Radiation Equilibrium, Inertia Moments, and the Nucleus Radius in the Electron-Proton Atom

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Abstract Bohr's Atom resolves the paradox of the lost energy that does not lead to the Atom collapse, by claiming that since there is no Atomic collapse, there cannot be lost radiation.

According to Bohr's Theory, Electrodynamics does not apply to the acceleration of the electron towards the proton. The Quantized Angular Momentum electron orbits allow no radiation, and consequently, no atomic energy loss.

Bohr proposed that the electron orbits can defy Electrodynamics because they have angular momentums that are discrete multiples of $\hbar$.

Consequently, the orbits are occupied by standing waves,

and no radiation takes place in them.

However, while the standing wave argument is pure hypothesis, the radiation by an accelerating charge is an experimental fact of electrodynamics.

Bohr's resolution failed to take into account the proton's radiation that compensates for the electron's radiation loss.

Thus, Bohr argument does not guarantee that the Atom will not collapse.

And it does place the electron-proton Atom out of the laws of Physics, and Electrodynamics.

Clearly, the Proton has its own orbits, in which it accelerates towards the electron, and radiates it.

Then, the electron-proton system satisfies a modified Kepler's 3rd law for the periods of the electron and the proton

$$\boxed{\left(\frac{T_{electron}}{T_{proton}}\right)^2 = \frac{m_{electron}}{M_{proton}}\left(\frac{r_{electron}}{\rho_{proton}}\right)^3}$$

Next, we assume that the power of the energy radiated by the proton into the electron field equals the power of the energy radiated by the electron into the proton field.

This leads to the equality of the Inertia Moments of the electron, and the proton: A purely Mechanical condition.

Namely, we show that

The electron-proton Atom is <u>*electrodynamically stable*</u> *if and only if the electron and proton inertia moments are equal*

$$\boxed{m_{electron} r_{electron}^2 \approx M_{proton} \rho_{proton}^2}$$

This equality determines the radius of the proton's orbit, which is the Nucleus Radius of the Electron-Proton Atom:

$$\boxed{\rho \approx r\sqrt{\frac{m_e}{M_p}} \approx 1.221173735 \times 10^{-12}}$$

The Proton's Period is approximately

$$\boxed{T_p \approx T_e \sqrt[4]{\frac{m_e}{M_p}} \sim 2 \times 10^{-15} \,/\, 6.546 \sim 3 \times 10^{-16}}$$

The Proton's Angular Velocity is approximately

$$\boxed{\Omega_{\mathrm{p}} \approx \omega_e \sqrt[4]{\frac{M_p}{m_e}} \sim 3 \times 10^{15} \times 6.546 \sim 2 \times 10^{16}}$$

The Proton's Quantized Angular Momentums are approximately

$$\boxed{M\Omega_n \rho_n^2 \sim n\hbar \sqrt[4]{\frac{M_p}{m_e}} \approx n\hbar(6.546)}$$

Keywords: Electromagnetic Radiation of Accelerated Charge, Relativistic, Radiation Loss, Quantized Angular Momentum, Atomic Orbits, Electron, Proton, Atom, Nucleus Radius,

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The Electron Spiraling onto the Proton

0.1 Electron's Acceleration towards the Proton

The Electron-Proton Atom is a planetary system, where the electron orbits the proton along some ellipse. For simplicity, we consider a circular orbit of radius r determined by a balance between the centripetal force

$$m_e \frac{v^2}{r} = m_e \omega^2 r\,,$$

and the electric attraction

$$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}.$$

That is,

$$m_e \omega^2 r = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}$$

Thus, the electron's acceleration towards the proton is

$$\boxed{\omega^2 r = \frac{1}{m_e}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}}.\square$$

0.2 Non-relativistic Electron

The velocity of the electron in its orbit is tangential to the orbit, and perpendicular to the electron's acceleration towards the proton.

