Teaching Upper-Division Electromagnetism

in the Paradigms Program

http://physics.oregonstate.edu/portfolioswiki

Corinne Manogue

& the whole

Paradigms Team

Support

- National Science Foundation
  - DUE-9653250, 0231194, 0618877
  - DUE-0837829, 1023120, 1323800
  - DUE-0088901, 0231032, 0837829
- Oregon State University
- Oregon Collaborative for Excellence in the Preparation of Teachers
- Grinnell College
- Mount Holyoke College
- Utah State University

Workshop on the Status of the Upper-Division Physics Curriculum

- Conference site, presentations, and report:

  http://compadre.org/SUPC/

- Many thanks to Ernest Behringer for an insightful conference summary.
- Upper-division collections on comPADRE are in process.

Workshop on the Status of the Upper-Division Physics Curriculum

- Griffiths E&M text is considered by most to be canonical.
- There is no common agreement about quantum textbooks or goals.
- 1/3 of participant faculty do not feel comfortable enough with thermal content to want to teach the course.

Colorado Goals

- Math/physics connection
- Visualize the problem
- Organized knowledge
- Communication
- Problem-solving strategy
- Expect & check solution
- Intellectual maturity
- Maxwell’s Equations
- Build on Earlier Material
- Problem-solving techniques
  - Approximations
  - Series expansions
  - Symmetries
  - Integration
  - Superposition

http://www.colorado.edu/sei/departments/physics_learning.htm

- Raise your hand if you believe that Green Functions are an advanced topic.

Griffiths 1: Vector Analysis

- Derivatives: Gradient, Divergence, Curl
- Product Rules, Second Derivatives
- Theorems
- Curvilinear Coordinates
- Dirac Delta Function
- Theory of Vector Fields

Griffiths 1: Vector Analysis

- Derivatives: Gradient, Divergence, Curl
  (in rectangular coordinates)
- Product Rules, Second Derivatives
- Theorems (maybe)
- Curvilinear Coordinates (not $\hat{r}, \hat{\theta}, \hat{\phi}$).
- Dirac Delta Function
- Theory of Vector Fields

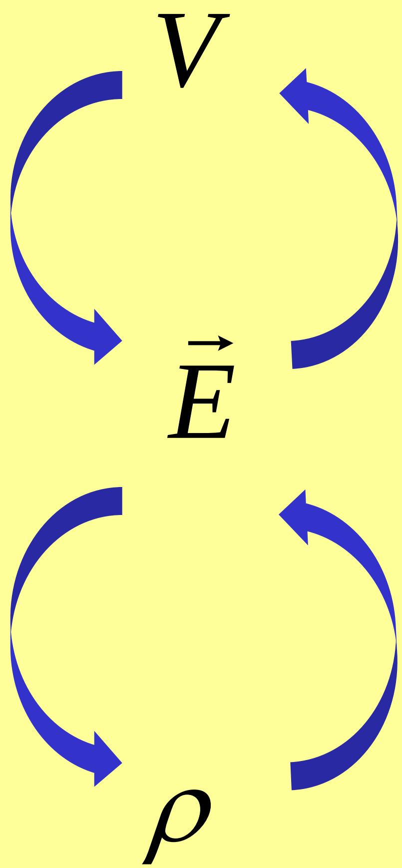
Griffiths 2 & 5: Electrostatics (Magnetostatics)

- Introduce $V \quad \vec{E} \quad \rho$
- Direct integration (chop up the source and accumulate) Use explicit coordinates.
- What is script r? Note that Coulomb's law is a Green function.
- Use the theorems to derive differential versions of Maxwell's eqns.

