The Neutron as a Collapsed-Hydrogen Atom: Zero Point Energy & Nuclear Binding Energy, X Rays & Gamma Rays, Nuclear Forces & Bonding Neutronic Electrons Orbitals, and Neutron Stars

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Abstract: A neutron may disintegrate into a proton, an electron, and an antineutrino. In Gravitational Collapse, electrons and protons combine into neutrons. We establish that the Neutron is a Collapsed-Hydrogen Atom composed of an electron and a proton:
The Electron has

$$1^{st} \text{ Orbit Radius} \sim 9.398741807 \times 10^{-14}\text{m},$$

$$\text{Speed} \sim 51,558,134 \text{ m/sec},$$

$$\sim 23.5 \times (\text{Speed in Hydrogen}).$$

Frequency $\sim 8.73067052 \times 10^{19}$ cycles/second ,

$\sim 13{,}271 \times$ (Hydrogen Electron Frequency) ,

in the range of Hard X Rays:

Quantum of Angular Momentum $\sim (0.043132065)\hbar$,

Zero Point Energy ~ -7671 eV ,

$\sim$ ***564***$\times$***(Hydrogen's Zero Point Energy)***

with photon frequency $\sim 3.708097094 \times 10^{18}$ cycles/sec .

The Proton has

1st Orbit Radius $\sim 2.209505336 \times 10^{-15}$m ,

Speed $\sim 7{,}905{,}145$ m/sec ,

$\sim 24 \times$ (Hydrogen Proton Speed).

Frequency $\sim 5.69422884 \times 10^{20}$ cycles/sec ,

$\sim 13{,}223 \times$ (Hydrogen Proton Frequency) ,

in the range of Gamma Rays:

Quantum of Angular Momentum $\sim (0.277007069)\hbar$,

Nuclear Binding Energy $\sim -326{,}308$ eV ,

$\sim$ ***553***$\times$***(Hydrogen's Nuclear Energy Binding)***

1) $\dfrac{\text{Electron Orbit Radius}}{\text{Proton Orbit Radius}} \sim 42.5$,

2) **Hard X Rays, and Gamma rays:**

Hard X Rays, are due to a Neutron's excited electron returning from a higher energy Orbit to a lower energy Orbit

Gamma Rays are due to a Neutron's excited proton returning from a higher energy Orbit to a lower energy Orbit

3) **Kepler's 3rd Law for the Neutron:**

$$\frac{T_{\mathrm{e}}}{T_{\mathrm{p}}} \approx \sqrt[4]{\frac{M_{\mathrm{p}}}{m_{\mathrm{e}}}}\sqrt[8]{\frac{1-\beta_{\mathrm{e}}^2}{1-\beta_{\mathrm{p}}^2}}, \quad \beta_{\mathrm{e}} = \frac{v_{\mathrm{e}}}{c}, \quad \beta_{\mathrm{p}} = \frac{V_{\mathrm{p}}}{c}$$

is consistent with the model of the Neutron as a Collapsed Hydrogen

4) **Neutron Spin Angular Momentum:**

$$\sim (0.319479146)\hbar$$

The Assumption of $(0.5)\hbar$ *is off by 57%*

5) **Electric Collapse Versus Gravitational Collapse:**

The Electric Force between the electron and the proton is

$$\sim 3\times 10^{19}(\text{Gravitational Force}).$$

Thus, *a Neutron star is created by Electric Collapse.*

6) **Total Binding Energy:**

The Neutron's Total Binding Energy is

$\sim$*553*$\times$*(Hydrogen's Total Binding Energy).*

7) **Nuclear Force:**

The Neutron's Nuclear Force is

$\sim$*305,000*$\times$*(Hydrogen's Nuclear Force), and*

$\sim$*1836*$\times$*(Neutron's Zero Point Energy Force)*

8) ***Mini Molecular Bonding*:**

A Nucleus composed of a Proton and a Neutron is a Mini One-Electron Molecule H_2^+, *with an electron that orbits the two protons.*

9) **Nucleus Bonding:**

The Neutrons supply electrons which Orbitals about the Nucleus bond the protons, and ensure the Nucleus Stability.

10) **Neutron Stars Bonding:**

In Neutron Stars, the Gravitational Forces are negligible compared to the Nuclear Bonding, which keeps the star packed together.

11) **Neutronic Electrons Orbitals:**

Neutron's electrons orbit the nucleus, and keep the protons bonded

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Point Energy, Electromagnetic Spectrum, X Ray, Gamma Ray, Bohr Model, Hydrogen Atom, Radiation Power Equilibrium, Neutron Star,

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Contents

The Neutron in Radioactivity, in Astrophysics, and in the Nucleus

0.1 The Neutron in Radioactivity

It is well-established that in the weak interaction, a neutron disintegrates into a proton, an electron, and an antineutrino. This may suggest that a Neutron is composed of these three stable particles. But if such suggestion was ever made, it has no trace in Atomic Physics Textbooks.

0.2 The Neutron in Astrophysics

It is firmly believed that in the Gravitational Collapse of a star into a Neutron Star, electrons and protons combine into neutrons. Again, this may suggest that a Neutron is a Collapsed-Hydrogen Atom, composed of an electron, and a Proton. But such suggestion has no trace in Astrophysics Textbooks.