To see that the velocity is non-relativistic, far slower than light speed c, approximate the electron orbit's radius by

$$r \sim 6 \times 10^{-11}\text{meter}$$

A wavelength in the mid optical spectrum is

$$\lambda \sim 6 \times 10^{-7}\text{meter.}$$

This wavelength corresponds to the optical frequency

$$\nu = \frac{c}{\lambda} \sim \frac{3 \times 10^{8}}{6 \times 10^{-7}} = 5 \times 10^{14}\text{cycles/sec.}$$

This frequency corresponds to angular velocity

$$\omega = 2\pi\nu \sim 6 \times 5 \times 10^{14} = 3 \times 10^{15}\text{radians/sec.}$$

The electron's velocity is

$$v = \omega r \sim 3 \times 10^{15} \times 6 \times 10^{-11} \sim 2 \times 10^{5}\text{meter/sec}$$

Thus,

$$\frac{v}{c} \sim \frac{2 \times 10^{5}}{3 \times 10^{8}} \ll 1,$$

Then, Lorentz Factor

$$\gamma(v) = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \approx 1$$

is non-relativistic.

0.3 The Electron Spiraling Time onto the Proton

Since
$$m\frac{v^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2},$$

The kinetic energy of the electron in its circular orbit is

$$\tfrac{1}{2}mv^2 = \frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}.$$

The electric energy of the Electron-Proton Atom is

$$-\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}.$$

Therefore, the total electron energy is

$$-\frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r},$$

and its rate of change is

$$\frac{d}{dt}\left\{-\frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}\right\} = \frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}\frac{dr}{dt}.$$

This equals the rate at which the electron radiates energy as it accelerates towards the proton,

$$\left[\frac{e^2}{6\pi\varepsilon_0 c^3}a^2\right]_{t-\frac{r}{c}},$$

where the retarded time is

$$t - \frac{r}{c} \approx t - \frac{6\times 10^{-11}}{3\times 10^{8}} = t - \frac{2}{10^{19}} \approx t\,,$$

and the acceleration of the electron is

$$a = \frac{[v\gamma(v)]^2}{r} \approx \frac{v^2}{r}.$$

Therefore, the Electron's Radiation rate is

$$\frac{e^2}{6\pi\varepsilon_0 c^3}\left(\frac{v^2}{r}\right)^2 \approx \frac{e^2}{6\pi\varepsilon_0 c^3}\left(\frac{1}{m}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}\right)^2.$$

Since it equals the rate of change of the electron's energy,

$$\frac{\cancel{e}^2}{6\cancel{\pi}\cancel{\varepsilon}_0 c^3}\left(\frac{1}{m}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}\right)^2 \approx \frac{1}{2}\frac{1}{4\cancel{\pi}\cancel{\varepsilon}_0}\frac{\cancel{e}^2}{r^2}\frac{dr}{dt},$$

$$dt \approx \frac{3}{4}c^3\left(4\pi\varepsilon_0\frac{m}{e^2}\right)^2 r^2 dr$$

Therefore, the electron will spiral into the proton in

$$t\Big|_{r=0}^{r=6\times 10^{-11}} \approx \frac{1}{4}c^3\left(4\pi\varepsilon_0\frac{m}{e^2}\right)^2 r^3\Big|_{r=0}^{r=6\times 10^{-11}} \text{ sec}$$

$$\approx \underbrace{\frac{1}{4}c^3(\underbrace{\frac{4\pi}{\mu_0}}_{10^7}\underbrace{\mu_0\varepsilon_0}_{c^{-2}})^2}_{\frac{1}{4c}10^{14}}\left(\frac{m}{e^2}\right)^2 6^3 10^{-33}$$

$$\approx \frac{1}{12\cdot 10^8}10^{14}\frac{(9.11\cdot 10^{-31})^2}{(1.6\cdot 10^{-19})^4}6^3\cdot 10^{-33}$$

$$\approx 18 \cdot \frac{(9.11)^2}{(1.6)^4} 10^{-13}$$

$$\approx 2.23 \times 10^{-11} \text{sec.}$$

0.4 Bohr's Resolution avoiding Electrodynamics

Bohr claimed that since there is no Atomic collapse, there cannot be lost radiation. According to Bohr, the electron that accelerates towards the proton in its orbit, does not radiate electromagnetic energy, defying Electrodynamics.