Griffiths 2 & 5: Electrostatics (Magnetostatics)

$$\vec{E} = -\vec{\nabla} V$$

$$V = -\int \vec{E} \cdot d\vec{r}$$



$$\frac{\rho}{\varepsilon_0} = \vec{\nabla} \cdot \vec{E}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho(\vec{r}-\vec{r}')}{\left|\vec{r}-\vec{r}'\right|^3} d\tau'$$

Griffiths 4 & 6: Electric (Magnetic) Fields in Matter

- Polarization
- The Field of a Polarized Object
- The Electric Displacement
- Linear Dielectrics

Derivation of Bound Charges

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int_V \frac{(\vec{r}-\vec{r}')\cdot\vec{P}(r')}{|\vec{r}-\vec{r}'|^3}\,d\tau'$$

$$= \frac{1}{4\pi\varepsilon_0}\int_V \vec{P}(r')\cdot\vec{\nabla}'\left(\frac{1}{|\vec{r}-\vec{r}'|}\right)d\tau'$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\int_V \vec{\nabla}'\cdot\left(\frac{\vec{P}}{|\vec{r}-\vec{r}'|}\right)d\tau' - \int_V \frac{1}{|\vec{r}-\vec{r}'|}\left(\vec{\nabla}'\cdot\vec{P}\right)d\tau'\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\int_S \frac{1}{|\vec{r}-\vec{r}'|}\,\vec{P}\cdot d\vec{a}' - \int_V \frac{1}{|\vec{r}-\vec{r}'|}\left(\vec{\nabla}'\cdot\vec{P}\right)d\tau'\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\int_S \frac{\sigma_b}{|\vec{r}-\vec{r}'|}\,da' - \int_V \frac{\rho_b}{|\vec{r}-\vec{r}'|}\,d\tau'\right]$$

Griffiths 3: Special Techniques

- Uniqueness Theorems
- Method of Images
- Separation of Variables
  - Rectangular Coordinates
  - Spherical Coordinates w/ Azimuthal Symmetry
- Multipole Expansions

Real Title of This Talk

It is impossible to teach upper-division E&M as defined by Griffiths.

(or—you can teach it but your students can't be expected to follow!)

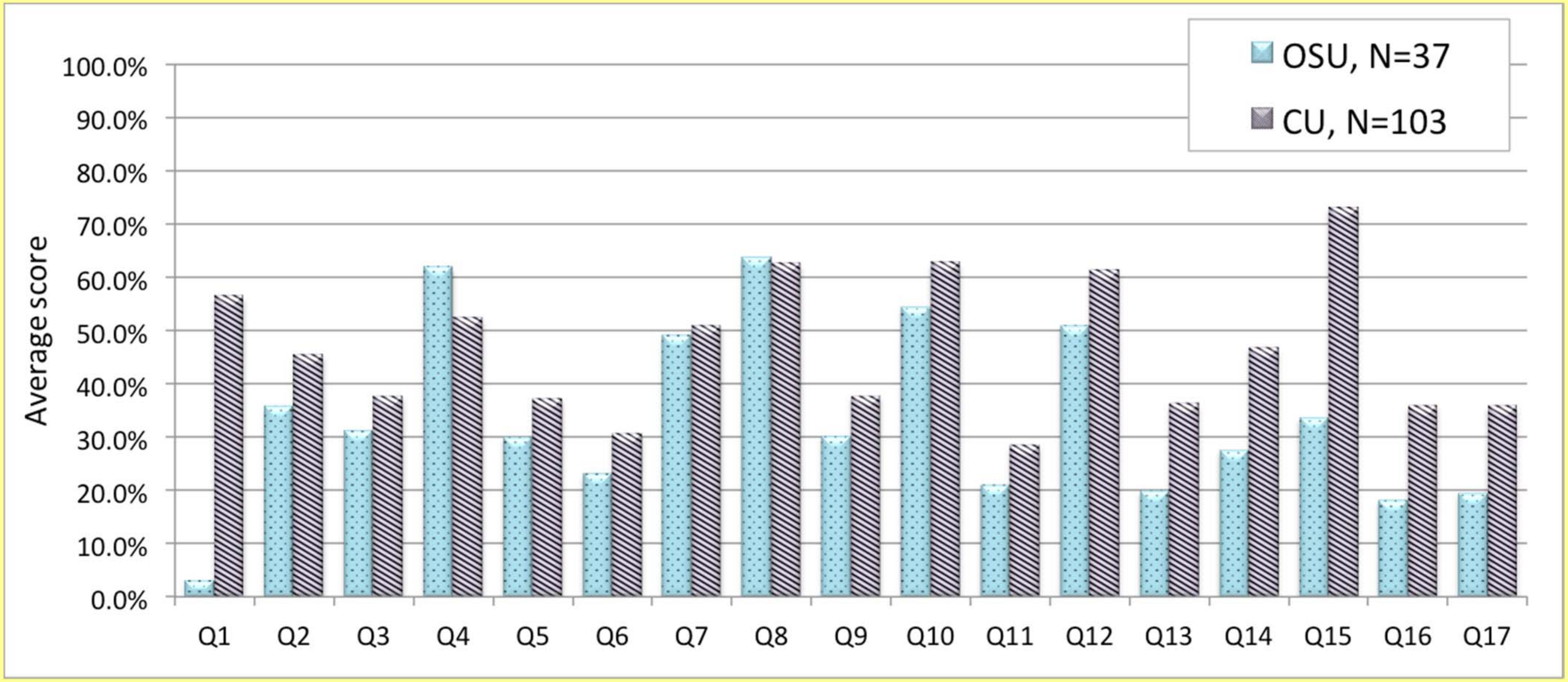
YOU HAVE TO MAKE CHOICES.