0.3 The Neutron as a Collapsed-Hydrogen Atom

Thus, the description of Neutron as a Collapsed-Hydrogen Atom dwells in the background of major Physics disciplines, and may not be considered a Hypothesis. It may be described better as a fact that has to be substantiated, and explored.

The fundamental theory of the Neutron is dismissed by assuming the Neutron to be a variation of the Proton.

However, X Rays Emission, and Computations confirm that the Neutron is a Collapsed-Hydrogen Atom.

This Model of the Neutron points to the origin of X Rays, reveals the source of the Nuclear Forces, and offers a believable explanation to the Stability of the Nucleus.

0.4 Nuclear Forces

Nuclear forces were suggested to account for the binding of the protons and the neutrons in the nucleus. But no progress was made in the understanding of these forces.

As Quarks were suggested as sub-nucleons, their binding was attributed to a sub-nuclear force, and the binding of nucleons was attributed to leftover sub-nuclear forces. This amounts to pure speculation about involved exchange of gluons, quarks, and a Pion for luck [Wikipedia, Nuclear Forces].

At the end, it remains unclear

1. What are the forces? Leftovers of the strong force do not tell us much.

2. How do they operate? Exchange of this, and that does not clarify how the forces operate.
3. How the electric repulsion between protons is resolved? At the distance between the protons in the Nucleus, the electric force is formidable too.
4. How do Neutrons contribute to the stability of the nucleus? We cannot tell the difference if in the Wikipedia animated fairytale about the exchange of Quarks, and Gluons, and Pion, the Neutron will be replaced with a Proton.

0.5 Nuclear Structure

It is believed that the more protons in the nucleus, the more neutrons are necessary to keep it stable. But there is no explanation to how that greater stability is achieved.

In particular, none of the models for the nucleus [Wikipedia, Nuclear Structure] accounts for the repulsion between the protons, and for the benign electric neutrality of the neutrons.

How does the neutral Neutron help to decrease the repulsion between the protons?

0.6 Nuclear Radiation

It is believed that X-Rays originate in the nucleus, but the process by which they are produced is not clear, [Wikipedia, X Ray].

In fact, X Rays Emission proves the existence of an orbiting electron in the Neutron, and Gamma Ray Emission proves the existence of an orbiting proton in the Neutron.

1.

X-Rays Line Spectrum Proves the Existence of an Orbiting Electron in the Neutron

It has been observed that X Rays are emitted when a material is bombarded with a beam of electrons or ions.

Continuous electromagnetic spectrum indicates that some electrons in the beam decelerate by electrons in the atomic shells, and some may accelerate towards the protons in the nucleus

But peaks of a line spectrum that show up in the X Ray Spectrum are produced just as the Optical spectrum is generated: Electrons in the beam push an electron to a higher orbit, and when that excited electron returns to a lower orbit, it emits a photon of X Ray Radiation.

It is clear that line spectrum of X Rays Emission mandates an orbiting electron.

What is disputed is that this electron may be in the nucleus itself.

Even the electron that comes out of a disintegrating neutron, is not enough evidence for the believers that there cannot be any electron in the neutron.

Thus, the electron that emits an X Rays photon has been speculated to belong to the inner shells of the heavy elements which bombardment generates X Rays, and Gamma Rays

However, those inner shells electrons don't move fast enough: The electron shells surrounding the nucleus are not energetic enough to yield more than optical frequencies, and at most, soft X Rays.

Our computations prove that Ground Orbit Electron Does not Radiate Gamma Rays. And even Hard X Rays must originate in the Nucleus.

Gamma Rays emission requires orbiting protons in the Nucleus.

2.

The Electron and Proton Orbits

The electron orbits the proton to generate the centripetal force that counters the electric attraction between the electron and the proton.

That attraction accelerates the electron towards the proton, and the electron radiates energy, that could cause it to spiral onto the proton, and have the atom collapse

In [Dan4] we explained that to compensate for the radiation energy lost by the electron, and prevent its spiraling onto the proton, the proton must be orbiting the electron, accelerating towards it, and radiating it with energy that keeps the Atom in equilibrium, and in existence.

The electron's orbit is larger than the proton's orbit, and to find the electron's orbit, and speed, we assume that the proton is at rest with respect to the electron. Thus, the equations for the electron's orbit and speed, in which the proton's motion will be neglected, are only approximate

2.1 Closed Orbits

To stay within the neutron's boundaries, *the electron, and the proton should have closed orbits*.

2.2 Central Forces

By [Routh, p. 274], a closed orbit results from a central force that is proportional to the inverse square of the distance,(such as the Coulomb electric force) or directly to the distance(such as the centripetal force).

The electron's charge, and the proton's charge supply
the electromagnetic forces to close their orbits.