Bohr proposed that the electron orbits have angular momentums that are discrete multiples of $\hbar$.

Consequently, according to Bohr, the orbits are occupied by standing waves, and no radiation takes place in them.

Clearly, the standing wave argument is pure hypothesis.

On the other hand, the radiation by an accelerating charge is an experimental fact of Electrodynamics.

Thus, Bohr's argument places the electron-proton Atom out of the laws of Physics, and Electrodynamics.

0.5 The Proton's Loss balances the Electron's loss

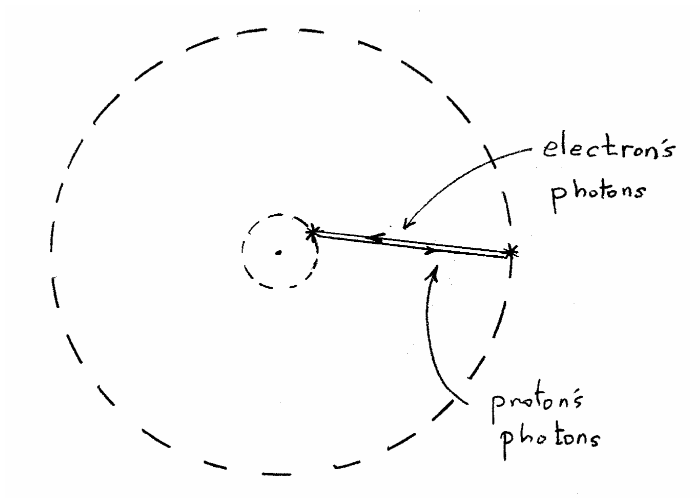
Clearly, the Proton has its own orbits, in which it accelerates towards the electron, and radiates it. Then, we'll assume that the electron and the proton exchange equal amounts of energy between them, and balance each others loss.

1. Photons Equilibrium and Radiation Equilibrium

1.1 Equilibrium of Exchanged Photons

The proton showers the electron with photons that compensate for the photons' energy lost by the electron, prevent the spiraling of the electron onto the proton, and keep the atom stable.

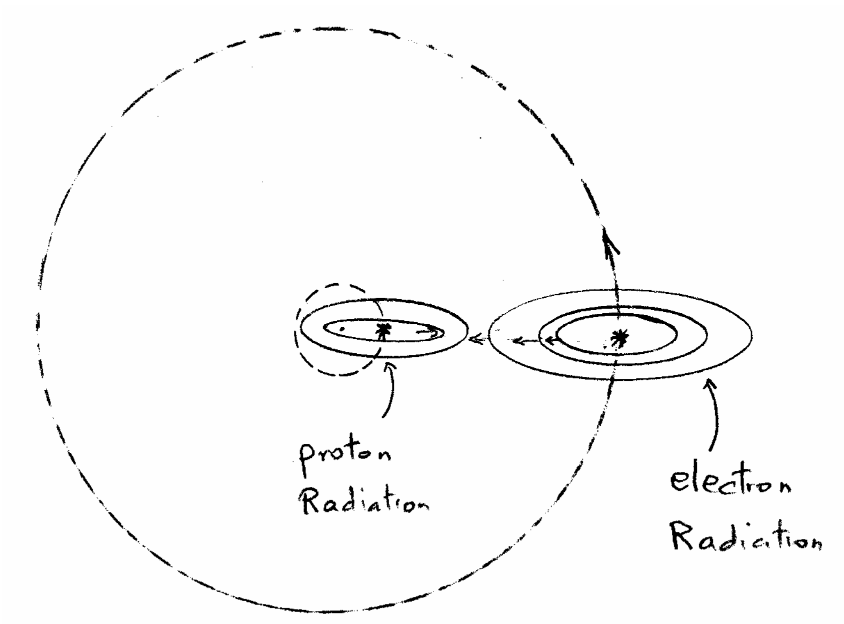
Clearly, that exchange of photons between the electron and the proton takes place along the line that separates them.