Our Choices

- In the equivalent of 12 weeks:
  - All of Chapter 1, but mixed in with the physics.
  - All of Chapters 2 & 4, emphasizing geometry.
  - An intro version of multipole expansions.
  - An intro version of vector algebra manipulations.
  - An intro version of the uniqueness theorems.
- In quantum mechanics:
  - Separation of variables
- In 10 weeks:
  - Everything else!!! (including Chaps 4 & 6)

CUE Data

What do you notice about this data?

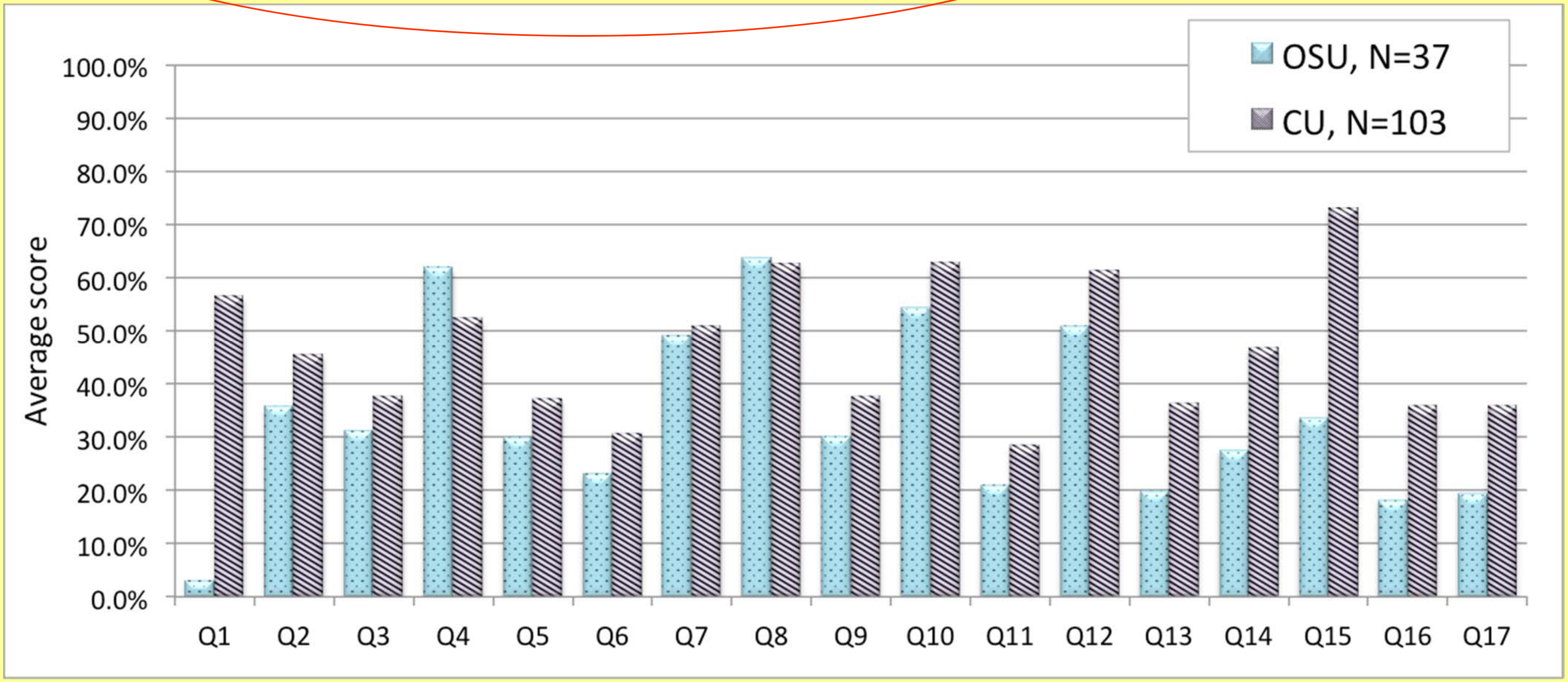


What I notice

- E&M I is hard!
- The Paradigms curriculum is not as successful as Colorado’s (maybe).
- Paradigms students can’t answer questions 1 & 15.
- Paradigms students tail off at the end.
- Otherwise the performance of Paradigms students tends to parallel the performance of Colorado students.

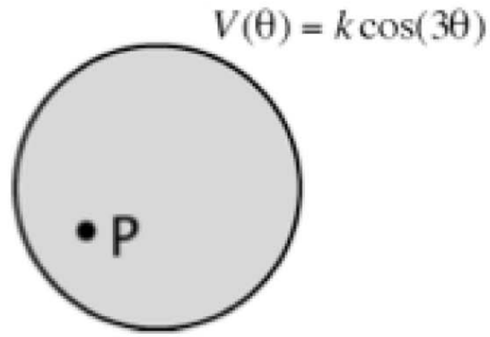
CUE Data

- OSU Normalized Gain 33%
- CU Normalized Gain 34%



CUE Separation of Variables

Q1. An insulating sphere with radius R, with a voltage on its surface $V(\theta) = k\cos(3\theta)$. Find E (or V) inside the sphere at point P.



Q15. Circle **all** of the following boundary conditions that are suitable for solving Laplace's equation for finding V(r,θ) everywhere due to a charge density σ on a spherical surface of radius R.

(I) $V_{in} = V_{out}$ at r=R

(II) $\vec{E}_{in} = \vec{E}_{out}$ at r=R

(III) $E_{in}^{\perp} - E_{out}^{\perp} = -\sigma/\varepsilon_0$ at r=R

(IV) $E_{in}^{\parallel} - E_{out}^{\parallel} = -\sigma/\varepsilon_0$ at r=R

Colorado Goals

- Math/physics connection
- Visualize the problem
- Organized knowledge
- Communication
- Problem-solving strategy
- Expect & check solution
- Intellectual maturity
- Maxwell's Equations
- Build on Earlier Material

- Problem-solving techniques
  - Approximations
  - Series expansions
  - Symmetries
  - Integration
  - Superposition

http://www.colorado.edu/sei/departments/physics_learning.htm

Our materials

- “Just in time” math.