2.3 Orbits' Stability

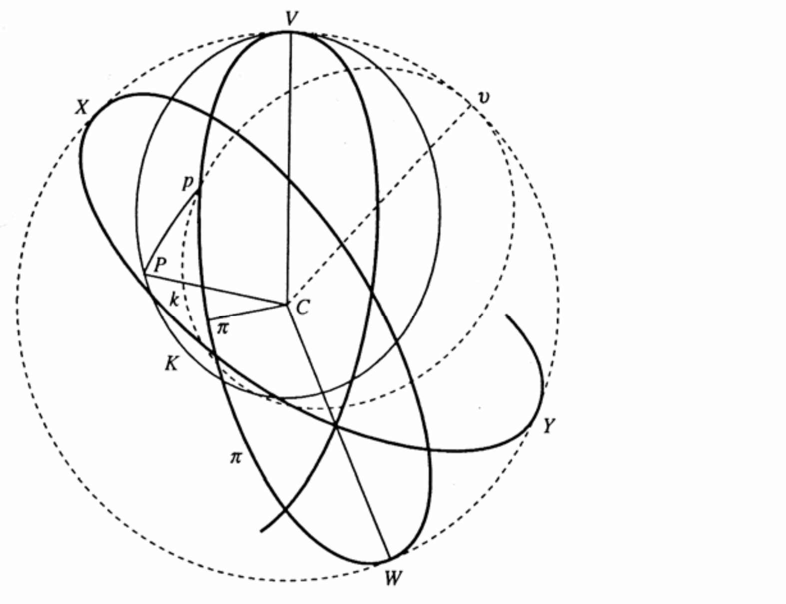
By [Routh, p.280] Central Force orbits are stable. That is, they are bounded in a ring between two circles. The stability of the neutron indicates a stable electron orbit, and a stable proton orbit.

2.4 Planar Motion

Since the electric force is inverse squared law force,

The electron's orbit, and the proton's orbit will stay in the same
plane, and will not generate a sphere

The plane of motion of a particle turns around to generate a sphere <u>only under a non-inverse squared law force.</u>



[Chandrasekhar, p. 195].

2.5 Speeds, and Orbits' Radii

Denote the proton speed in its orbit by

$$V_{\mathrm{p}}$$

$$\beta_{\mathrm{p}} = V_{\mathrm{p}} / c.$$

The radius of the proton's orbit is

$$\rho_{\mathrm{p}}.$$

The electron's speed in its orbit

$$v_{\mathrm{e}}$$

$$\beta_{\mathrm{e}} = v_{\mathrm{e}} / c.$$

The radius of the electron's orbit, which is the neutron radius,

$$r_{\mathrm{N}}.$$

3.

Radiation Power Equilibrium, and the Neutron's Inertia Moments

The electron is attracted to the proton by the force

$$\frac{m_e}{\sqrt{1-\beta_e^2}} a_e = \frac{1}{4\pi\varepsilon_0} \frac{e^2}{r_N{}^2}.$$

It is accelerated towards the proton by

$$a_e = \frac{\sqrt{1-\beta_e^2}}{m_e} \frac{1}{4\pi\varepsilon_0} \frac{e^2}{r_N{}^2},$$

and radiates photons into the proton field.

3.1 The Electron's Radiation Power is

$$\boxed{\frac{e^2}{6\pi\varepsilon_0 c^3} a_e{}^2 = \frac{e^2}{6\pi\varepsilon_0 c^3}\left(\frac{\sqrt{1-\beta_e^2}}{m_e} \frac{1}{4\pi\varepsilon_0} \frac{e^2}{r_N{}^2}\right)^2}$$

The Proton has an orbit within the electron's orbit, with radius ρ_p, far smaller than the electron's orbit radius.

The proton is attracted to the electron, with acceleration

$$A_{\mathrm{p}} = \frac{\sqrt{1-\beta_{\mathrm{p}}^2}}{M_p}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{{\rho_{\mathrm{p}}}^2},$$

and radiates photons into the electron's field.

3.2 The Proton's Radiation Power is

$$\boxed{\frac{e^2}{6\pi\varepsilon_0 c^3}{A_{\mathrm{p}}}^2 = \frac{e^2}{6\pi\varepsilon_0 c^3}\left(\frac{\sqrt{1-\beta_{\mathrm{p}}^2}}{M_p}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{{\rho_{\mathrm{p}}}^2}\right)^2}$$

At equilibrium, the Radiation Power absorbed by the proton, equals the Radiation Power absorbed by the electron.

$$\frac{e^2}{6\pi\varepsilon_0 c^3}\left(\frac{\sqrt{1-\beta_{\mathrm{e}}^2}}{m_{\mathrm{e}}}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{{r_{\mathrm{N}}}^2}\right)^2 \approx \frac{e^2}{6\pi\varepsilon_0 c^3}\left(\frac{\sqrt{1-\beta_{\mathrm{p}}^2}}{M_p}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{{\rho_{\mathrm{p}}}^2}\right)^2$$

Here, we use $\approx$ because each formula assumes that one particle is moving while the other particle is stationary.

This Electrodynamics equality leads to a surprisingly purely mechanical condition: the equality of the Relativistic Inertia Moments of the electron and the proton.

That is,

3.3 The Neutron's Inertia Moments Balance

The Neutron is Electrodynamically stable if and only the Inertia Moments of its electron and proton are equal

$$\boxed{\frac{m_{\mathrm{e}}}{\sqrt{1-\beta_{\mathrm{e}}^2}}\, {r_{\mathrm{N}}}^2 \approx \frac{M_{\mathrm{p}}}{\sqrt{1-\beta_{\mathrm{p}}^2}}\, {\rho_{\mathrm{p}}}^2}$$

4.

$$\frac{r_{\mathrm{N}}}{\rho_{\mathrm{p}}}$$

4.1 The Neutron's Force Balance

$$\boxed{\frac{m_{\mathrm{e}}}{\sqrt{1-\beta_{\mathrm{e}}{}^2}}\beta_{\mathrm{e}}{}^2 r_{\mathrm{N}} = \frac{e^2}{10^7} = \frac{M_{\mathrm{p}}}{\sqrt{1-\beta_{\mathrm{p}}{}^2}}\beta_{\mathrm{p}}{}^2\rho_{\mathrm{p}}}$$