However, we do not have a formula that involves the number of photons exchanged, and the radii of the orbits. To derive such formula, we have to describe the exchanged photons as exchanged radiation.

1.2 Equilibrium of Exchanged Radiation

The electromagnetic radiation of an accelerating charge is in a plane perpendicular to its orbit. Thus, the radiation of the electron and the proton are along circles perpendicular to their orbits



We shall assume that the power of the energy radiated by the proton into the electron field equals the power of the energy radiated by the electron into the proton field.

Each compensates for the other's loss, to keep the orbits energies in a dynamic equilibrium.

Requiring the

$$\text{Power} = \frac{d}{dt}\left\{\text{radiation energy}\right\}$$

radiated by the accelerating proton into the electron field, to equal the Power lost by the accelerating electron, leads to an estimate for the radius of the proton's orbit.

The electron with acceleration $a_{electron}$ radiates the power

$$\frac{e^2}{6\pi\varepsilon_0 c^3} a_{electron}^2 .$$

The proton with acceleration A_{proton} radiates the power

$$\frac{e^2}{6\pi\varepsilon_0 c^3} A_{\text{proton}}^2 .$$

The equality of the two yields an estimate for the proton's orbit, which is the nucleus' radius.

2.

The accelerations of the Electron and the Proton

Denote

$$m = \text{ electron's mass,}$$

$$M = \text{proton's mass,}$$

$$r = \text{electron's orbit radius,}$$

$$\rho = \text{proton's orbit radius,}$$

$$\omega = \text{electron's angular velocity in its orbit}$$

$$\Omega = \text{proton's angular velocity in its orbit}$$

$$a = \text{electron acceleration in its orbit towards the proton}$$

$$A = \text{proton acceleration in its } n^{\text{th}} \text{ orbit towards the electron}$$

$$\gamma(v) = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \text{ Lorentz Factor for the Electron orbit}$$

$$\gamma(V) = \frac{1}{\sqrt{1 - \frac{V^2}{c^2}}} = \text{ Lorentz Factor for Proton orbit}$$

Due to the nonrelativistic speed of the electron,

$$\gamma(v) \approx 1\,,$$

$$m(v) \approx m\,.$$

We approximate the proton in its orbit by a charge $+e$ concentrated at the center of the electron's orbit.

Then, the electric attraction on the electron

$$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}\ ,$$

is balanced by the centripetal force

$$m\omega^2 r\,.$$

Therefore, the balance of forces on the Electron is

$$m\omega^2 r = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}\,,$$

Hence,

2.1 The Electron Acceleration in its Orbit is

$$a = \omega^2 r = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{mr^2}$$

Similarly, we approximate the electron in its orbit by a charge $-e$ concentrated at the center of the proton's orbit. Then, the electric attraction on the proton

$$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{\rho^2}\ ,$$

is balanced by the centripetal force

$$M\Omega^2\rho\,.$$

Therefore, the balance of forces on the proton is

$$M\Omega^2\rho = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{\rho^2}\,,$$

Hence,

2.2 The Proton acceleration in its Orbit is

$$A = \Omega^2\rho = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{M\rho^2}$$

3.

Kepler's 3rd Law for the Electron-Proton Atom

Newton's Gravitational force on a planet orbiting the sun at radius r, with period $T = \frac{2\pi}{\omega}$ is

$$m_{Planet}\omega_{Planet}^2 r = G\frac{m_{Planet}M_{Sun}}{r^2}.$$

Therefore,

$$\omega_{Planet}^2 r^3 = GM_{Sun} = \text{constant}.$$

Or

$$T^2 = \frac{4\pi^2}{GM}r^3 = kr^3,$$

which is Kepler's 3rd Law for Gravitation.

