Physics Challenge for Teachers and Students

***Boris Korsunsky*, Column Editor**
Weston High School, Weston, MA 02493
korsunbo@post.harvard.edu

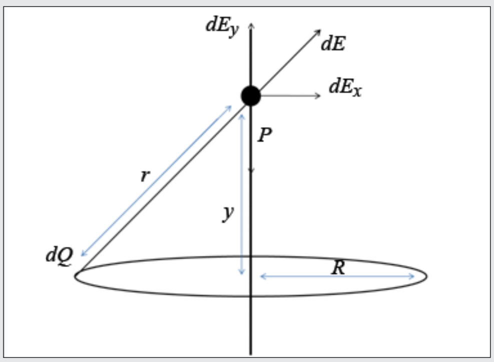
Solution to March 2016 *Challenge*

A ring on a string

A small marble of charge q and mass m can slide without friction along a long, thin vertical rod passing through the center of a horizontal conducting ring of radius r, mounted on an insulating support. What is the magnitude of the minimum charge Q placed on the ring that would allow the marble to oscillate along the rod?

Solution:

In order for the mass m to oscillate along the rod at some point y, it is necessary that the following conditions are met (figure):



- The equilibrium condition, i.e.:

$$F_{\text{total}-y} = \sum F_y = 0$$
$$F_{\text{total}-y} = E_y q - mg$$
$$E_y q = mg,$$

where E_y is the electric field of the ring at the location of the mass

$$E_y = \frac{1}{4\pi\varepsilon_0}\frac{Qy}{(y^2+R^2)^{3/2}}$$
$$\frac{1}{4\pi\varepsilon_0}\frac{qQy}{(y^2+R^2)^{3/2}} = mg. \qquad (1)$$

- It is also necessary to have a *stable* equilibrium, i.e.:

$$\frac{dF_{\text{total}-y}}{dy} = 0$$
$$\frac{dE_y}{dy} = 0 \qquad (2)$$

From Eq. (2) we obtain: $-2y^2 + R^2 = 0.$ (3)

From Eq. (3) we obtain the value of y

$$y = \pm R\sqrt{\frac{1}{2}}.$$

We use the obtained value of y in Eq. (1) and finally we obtain

$$Q = \pm\frac{mg2\pi\varepsilon_0 R^2\sqrt{27}}{q}.$$

So, we have two possible solutions:

$$\text{if } y > 0 \text{ then } Q = \frac{mg2\pi\varepsilon_0 R^2\sqrt{27}}{q}.$$

$$\text{if } y < 0 \text{ then } Q = -\frac{mg2\pi\varepsilon_0 R^2\sqrt{27}}{q}.$$

The final solution can be written in the form:

$$Q_{\min} = -\frac{mg2\pi\varepsilon_0 R^2\sqrt{27}}{|q|}.$$

Submitted by Manuel Rivero Pasca, student, Escuela Politécnica Superior, University of Seville, Seville, Spain)

We also recognize the following successful contributors:

Phil Cahill (The SI Organization, Inc., Rosemont, PA)
Don Easton (Lacombe, Alberta, Canada)
Fredrick P. Gram (Cuyahoga Community College, Cleveland, OH)
Art Hovey (Galvanized Jazz Band, Milford, CT)
José Ignacio Íñiguez de la Torre (Universidad de Salamanca, Salamanca, Spain)
John Mallinckrodt (Cal Poly Pomona, Pomona, CA)
Carl E. Mungan (U. S. Naval Academy, Annapolis, MD)
Thomas Olsen (Dar Al Uloom University, College of Medicine, Riyadh, Saudi Arabia)
Pascal Renault (John Tyler Community College, Midlothian, VA)
Joseph Rizcallah (School of Education, Lebanese University, Beirut, Lebanon)
Daniel Schumayer (University of Otago, Dunedin, New Zealand)

Asif Shakur (Salisbury University, Salisbury, MD)

Jason L. Smith (Richland Community College, Decatur, IL)

Clint Sprott (University of Wisconsin – Madison, WI)

Michael Wood (National Institute of Standards and Technology)

Guidelines for contributors:

- We ask that all solutions, preferably in Word format, be submitted to the dedicated email address *challenges@aapt.org*. Each message will receive an automatic acknowledgment.
- The subject line of each message should be the same as the name of the solution file.
- The deadline for submitting the solutions is the last day of the corresponding month.
- Each month, a representative selection of the successful solvers' names will be published in print and on the web.
- If your name is—for instance—Donald Duck, please name the file "**Duck16May**" (do not include your first initial) when submitting the May 2016 solution.
- If you have a message for the Column Editor, you may contact him at ***korsunbo@post.harvard.edu***; however, please do not send your solutions to this address.

Many thanks to all contributors; we hope to hear from many more of you in the future. We also hope to see more submissions of the original problems – thank you in advance!

Boris Korsunsky, Column Editor