Proof:

The Force balance equation for the electron is

$$\frac{m_{\mathrm{e}}}{\sqrt{1-\beta_{\mathrm{e}}{}^2}}\frac{v_{\mathrm{e}}{}^2}{r_{\mathrm{N}}} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_{\mathrm{N}}{}^2}.$$

Dividing both sides by c^2,

$$\frac{m_{\mathrm{e}}}{\sqrt{1-\beta_{\mathrm{e}}{}^2}}\frac{\beta_{\mathrm{e}}{}^2}{r_{\mathrm{N}}} = \frac{1}{4\pi\varepsilon_0 c^2}\frac{e^2}{r_{\mathrm{N}}{}^2},$$

$$= \frac{\mu_0}{4\pi}\frac{e^2}{r_{\mathrm{N}}{}^2},$$

$$= \frac{1}{10^7}\frac{e^2}{r_{\mathrm{N}}{}^2},$$

$$\frac{m_{\mathrm{e}}}{\sqrt{1-\beta_{\mathrm{e}}{}^2}}\beta_{\mathrm{e}}{}^2 r_{\mathrm{N}} = \frac{e^2}{10^7}.$$

The Force balance equation for the proton is

$$\frac{M_{\mathrm{p}}}{\sqrt{1-\beta_{\mathrm{p}}{}^2}}\frac{V_{\mathrm{p}}{}^2}{\rho_{\mathrm{p}}} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{\rho_{\mathrm{p}}^2}$$

Dividing both sides by c^2,

$$\frac{M_{\mathrm{p}}}{\sqrt{1-\beta_{\mathrm{p}}{}^2}}\frac{\beta_{\mathrm{p}}{}^2}{\rho_{\mathrm{p}}} = \frac{1}{4\pi\varepsilon_0 c^2}\frac{e^2}{\rho_{\mathrm{p}}^2},$$

$$= \frac{1}{10^7}\frac{e^2}{\rho_{\mathrm{p}}^2},$$

$$\frac{M_{\mathrm{p}}}{\sqrt{1-\beta_{\mathrm{p}}{}^2}}\beta_{\mathrm{p}}{}^2\rho_{\mathrm{p}} = \frac{e^2}{10^7}.$$

Therefore,

$$\frac{m_{\mathrm{e}}}{\sqrt{1-\beta_{\mathrm{e}}{}^2}}\beta_{\mathrm{e}}{}^2 r_{\mathrm{N}} = \frac{e^2}{10^7} = \frac{M_{\mathrm{p}}}{\sqrt{1-\beta_{\mathrm{p}}{}^2}}\beta_{\mathrm{p}}{}^2\rho_{\mathrm{p}}.\square$$

4.2 $$\boxed{\frac{r_{\mathrm{N}}}{\rho_{\mathrm{p}}} \approx \frac{\beta_{\mathrm{e}}{}^2}{\beta_{\mathrm{p}}{}^2}}$$

Proof: Dividing the Neutron's Inertia Moments Balance 3.3,

$$\frac{m_{\mathrm{e}}}{\sqrt{1-\beta_{\mathrm{e}}^2}} r_{\mathrm{N}}{}^2 \approx \frac{M_{\mathrm{p}}}{\sqrt{1-\beta_{\mathrm{p}}^2}}\rho_{\mathrm{p}}{}^2,$$

by the Neutron's Force balance 4.1,

$$\frac{m_e}{\sqrt{1-\beta_e{}^2}}\beta_e{}^2 r_N = \frac{M_p}{\sqrt{1-\beta_p{}^2}}\beta_p{}^2 \rho_p,$$

we obtain

$$\frac{r_N}{\beta_e{}^2} \approx \frac{\rho_p}{\beta_p{}^2},$$

$$\frac{r_N}{\rho_p} \approx \frac{\beta_e{}^2}{\beta_p{}^2}.\square$$

$\frac{r_N}{\rho_p}$ may be estimated roughly by $\sqrt{\frac{M_p}{m_e}}$, as in [Dan4].

4.3 $$\boxed{\frac{r_N}{\rho_p} \sim \sqrt{\frac{M_p}{m_e}} \approx 42.5}$$

Proof: From the Inertia Moments Balance, 3.3,

$$\frac{m_e}{\sqrt{1-\beta_e^2}} r_N{}^2 \approx \frac{M_p}{\sqrt{1-\beta_p^2}} \rho_p{}^2,$$

$$\frac{r_N}{\rho_p} \approx \sqrt{\frac{M_p}{m_e}} \sqrt[4]{\frac{1-\beta_e^2}{1-\beta_p^2}}.$$

Even if $\beta_e = 0.5$, and $\beta_p = 0.1$, then,

$$\sqrt[4]{\frac{1-\beta_e^2}{1-\beta_p^2}} \approx 0.933 \sim 1.$$

In fact, we'll have $\beta_{\mathrm{e}} \approx 0.173$, $\beta_{\mathrm{p}} \approx 0.02635$, and

$$\sqrt[4]{\frac{1-\beta_{\mathrm{e}}^2}{1-\beta_{\mathrm{p}}^2}} \approx 0.99 \approx 1\,.$$

Therefore,

$$\begin{aligned}\frac{r_{\mathrm{N}}}{\rho_{\mathrm{p}}} &\approx \sqrt{\frac{M_{\mathrm{p}}}{m_{\mathrm{e}}}} \times 0.99 \\ &= \sqrt{1836.1527} \times 0.99 \\ &\approx 42.5\,.\square\end{aligned}$$

5.