For two planets,

$$\omega_1^2 r_1^3 = GM_{Sun} = \omega_2^2 r_2^3.$$

Or

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3.$$

For the Electron-Proton Atom,

$$m_{electron}\omega^2_{electron}r = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}$$

$$M_{\text{Proton}}\Omega^2_{\text{Proton}}\rho = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{\rho^2}$$

yield

3.1 $$\boxed{m\omega^2 r^3 = \frac{e^2}{4\pi\varepsilon_0} = M\Omega^2\rho^3}$$

Hence,

3.2 Kepler's 3rd Law for the Electron-Proton Atom is

$$\boxed{\left(\frac{T_{electron}}{T_{proton}}\right)^2 = \frac{m}{M}\left(\frac{r}{\rho}\right)^3}$$

4.

Radiation Power Equilibrium, Inertia Moments, and the Nucleus Radius

We consider an electron orbiting the proton in a circle with radius r.

The electron is attracted to the proton with acceleration

$$a = \frac{1}{m_e}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}$$

and radiates photons into the proton field. Therefore,

4.1 The Electron's radiation Power is

$$\boxed{\frac{e^2}{6\pi\varepsilon_0 c^3}a^2 = \frac{e^2}{6\pi\varepsilon_0 c^3}\left(\frac{1}{m_e}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}\right)^2}$$

The Proton in the Electron-Proton Atom, has an orbit within the electron's orbit, with radius ρ, far smaller than the electron's orbit radius.

The proton is attracted to the electron, with acceleration

$$A = \frac{1}{M_p}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{\rho^2},$$

and radiates photons into the electron's field.

Therefore,

4.2 The Proton's Radiation Power is

$$\boxed{\frac{e^2}{6\pi\varepsilon_0 c^3}A^2 = \frac{e^2}{6\pi\varepsilon_0 c^3}\left(\frac{1}{M_p}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{\rho^2}\right)^2}$$

At equilibrium, the Radiation Power absorbed by the proton, equals the Radiation Power absorbed by the electron.

$$\frac{e^2}{6\pi\varepsilon_0 c^3}\left(\frac{1}{m_e}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}\right)^2 \approx \frac{e^2}{6\pi\varepsilon_0 c^3}\left(\frac{1}{M_p}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{\rho^2}\right)^2$$

Here, we use $\approx$ because of the slightly relativistic character of the speeds and the masses.

This electrodynamic equality leads to a surprisingly purely mechanical condition: the equality of the Inertia Moments of the electron and the proton.

That is,

4.3 The electron-proton Atom is Electrodynamically stable if and only if the inertia Moments of the electron and the proton are equal

$$\boxed{m_e r^2 \approx M_p \rho^2}$$

Consequently,

4.4 The proton orbit radius, which is the nucleus radius is

$$\boxed{\frac{r}{\rho} \approx \sqrt{\frac{M_p}{m_e}} \approx \sqrt{1836.152701} \approx 42.8503524}$$

We compute

$$\rho \approx r\sqrt{\frac{m_e}{M_p}}$$

$$= 5.29277249 \times 10^{-11} \;/\; 42.8503524$$

$$= 1.221173735 \times 10^{-12}.$$

Therefore,

4.4 The Nucleus Radius of the Electron-Proton Atom

$$\boxed{\rho \approx r\sqrt{\frac{m_e}{M_p}} \approx 1.221173735 \times 10^{-12}}$$

5.

Proton's Period, and Angular Velocity

By Kepler's 3rd Law for the Electron-Proton Atom,

$$\left(\frac{T_e}{T_p}\right)^2 = \frac{m}{M}\left(\frac{r}{\rho}\right)^3$$

$$= \frac{m}{M}\left(\sqrt{\frac{M}{m}}\right)^3$$

$$= \sqrt{\frac{M}{m}}$$

That is,

5.1 The Electron and the Proton Periods

$$\boxed{\frac{\Omega_p}{\omega_e} = \frac{T_e}{T_p} \approx \sqrt[4]{\frac{M_p}{m_e}} \approx (1836.152701)^{\frac{1}{4}} \approx 6.546018057}$$

