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Similar Density Questions with Very Different Results

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ABSTRACT

While developing a standardized fluids assessment covering buoyancy and pressure, we discovered deficiencies in student understanding of density. In particular, many college students do not recognize that density is a fixed property of a solid substance, such as aluminum or gold. We added questions to our diagnostic exam to probe the extent of student difficulties. In one of our questions, only 50-60% of students (depending upon class) recognize that the density of gold is a fixed value. When similar questions from an existing diagnostic [1] are used, however, 88-100% of students correctly identify the density of a piece of wood and of a diamond as fixed values. In this paper we discuss the differences between these questions and how those differences affect student responses.

INTRODUCTION

While conducting interviews during the development of a fluids statics assessment, we discovered that many of our students do not have a robust understanding of mass density. To probe the prevalence of this difficulty, we wrote a question requiring students to recognize that the density of 24-karat gold is independent of the size or mass of a particular piece. Results were poor, with typically 50-60% of students answering correctly.

In fall, 2012 (F12), pre-instruction, students performed far worse on our "Rho Gold" question than on the original form of similar questions from Yeend's density survey [1]: "Three Piece" and "Diamond." Two differences between Yeend's questions and our question are (1) Rho Gold specifies the ratio of masses of the two pieces whose density are to be compared, and (2) Rho Gold presents the two pieces as initially separate, rather than as a single object which is then cut. In F12 and spring, 2013 (S13), we modified questions from Yeend's density survey to further explore this topic. Each semester's modifications targeted a single difference.

The questions from this study were included in assessments taken by students in all three introductory physics classes at Grove City College (GCC). These classes include a calculus-based ("calc") course, a trigonometry-based ("trig"), and a concept-based ("cnpt") course.

METHODS

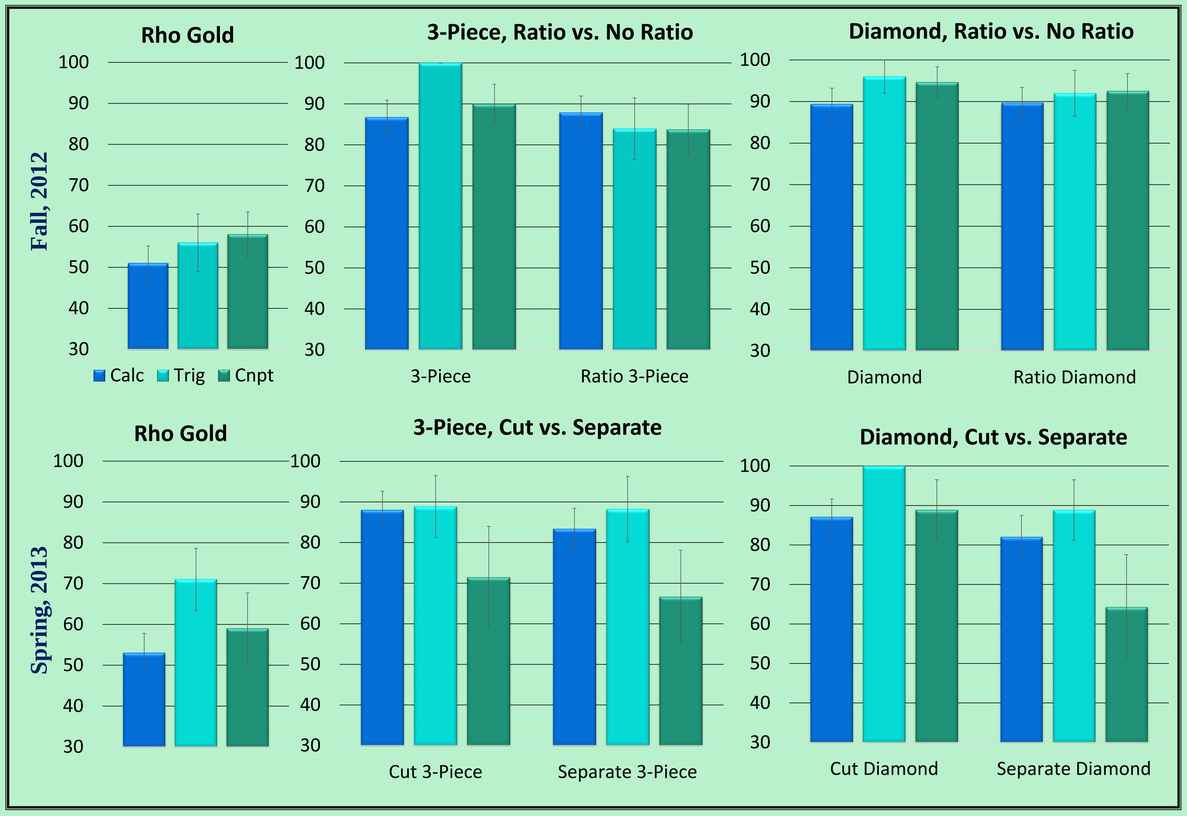
- **Two versions of Assessment F12 and S13 (full text of questions in rightmost column)**
  - **v. A: Original Three-Piece, Modified Diamond**
  - **v. B: Original Diamond, Modified Three-Piece**
- **F12 Modifications added size ratio**
- **S13 Modifications used different pieces that did not start as single object**
  - **Also used aluminum instead of wood in Three Piece, to keep density constant**
  - **Added "not enough information" to Three Piece**
- **Analyzed results with 3D ANOVA**
  - **Course and test version "between subject"**
  - **Whether question was modified "within subject"**

REFERENCES

1. Yeend's Density Survey, WWW Document, (http://www.compadre.org/Repository/document/ServeFile.cfm?ID=4313&DocID=1045).

2. W. V. Dooren, D. De Brock, D. Janssens and L Verschaffel, *Journal for Research in Mathematics Education*, 39 (3), 331-342, (2008).