The Electron's Zero Point Energy

5.1 The Electron's Electric Binding Energy

$$U_{\text{electric}} = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_{\text{N}}}$$

$$= -\frac{1}{10^7}c^2\frac{e^2}{r_{\text{N}}}$$

5.2 The Electron's Magnetic Energy

$$U_{\text{magnetic}} = \tfrac{1}{4}\mu_0 r_{\text{N}} e^2 \nu_e^2$$

$$= \frac{1}{(4\pi)^2}\mu_0 v_e^2 e^2 \frac{1}{r_{\text{N}}}$$

Proof: The current due to the electron's charge e that turns ν_{e} cycles/second is

$$I = e\nu_{\text{e}}$$

The Magnetic Energy of this current is [Benson, p.486]

$$\tfrac{1}{2}LI^2.$$

By [Fischer, p.97]

$$L = \mu_0\frac{\pi r_{\text{N}}{}^2}{2\pi r_{\text{N}}} = \tfrac{1}{2}\mu_0 r_{\text{N}}.$$

Thus, the magnetic energy due to the electron charge is

$$\tfrac{1}{2}\tfrac{1}{2}\mu_0 r_{\mathrm{N}}(e\nu_e)^2 = \tfrac{1}{4}\mu_0 r_{\mathrm{N}} e^2 \underbrace{\nu_{\mathrm{e}}^2}_{\frac{1}{4\pi^2}\omega_{\mathrm{e}}^2}$$

$$= \frac{1}{(4\pi)^2}\mu_0 v_{\mathrm{e}}^2 e^2 \frac{1}{r_{\mathrm{N}}}.$$

5.3 The Electron's Magnetic Energy in its Neutron's Orbit is negligible compared to its Electric Energy

Proof:

$$\frac{U_{\text{magnetic}}}{U_{\text{electric}}} = \frac{\frac{1}{(4\pi)^2}\mu_0 v_{\mathrm{e}}^2 e^2 \frac{1}{r_{\mathrm{N}}}}{\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_{\mathrm{N}}}}$$

$$= \frac{1}{4\pi}\frac{v_{\mathrm{e}}^2}{c^2}$$

$$= \frac{1}{4\pi}\beta_{\mathrm{e}}{}^2$$

In the following we'll approximate $\beta_{\mathrm{e}}{}^2 \approx 0.03$. Therefore,

$$\approx \frac{0.03}{4\pi} \approx 2.4\times 10^{-3}.\square$$

5.4 The Electron's Rotation Energy

$$\frac{1}{2}\frac{m_{\mathrm{e}}}{\sqrt{1-\beta_{\mathrm{e}}{}^2}}v_{\mathrm{e}}{}^2 = \frac{1}{2}\frac{c^2}{10^7}\frac{e^2}{r_{\mathrm{N}}}$$

Proof: From the balance between the Centripetal and Electric forces on the electron in its Neutron orbit,

$$\frac{m_{\mathrm{e}}}{\sqrt{1-{\beta_{\mathrm{e}}}^2}}\frac{{v_{\mathrm{e}}}^2}{r_{\mathrm{N}}} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{{r_{\mathrm{N}}}^2},$$

$$\frac{1}{2}\frac{m_{\mathrm{e}}}{\sqrt{1-\beta^2}}{v_{\mathrm{e}}}^2 = \frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_{\mathrm{N}}}.$$

Substituting $\frac{1}{\varepsilon_0} = c^2\mu_0$, and $\mu_0 = \frac{4\pi}{10^7}$,

$$= \frac{1}{2}\frac{c^2}{10^7}\frac{e^2}{r_{\mathrm{N}}}.\square$$

5.5 The Electron's Total Binding Energy

(= Zero Point Energy)

$$U_{\text{electron binding}} \approx \frac{1}{2}\frac{1}{10^7}c^2\frac{e^2}{r_{\mathrm{N}}}$$

$$= \frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_{\mathrm{N}}}$$

Proof: $U_{\text{electron binding}} \approx \frac{1}{10^7}c^2\frac{e^2}{r_{\mathrm{N}}} - \frac{1}{2}\frac{c^2}{10^7}\frac{e^2}{r_{\mathrm{N}}}$

$$= \frac{1}{2}\frac{1}{10^7}c^2\frac{e^2}{r_{\mathrm{N}}}.\square$$

5.6 The Electron's Total Energy

$$U_{\text{electron}} \approx m_{\text{e}}c^2 + \frac{1}{2}\frac{1}{10^7}c^2\frac{e^2}{r_{\text{N}}}$$

$$= m_{\text{e}}c^2 + \frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_{\text{N}}}$$

6.

