Physics Challenge for Teachers and Students

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Solution to September 2016 *Challenge*

Wait a second!

A rock is launched vertically upward. Let d_1 be the distance traveled during the first second of the flight and d_2 the distance traveled during the second second. What is the maximum possible ratio of d_1/d_2? What is the initial speed of the rock that corresponds to that maximum ratio? Neglect the air resistance and assume that the flight lasts longer than two seconds. The acceleration due to gravity is g.

Solution:

If the rock passes through its highest point during the first second when its average speed is low, then d_1 will be smaller than d_2 and so the ratio d_1/d_2 will be less than 1, which is no good. To instead make d_2 as small as possible, the rock should pass through its highest point during the second second. In that case, the rock travels a distance d_1 during the first second $t = 1$ s given by

$$d_1 = v_0 t - \tfrac{1}{2} g t^2, \tag{1}$$

where v_0 is the launch speed. Split the second second into two intervals. The rock first travels upward a distance y_1 during time t_1 that can be conveniently found by considering the rock to fall downward during that time starting from rest at the top, so that

$$y_1 = \tfrac{1}{2} g t_1^2. \tag{2}$$

The rock next travels downward a distance y_2 during time $t_2 = t - t_1$ which likewise can be found as

$$y_2 = \tfrac{1}{2} g t_2^2 = \tfrac{1}{2} g (t - t_1)^2. \tag{3}$$

Adding together Eqs. (2) and (3) gives

$$d_2 = \tfrac{1}{2} g t_1^2 + \tfrac{1}{2} g (t - t_1)^2 = \tfrac{1}{2} g^2 - g t t_1 + g t_1^2. \tag{4}$$

If we again consider the rock to fall downward from rest at the top to its launch point, we find that the launch speed is

$$v_0 = g(t + t_1), \tag{5}$$

which we substitute into Eq. (1) to get

$$d_1 = \tfrac{1}{2} g t^2 + g t t_1. \tag{6}$$

The ratio of Eq. (6) to (4) is now a function only of t_1. So differentiate it with respect to that independent variable and equate the result to zero to find the maximum. After simplifying, one obtains the quadratic equation

$$t_1^2 + t t_1 - t^2 = 0, \tag{7}$$

whose positive root is

$$t_1 = \frac{\sqrt{5} - 1}{2} t. \tag{8}$$

(Interestingly, this time is *not* half of t but is instead a bit larger than half a second, apparently in order to make d_1 a bit larger than it would be if the second interval were distributed symmetrically about the peak of the rock's trajectory.)

Substituting this value of t_1 back into Eqs. (4) and (6), and taking their ratio gives the final answer

$$d_1 / d_2 = 2 + \sqrt{5} \approx 4.236 \tag{9}$$

and substituting Eq. (8) into (5) gives the requested launch speed as

$$v_0 = \tfrac{1}{2} g t (1 + \sqrt{5}) \approx (1.618 \text{ s}) g.$$

(Submitted by Carl E. Mungan, U. S. Naval Academy, Annapolis, MD)

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Guidelines for contributors:

– We ask that all solutions, preferably in Word format, be submitted to the dedicated email address *challenges@aapt.org*. Each message will receive an automatic acknowledgment.

– If your name is—for instance—Kate Middleton, please name the file "**Middleton16Dec**" (do not include your first initial) when submitting the December 2016 solution.

– The subject line of each message should be the same as the name of the solution file.

– The deadline for submitting the solutions is the last day of the corresponding month.

– Each month, a representative selection of the successful solvers' names will be published in print and on the web.

– If you have a message for the Column Editor, you may contact him at ***korsunbo@post.harvard.edu***; however, please do not send your solutions to this address.

Many thanks to all contributors and we hope to hear from many more of you in the future!

Note: as always, we would very much appreciate reader-contributed original *Challenges*.

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