can also calculate the maximum (upwards) acceleration of the student at the bottom of her motion as,

$$|a_{\max}| = A\omega^2 = h\frac{k}{m} = gh\frac{k}{mg} = \frac{gh}{d-L} = g\left(\frac{20}{60-40\sqrt{2}}\right) = \left(3+2\sqrt{2}\right)g \approx 5.83g \ ,$$

which is an "unpleasant" experience, especially if head-down!

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Guidelines for contributors.

- We ask that all solutions, preferably in Word format, be submitted to the dedicated email address *challenges@aapt.org*. Each message will receive an automatic acknowledgment.
- If your name is—for instance—Michael Phelps, please name the file "Phelps17March" (do not include your first initial) when submitting the March 2017 solution.
- The subject line of each message should be the same as the name of the solution file.
- The deadline for submitting the solutions is the last day of the corresponding month.
- Each month, a representative selection of the successful solvers' names will be published in print and on the web.
- If you have a message for the Column Editor, you may contact him at *korsunbo@post.harvard.edu*; however, please do not send your solutions to this address.

Note: as always, we would very much appreciate reader-contributed original *Challenges.*

Many thanks to all contributors; we hope to hear from many more of you.. We also hope to see more submissions of the original problems.--- *Boris Korsunsky, Column Editor*