The Proton's Nuclear Energy

6.1 The Proton's Electric Binding Energy

$$U_{\text{electric}} = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{\rho_{\text{p}}}$$

$$= -\frac{1}{10^7}c^2\frac{e^2}{\rho_{\text{p}}}$$

6.2 The Proton's Magnetic Energy

$$U_{\text{magnetic}} = \tfrac{1}{4}\mu_0\rho_{\text{p}}e^2\nu_p^2$$

$$= \frac{1}{(4\pi)^2}\mu_0 {V_{\text{p}}}^2 e^2 \frac{1}{\rho_{\text{p}}}$$

__Proof__: The current due to the Proton's charge $-e$ that turns ν_{p} cycles/second is

$$I = -e\nu_{\text{p}}$$

The Magnetic Energy of this current is [Benson, p.486]

$$\tfrac{1}{2}LI^2.$$

By [Fischer, p.97]

$$L = \mu_0\frac{\pi{\rho_{\text{p}}}^2}{2\pi\rho_{\text{p}}} = \tfrac{1}{2}\mu_0\rho_{\text{p}}.$$

Thus, the magnetic energy due to the Proton's charge is

$$\tfrac{1}{2}\tfrac{1}{2}\mu_0\rho_{\mathrm{p}}(e\nu_{\mathrm{p}})^2 = \tfrac{1}{4}\mu_0\rho_{\mathrm{p}}e^2 \underbrace{\nu_{\mathrm{p}}^2}_{\frac{1}{4\pi^2}\Omega_{\mathrm{p}}^2}$$

$$= \frac{1}{(4\pi)^2}\mu_0 V_{\mathrm{p}}^2 e^2 \frac{1}{\rho_{\mathrm{p}}}.$$

6.3 The Proton's Magnetic Energy in its Neutron Orbit is negligible compared to its Electric Energy

Proof:

$$\frac{U_{\text{magnetic}}}{U_{\text{electric}}} = \frac{\dfrac{1}{(4\pi)^2}\mu_0 V_{\mathrm{p}}^2 e^2 \dfrac{1}{\rho_{\mathrm{p}}}}{\dfrac{1}{4\pi\varepsilon_0}\dfrac{e^2}{\rho_{\mathrm{p}}}}$$

$$= \frac{1}{4\pi}\frac{V_p^2}{c^2}$$

$$= \frac{1}{4\pi}\beta_{\mathrm{p}}{}^2$$

Approximating $\beta_{\mathrm{p}}{}^2 \approx 0.0007$,

$$\frac{U_{\text{magnetic}}}{U_{\text{electric}}} \approx \frac{0.0007}{4\pi} \approx 5.6\times 10^{-5}.\square$$

6.4 The Proton's Rotation Energy

$$\frac{1}{2}\frac{M_{\mathrm{p}}}{\sqrt{1-\beta_{\mathrm{p}}{}^2}}V_{\mathrm{p}}{}^2 = \frac{1}{2}\frac{c^2}{10^7}\frac{e^2}{\rho_{\mathrm{p}}}$$

Proof: From the balance between the Centripetal and Electric forces on the Proton in its Neutron orbit,

$$\frac{M_{\mathrm{p}}}{\sqrt{1-\beta_{\mathrm{p}}{}^2}}\frac{V_{\mathrm{p}}{}^2}{\rho_{\mathrm{p}}} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{\rho_{\mathrm{p}}{}^2},$$

$$\frac{1}{2}\frac{M_{\mathrm{p}}}{\sqrt{1-\beta_{\mathrm{p}}{}^2}}V_{\mathrm{p}}{}^2 = \frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{\rho_{\mathrm{p}}}.$$

Substituting $\frac{1}{\varepsilon_0} = c^2\mu_0$, and $\mu_0 = \frac{4\pi}{10^7}$,

$$= \frac{1}{2}\frac{c^2}{10^7}\frac{e^2}{\rho_{\mathrm{p}}}.\square$$

6.5 The Proton's Total Binding Energy is the Nuclear Energy

$$U_{\text{proton binding}} \approx \frac{1}{2}\frac{1}{10^7}c^2\frac{e^2}{\rho_{\mathrm{p}}}$$

$$\approx \frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{\rho_{\mathrm{p}}}$$

Proof: $U_{\text{proton}} \approx \frac{1}{10^7}c^2\frac{e^2}{\rho_{\mathrm{p}}} - \frac{1}{2}\frac{c^2}{10^7}\frac{e^2}{\rho_{\mathrm{p}}}$

$$= \frac{1}{2}\frac{1}{10^7}c^2\frac{e^2}{\rho_{\mathrm{p}}}.\square$$

6.6 The Proton's Total Energy

$$U_{\text{proton}} \approx M_{\mathrm{p}}c^2 + \frac{1}{2}\frac{1}{10^7}c^2\frac{e^2}{\rho_{\mathrm{p}}}$$

$$\approx M_{\mathrm{p}}c^2 + \frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{\rho_{\mathrm{p}}}$$

6.7 $$\boxed{\frac{U_{\text{proton binding}}}{U_{\text{electron binding}}} = \frac{r_{\mathrm{N}}}{\rho_{\mathrm{p}}} \approx 42.5}$$

Proof: $$\frac{U_{\text{proton binding}}}{U_{\text{electron binding}}} = \frac{\frac{1}{2}\frac{1}{10^7}c^2\frac{e^2}{\rho_{\mathrm{p}}}}{\frac{1}{2}\frac{1}{10^7}c^2\frac{e^2}{r_{\mathrm{N}}}}$$

$$= \frac{r_{\mathrm{N}}}{\rho_{\mathrm{p}}} \approx 42.5\,.\square$$

7.