5.2 The Proton Period is

$$\boxed{T_p \approx T_e \sqrt[4]{\frac{m_e}{M_p}}}$$

To approximate T_e, and T_p, note that a wavelength in the mid optical spectrum is

$$\lambda \sim 6 \times 10^{-7} \text{meter.}$$

This wavelength corresponds to the optical frequency

$$\nu = \frac{c}{\lambda} \sim \frac{3 \times 10^{8}}{6 \times 10^{-7}} = 5 \times 10^{14} \text{cycles/sec.}$$

This frequency corresponds to a period

$$T_{electron} = \frac{1}{\nu} \sim \frac{1}{5 \times 10^{14}} = 2 \times 10^{-15} \text{sec/cycle}$$

Therefore,

5.3 The Proton's Period is approximately

$$\boxed{T_p \approx T_e \sqrt[4]{\frac{m_e}{M_p}} \sim 2 \times 10^{-15} \ / \ 6.546 \sim 3 \times 10^{-16}}$$

The frequency ν corresponds to the angular velocity

$$\omega_{electron} = 2\pi\nu \sim 6 \times 5 \times 10^{14} = 3 \times 10^{15} \text{radians/sec.}$$

Therefore,

5.4 The Proton's Angular Velocity is approximately

$$\boxed{\Omega_{\mathrm{p}} \approx \omega_e \sqrt[4]{\frac{M_p}{m_e}} \sim 3 \times 10^{15} \times 6.546 \sim 2 \times 10^{16}}$$

6.

Proton's Orbits with Quantized Angular Momentums

The electron's orbits determine the proton's corresponding orbits, with Quantized Angular Momentums:

Since by 3.1, $m_e\omega^2 r^3 = M_p\Omega^2\rho^3$,

The Proton's Quantized Angular Momentums are

$$M\Omega_n\rho_n^2 = \underbrace{m\omega_n r_n^2}_{n\hbar}\underbrace{\frac{\omega_n}{\Omega_n}}_{\sim\sqrt[4]{\frac{m}{M}}}\underbrace{\frac{r_n}{\rho_n}}_{\sim\sqrt{\frac{M}{m}}}$$

$$\sim n\hbar\sqrt[4]{\frac{M_p}{m_e}}$$

6.1 The Proton's Quantized Angular Momentums are approximately

$$\boxed{M\Omega_n\rho_n^2 \sim n\hbar(6.546)}$$

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References

[Bohm] David Bohm, "*The Special Theory of Relativity*" Routledge, 1996.

[Feynman] Richard Feynman, "*The Feynman Lectures of Physics*", Edison Wesley, 1965.

[Kovetz] Attay Kovetz, "*Electromagnetic Theory*", Oxford, 2000.

[Marion] Jerry Marion; Mark Heald, "*Classical Electromagnetic Radiation*", Second Edition, Academic Press, 1980.

[Panofsky] Wolfgang Panofsky; Melba Phillips, "*Classical Electricity and Magnetism*", Second Edition, Addison Wesley, 1962.

[Parker] Sybil P. Parker, "*McGraw-Hill Encyclopedia of*

Physics", Second Edition, McGraw-Hill, 1993.

[Polyanin] Andrei Polyanin; Alexei Chernoutsan, "*A Concise Handbook of Mathematics, Physics, and Engineering Sciences*", CRC, 2011.

[Poole] Charles P. Poole, "*The Physics Handbook Fundamentals and Key Equations*", Wiley, 1998.

[Rindler], Wolfgang Rindler, "*Relativity Special, General, and Cosmological*", Oxford, 2001.

[Skinner], Ray Skinner, "*Relativity for Scientists and Engineers*", Dover, 1982.

[Smith] Glenn S. Smith, "*An Introduction to Classical Electromagnetic Radiation*" Cambridge, 1997.

[Stannard], Russell Stannard, "*Relativity*", Sterling, 2008.

[Woan] Graham Woan, "*The Cambridge Handbook of Physics Formulas*", Cambridge, 2000.

http://en.wikipedia.org/wiki/Lorentz_transformation

http://en.wikipedia.org/wiki/Relativity_theory

http://en.wikipedia.org/wiki/Relativity_theory