Five Ring Problems

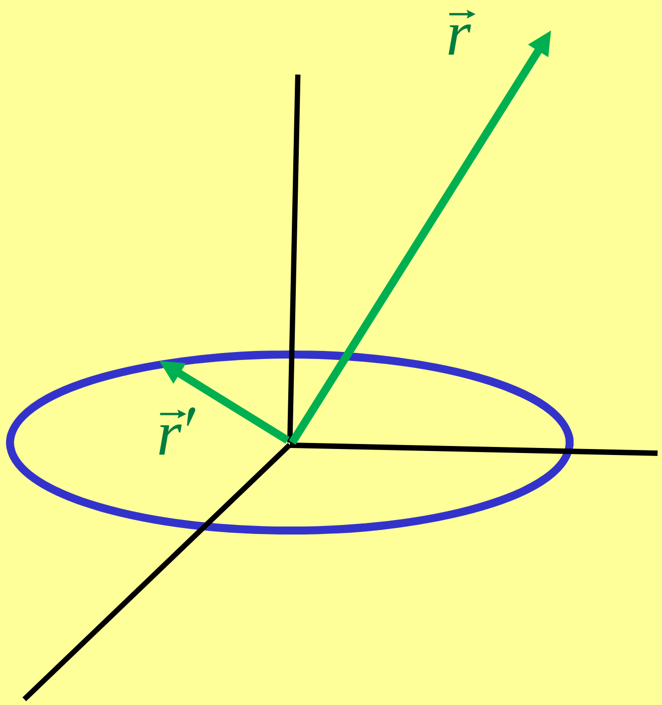
$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{|\vec{r}-\vec{r}'|}$$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(r')}{|\vec{r}-\vec{r}'|}\,d\tau'$$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(r')(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^2}\,d\tau'$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(r')}{|\vec{r}-\vec{r}'|}\,d\tau'$$

$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(r')\times(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^2}\,d\tau'$$



Our materials

- Spiral approach with reuseable content (distance between points, charge/current density)

Five Ring Problems

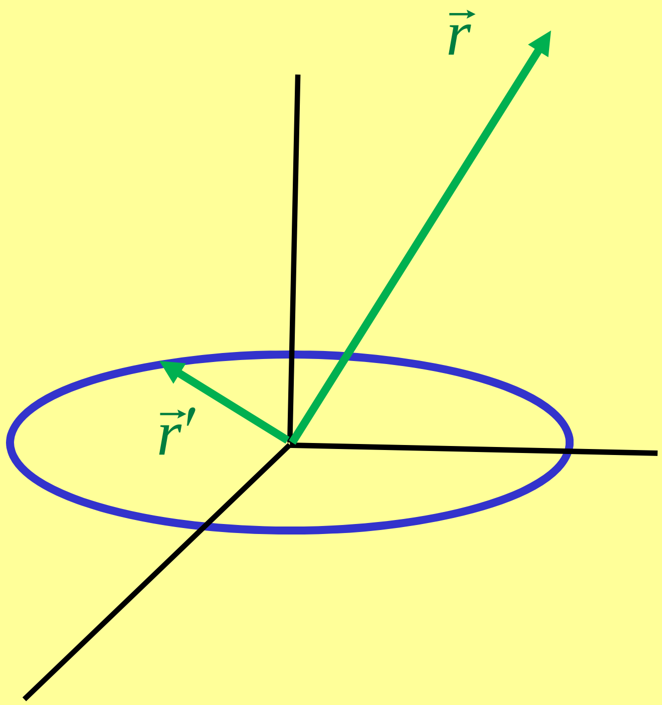
$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{|\vec{r}-\vec{r}'|}$$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(r')}{|\vec{r}-\vec{r}'|}\,d\tau'$$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(r')\widehat{(\vec{r}-\vec{r}')}}{|\vec{r}-\vec{r}'|^2}\,d\tau'$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(r')}{|\vec{r}-\vec{r}'|}\,d\tau'$$

$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(r')\times\widehat{(\vec{r}-\vec{r}')}}{|\vec{r}-\vec{r}'|^2}\,d\tau'$$



Our materials

- Communication from students to faculty
- Respect for student difficulties.

Prompt

- Find the magnetic vector potential everywhere in space from a spinning ring of charge with radius R, charge Q, and period T. Continue until you get an expression that *Maple* or *Mathematica* could evaluate.