The Neutron's Mass-Energy

7.1 The Neutron's Mass-Energy Equation

$$\boxed{\underbrace{M_{\mathrm{N}} - M_{\mathrm{p}} - m_{\mathrm{e}}}_{\Delta m} \approx \frac{1}{2}\frac{1}{10^7}\frac{e^2}{\rho_{\mathrm{p}}}\left(1 + \frac{\rho_{\mathrm{p}}}{r_{\mathrm{N}}}\right)}$$

Proof:

$$\begin{aligned} M_{\mathrm{N}}c^2 &= U_{\mathrm{neutron}}, \\ &= U_{\mathrm{proton}} + U_{\mathrm{electron}}, \\ &\approx M_{\mathrm{p}}c^2 + \frac{1}{2}\frac{1}{10^7}c^2\frac{e^2}{\rho_{\mathrm{p}}} + m_{\mathrm{e}}c^2 + \frac{1}{2}\frac{1}{10^7}c^2\frac{e^2}{r_{\mathrm{N}}}. \end{aligned}$$

Dividing by c^2,

$$M_{\mathrm{N}} - M_{\mathrm{p}} - m_{\mathrm{e}} \approx \frac{1}{2}\frac{1}{10^7}\frac{e^2}{\rho_{\mathrm{p}}}\left(1 + \frac{\rho_{\mathrm{p}}}{r_{\mathrm{N}}}\right).\square$$

8.

The Proton's Orbit Radius

8.1
$$\boxed{\rho_{\mathrm{p}} \approx \frac{1}{2}\frac{1}{10^7}\frac{e^2}{\Delta m}\left(1+\frac{\rho_{\mathrm{p}}}{r_{\mathrm{N}}}\right)}$$

Substituting

$$e = -1.60217733\times 10^{-19}\mathrm{C},$$

$$M_{\mathrm{N}} \approx 1.674\ 128\ 6\times 10^{-27}\mathrm{Kg},$$

$$M_{\mathrm{p}} \approx 1.672\ 623\ 1\times 10^{-27}\mathrm{Kg},$$

$$m_{\mathrm{e}} \approx 9.109\ 389\ 7\times 10^{-31}\mathrm{Kg},$$

$$\frac{1}{2}\frac{1}{10^7}\frac{e^2}{\Delta m} \approx \frac{1}{2}\frac{1}{10^7}\underbrace{\frac{(1.60217733)^2\times 10^{-38}}{5.9456103\times 10^{-31}}}_{0.431742422\ \times\ 10^{-7}}$$

$$= 2.15871211\times 10^{-15}.$$

By 4.3, $\dfrac{\rho_{\mathrm{p}}}{r_{\mathrm{N}}} \approx \dfrac{1}{42.5} = 0.023529411$

$$\rho_{\mathrm{p}} \approx 2.15871211\times 10^{-15}(1+0.023529411)$$

$$= 2.209505336\times 10^{-15}$$

8.2
$$\boxed{\rho_{\mathrm{p}} \approx 2.209505336\times 10^{-15}}$$

8.3 The Hydrogen Proton Orbit Radius is $\sim$553 $\times$ (Neutron's Proton Orbit Radius)

Proof: By [Dan4, p.21], the Hydrogen Proton Orbit Radius is

$$\rho_{\text{p in H}} = r_{\text{e}}\sqrt{\frac{m_e}{M_p}}$$

$$= 5.29277249\times 10^{-11}\sqrt{5.44617013\times 10^{-4}}$$

$$= 1.221173735\times 10^{-12}.$$

$$\frac{\rho_{\text{p in H}}}{\rho_{\text{p in N}}} = \frac{1.221173735\times 10^{-12}}{2.209505336\times 10^{-15}}$$

$$\approx 553.\square$$

9.

The Proton's Speed

9.1 $$\beta_{\mathrm{p}}{}^{4} + \left(\frac{1}{10^7}\frac{1}{M_{\mathrm{p}}}\frac{e^2}{\rho_{\mathrm{p}}}\right)^2 \beta_{\mathrm{p}}{}^{2} - \left(\frac{1}{10^7}\frac{1}{M_{\mathrm{p}}}\frac{e^2}{\rho_{\mathrm{p}}}\right)^2 \approx 0$$

Proof: From the Force Balance, 4.1,

$$\frac{M_{\mathrm{p}}}{\sqrt{1-\beta_{\mathrm{p}}{}^{2}}}\beta_{\mathrm{p}}{}^{2}\rho_{\mathrm{p}} = \frac{e^2}{10^7}$$

$$\beta_{\mathrm{p}}{}^{2} \approx \frac{1}{10^7}\frac{1}{M_{\mathrm{p}}}\frac{e^2}{\rho_{\mathrm{p}}}\sqrt{1-\beta_{\mathrm{p}}{}^{2}}$$

$$\beta_{\mathrm{p}}{}^{4} + \left(\frac{1}{10^7}\frac{1}{M_{\mathrm{p}}}\frac{e^2}{\rho_{\mathrm{p}}}\right)^2 \beta_{\mathrm{p}}{}^{2} - \left(\frac{1}{10^7}\frac{1}{M_{\mathrm{p}}}\frac{e^2}{\rho_{\mathrm{p}}}\right)^2 \approx 0 . \square$$