RESULTS



- **Students generally performed better on the unmodified questions.**
- **Ratio had a significant negative effect on student performance.**
  - **Wilks' Lambda = .984, $F(1, 255) = 4.074$, $p = .045$**
  - **Partial Eta Squared = 1.6%**
    - **Partial Eta Squared is essentially a measure of what percentage of variance in student responses can be attributed to a particular factor. A value over 1% is considered a moderate effect.**
- **Using separate pieces also had a significant negative effect on student performance.**
  - **Wilks' Lambda = .971, $F(1,165) = 4.986$, $p = 0.027$**
  - **Partial Eta Squared = 2.9%**

CONCLUSIONS

- **Students' identification of density as an intensive property of a material is dependent upon many features of the question.**
- **When a ratio, or a specific number, is given in a problem, many students will try to use it.**
- **Students are more likely to treat the density of two pieces of a single object as fixed than to treat the density of any object made of a particular material as fixed.**
- **These modifications alone did not account for the entire difference in performance between our question and Yeend's questions.**
  - **Possible factors?**
    - **Position in test**
    - **Is Rho Gold a missing-value format question?**
      - **Students often answer missing-value format questions (e.g.. One apple costs $0.30, how much do 7 cost?) using linear or proportional reasoning, even when it is not appropriate [2]. This is likely because missing-value questions are used extensively to test linear reasoning.**
    - **Surface features such as context or answer letter**
      - **Perhaps some students are more familiar with aluminum, wood and diamonds than they are with 24-karat gold.**
- **Use solely of questions such as those on Yeend's assessment could lead to an over-estimation of students' mastery of density as an intensive quality of a substance.**
- **Our Rho Gold question may under-estimate students' mastery of density as an intensive quantity, by including distracting aspects. We will continue to test additional revisions.**

QUESTIONS

"Rho Gold": Alberta has several pieces of 24-karat gold, of various shapes and sizes. She measures the density of the first pieces and calls that value *d*. The second piece of gold she picks up has four times the mass of the first piece. What is its density?

A. $d \div 16$.
B. $d \div 4$.
C. *d*.
D. $2 \times d$.
E. $4 \times d$.
F. Alberta needs more information to determine the density.

"Three Piece": A straight, uniform board is cut into three differently sized pieces. Each pieces has identical width and thickness. Which piece has the greatest density?

A. Piece A
B. Piece B
C. Piece C
D. They are all the same.

"Ratio Three Piece": A straight, uniform board is cut into three pieces of different length: B is twice as long as A, and C is three times as long as A. Each pieces has identical width and thickness. How does the density of piece C compare to the density of piece A?

A. Piece C has 27 times the density of A.
B. Piece C has 3 times the density of A.
C. Piece C has the same density as A.
D. Piece C has 1/3 the density of A.
E. Piece C has 1/27 the density of A.

"Cut Three Piece": A straight, rectangular piece of aluminum is cut into three differently sized pieces. Each pieces has identical width and thickness. Which piece has the greatest density?

A. Piece A
B. Piece B
C. Piece C
D. They are all the same.
E. Impossible to tell without more information

"Separate Three Piece": Consider three rectangular pieces of aluminum with different length. Each piece has identical width and thickness. Which piece has the greatest density?

A. Piece A
B. Piece B
C. Piece C
D. They are all the same.
E. Impossible to tell without more information.

"Diamond": A jeweler cut a small chip off a large, uncut diamond. How does the density of the chip compare with the density of the original diamond?

A. The density of the chip is the same as the density of the original diamond.
B. The density of the chip is smaller than the density of the original diamond.
C. The density of the chip is larger than the density of the original diamond.
D. Impossible to tell unless the volume and mass of each piece are given.

"Ratio Diamond": A jeweler cut a small chip off a large, uncut diamond. The chip has a mass 1/100 as large as the original diamond. How does the density of the chip compare with the density of the original diamond?

A. The density of the chip is the same as the density of the original diamond.
B. The density of the chip is 1/100 of the density of the original diamond.
C. The density of the chip is 1/1,000,000 of the density of the original diamond.
D. The density of the chip is 100 times larger than the density of the original diamond.
E. The density of the chip is 1,000,000 times larger than the density of the original diamond.
F. Impossible to tell unless the volume and mass of each piece are given.

"Separate Diamond": A jeweler is considering various large diamonds and small diamond chips when making a necklace. How does the density of a small diamond chip compare with the density of a large diamond?

A. The density of the small chip is the same as the density of the large diamond.
B. The density of the small chip is smaller than the density of the large diamond.
C. The density of the small chip is larger than the density of the large diamond.
D. Impossible to tell unless the volume and mass of each piece are given.