Five Ring Problems

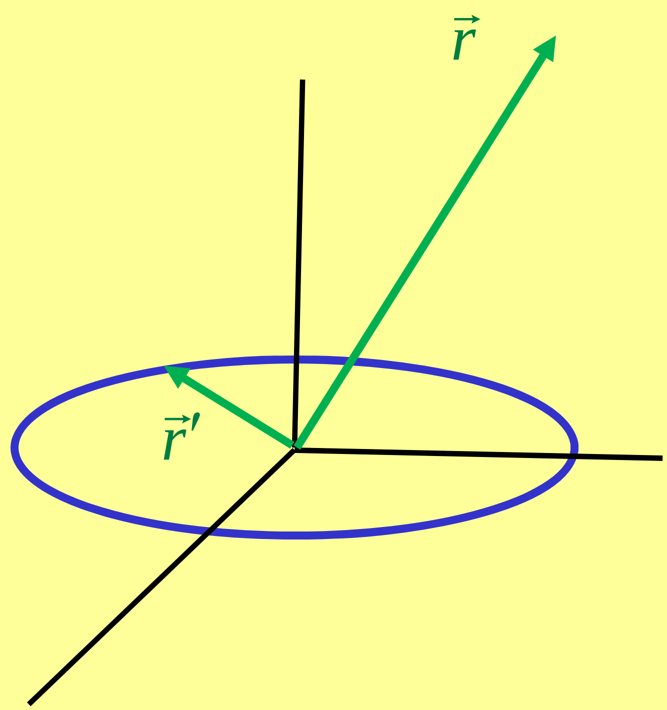
$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{|\vec{r}-\vec{r}'|}$$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(r')}{|\vec{r}-\vec{r}'|}\,d\tau'$$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(r')(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^2}\,d\tau'$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(r')}{|\vec{r}-\vec{r}'|}\,d\tau'$$

$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(r')\times(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^2}\,d\tau'$$



Our materials

- Students do hard examples in groups during class with faculty support but no prior lecture/reading. (back flip).

Five Ring Problems

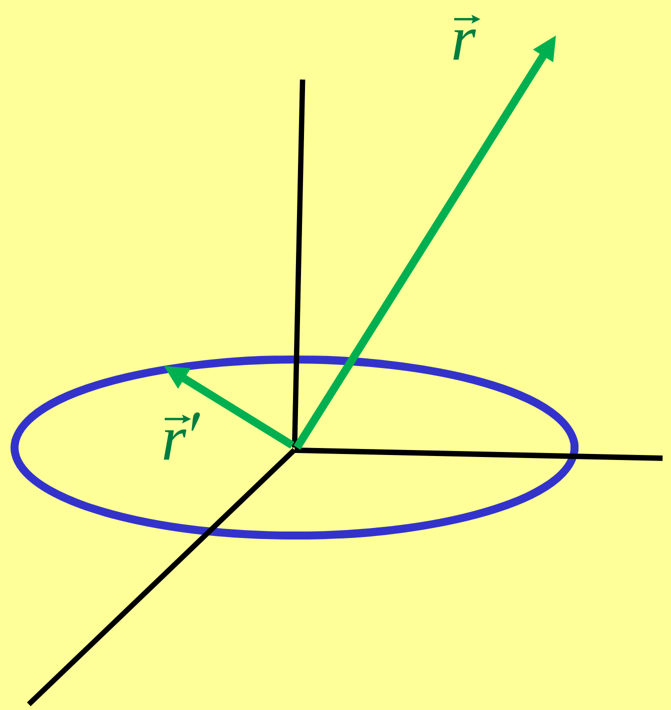
$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{\left|\vec{r}-\vec{r}'\right|}$$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(r')}{\left|\vec{r}-\vec{r}'\right|}\,d\tau'$$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(r')\widehat{(\vec{r}-\vec{r}')}}{\left|\vec{r}-\vec{r}'\right|^2}\,d\tau'$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(r')}{\left|\vec{r}-\vec{r}'\right|}\,d\tau'$$

$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(r')\times\widehat{(\vec{r}-\vec{r}')}}{\left|\vec{r}-\vec{r}'\right|^2}\,d\tau'$$



The Purpose of Education
—A challenging vision

"…the purpose of your education is your growth as an individual and the development of your capacity to contribute to the transformation of society."

—FUNDAEC

Fundación para la Aplicación y Enseñanza de las Ciencias