Physics Challenge for Teachers and Students

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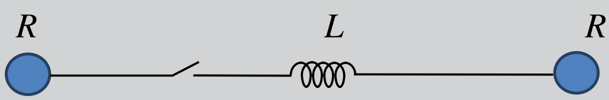
Solution to May 2017 *Challenge*

May we split that charge?

In a circuit shown below, two conducting spheres of radius R each are located far away from each other and are connected by thin wires through a solenoid with inductance L. Initially, one of the spheres is charged and the other is neutral. How long after the switch is closed do the charges on the spheres become equal?



Solution:

From Coulomb's law, electric field outside of a charged sphere is kQ/r^2. Integrating the field strength from an infinite distance down to the surface gives us the electric potential of the sphere:

$$V = kQ/R.$$

The capacitance of the sphere is $C = Q/V = R/k$. Because of the inductance, current in the wire increases from zero at a rate proportional to the potential difference between the spheres after the switch is closed:

$$dI/dt = d^2q/dt^2 = \Delta V/L.$$

In that equation "q" represents the charge on the second sphere. Since charge is conserved, the charge on the first sphere must always be $Q - q$. The potentials of the two spheres are then q/C and $(Q - q)/C$. Their potential difference is then

$$\Delta V = [(Q - q) - q]/C = [Q - 2q]/C$$

and, therefore,

$$(LC)[d^2q/dt^2] = Q - 2q.$$

The solution of that differential equation is sinusoidal. To satisfy the initial conditions, the graph of q vs. time must be a negative cosine curve above the time axis, and the time when the two charges become equal is one quarter of the period:

$$q = (Q/2)\,[1 - \cos(\omega t)] = Q/2 - (Q/2)\cos(\omega).$$

Differentiating twice with respect to time gives

$$d^2q/dt^2 = (\omega^2 Q/2)\cos(\omega t).$$

To find the value of ω, substitute those formulas into the differential equation and solve:

$(LC)(\omega^2 Q/2)\cos(\omega t) = Q - 2[Q/2 - (Q/2)\cos(\omega t)] = Q\cos(\omega t)$

$$\rightarrow \omega^2 LC/2 = 1 \;\rightarrow\; \omega^2 = 2/LC = 2k/RL$$

But $\omega = 2\pi/T$, so the period is $T = 2\pi/\omega = 2\pi(RL/2k)^{1/2}$.

The charges become equal after one-quarter of a cycle, so $t = T/4 = (\pi/2)(RL/2k)1/2^{1/2}$.

(Submitted by Art Hovey, Galvanized Jazz Band, Milford, CT)

We also recognize the following successful contributors:

- Alex M. Barr (Howard Community College, Columbia, MD)
- Pablo Bueno Martínez, student (Escuela Politécnica Superior, University of Seville, Seville, Spain)
- Phil Cahill (The SI Organization, Inc., Rosemont, PA)
- David A. Cornell (emeritus, Principia College, Elsah, IL)
- Norman Derby (Southwestern Oregon Community College, Brookings, Oregon)
- Don Easton (Lacombe, Alberta, Canada)
- Supriyo Ghosh (Kolkata, India)
- Andrew Hogan (Ramapo High School, Franklin Lakes, NJ)
- Omar Khan (GIK Institute, Pakistan)
- José Ignacio Íñiguez de la Torre (Universidad de Salamanca, Salamanca, Spain)
- Stephen McAndrew (Sydney, Australia)

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Yifan Zhou, student (Wuxi Big Bridge Academy, Wuxi, China)

Guidelines for contributors

- We ask that all solutions, preferably in Word format, be submitted to the dedicated email address *challenges@aapt.org*. Each message will receive an automatic acknowledgment.
- If your name is—for instance—Sean Spicer, please name the file "**Spicer17Sept**" (do not include your first initial) when submitting the September 2017 solution.
- The subject line of each message should be the same as the name of the solution file.
- The deadline for submitting the solutions is the last day of the corresponding month.
- Each month, a representative selection of the successful solvers' names will be published in print and on the web.
- If you have a message for the Column Editor, you may contact him at *korsunbo@post.harvard.edu*; however, please do not send your solutions to this address.

Many thanks to all contributors; we hope to hear from many more of you in the fall.
We also hope to see more submissions of the original problems – thank you in advance!

– Boris Korsunsky, Column Editor