Substituting

$$e = -1.60217733 \times 10^{-19}\mathrm{C},$$

$$M_{\mathrm{p}} \approx 1.672\ 623\ 1 \times 10^{-27}\mathrm{Kg},$$

$$\rho_{\mathrm{p}} \approx 2.209505336 \times 10^{-15}\mathrm{m}$$

$$\frac{1}{10^7}\frac{1}{M_{\mathrm{p}}}\frac{e^2}{\rho_{\mathrm{p}}} \approx \frac{1}{10^7}\frac{(1.60217733)^2 \times 10^{-38}}{1.672\ 623\ 1 \times 10^{-27}}\frac{1}{2.209505336 \times 10^{-15}}$$

$$= 6.94589189 \times 10^{-4}.$$

$$\left(\frac{1}{10^7}\frac{1}{M_{\text{p}}}\frac{e^2}{\rho_{\text{p}}}\right)^2 \approx 4.824541415 \times 10^{-7}$$

9.2 $$\boxed{\beta_{\text{p}}{}^4 + 4.824541415 \times 10^{-7}\beta_{\text{p}}{}^2 - 4.824541415 \times 10^{-7} \approx 0}$$

Using MAPLE

```
> with(RootFinding):

> r0 := NextZero(x → x^2 + 4.824541415·10^-7 x
      − 4.824541415·10^-7, 0.0001);

                          r0 := 0.0006943480038
> sqrt(r0)
                              0.02635048394
```

9.3 $$\boxed{\beta_{\text{p}}{}^2 = 6.943480038 \times 10^{-4}}$$

9.4 $$\boxed{\beta_{\text{p}} \approx 0.02635048394}$$

9.5 The Neutron's Proton's Speed

$$\boxed{V_{\text{p}} \sim 7,905,145 \text{ m/sec}}.$$

Proof: $V_{\text{p}} = \beta_{\text{p}}c$

$$\approx (0.02635048394)c$$

$$\approx 7,905,145 \text{ m/sec}\,.\square$$

9.6 The Hydrogen Proton's Speed

$$\boxed{V_{\text{p in H}} \sim 330,420 \text{ m/sec}}\,.$$

Proof: In Hydrogen Atom,

$$m_{\text{e}} \frac{v_{\text{H}}^2}{r_{\text{H}}} = \frac{1}{4\pi\varepsilon_0} \frac{e^2}{r_{\text{H}}^2}\,.$$

$$\underbrace{m_{\text{e}} v_{\text{H}} r_{\text{H}}}_{\hbar} \underbrace{v_{\text{H}}}_{\omega_{\text{H}} r_{\text{H}}} = \frac{1}{4\pi\varepsilon_0} e^2\,,$$

$$\omega_{\text{e in H}} = \omega_{\text{H}} = \frac{1}{r_{\text{H}}} \underbrace{\frac{1}{4\pi\varepsilon_0} \frac{e^2}{\hbar c}}_{\alpha \approx \frac{1}{137}} c\,,$$

$$= \frac{1}{5.2977249 \times 10^{-11}} \frac{1}{137} c$$

$$= 4.1334366 \times 10^{16} \text{ radians/sec}$$

By [Dan4, p.22], the Hydrogen Proton Angular Velocity is

$$\Omega_{\text{p-in-H}} \approx \omega_{\text{e-in-H}} \sqrt[4]{\frac{M_{\text{p}}}{m_{\text{e}}}}\,.$$

$$\sim 4.1334366 \times 10^{16} \times \sqrt[4]{1836.152701}$$

$$= 4.1334366 \times 10^{16} \times 6.546018057$$

$$= 2.705755062 \times 10^{17}$$

$$
\begin{aligned}
V_{\text{p-in-H}} &= \Omega_{\text{p-in-H}} \rho_{\text{p-in-H}} \\
&\approx 2.705755062 \times 10^{17} \times 1.221173735 \times 10^{-12} \\
&= 330,420 \text{ m/sec}
\end{aligned}
$$

9.7 The Neutron's Proton Speed is $\sim$24$\times$(Hydrogen Proton Speed)

Proof:

$$
\begin{aligned}
\frac{V_{\text{p-in-N}}}{V_{\text{p-in-H}}} &\sim \frac{7,905,145}{330,420} \\
&\approx 24 . \square
\end{aligned}
$$

10.

The Proton Frequency

10.1 The Neutron's Proton Angular Velocity

$$\boxed{\Omega_{\mathrm{p}} = \frac{V_{\mathrm{p}}}{\rho_{\mathrm{p}}} = 3.577789504 \times 10^{21} \text{ radians/sec}}$$

Proof: $\Omega_{\mathrm{p}} = \dfrac{V_{\mathrm{p}}}{\rho_{\mathrm{p}}}$

$$\approx \frac{7,905,145 \text{ m/sec}}{2.209505336 \times 10^{-15}\,\mathrm{m}}$$

$$= 3.577789504 \times 10^{21} \text{ radians/sec}.\Box$$

10.2 The Hydrogen Electron Angular Velocity

$$\boxed{\omega_{\mathrm{H}} = 4.1334366 \times 10^{16} \text{ radians/sec}}$$

Proof: In the Hydrogen Atom,

$$m_{\mathrm{e}} \frac{v_{\mathrm{H}}^2}{r_{\mathrm{H}}} = \frac{1}{4\pi\varepsilon_0} \frac{e^2}{r_{\mathrm{H}}^2}.$$

$$\underbrace{m_{\mathrm{e}} v_{\mathrm{H}} r_{\mathrm{H}}}_{\hbar} \underbrace{v_{\mathrm{H}}}_{\omega_{\mathrm{H}} r_{\mathrm{H}}} = \frac{1}{4\pi\varepsilon_0} e^2,$$

$$\omega_{\mathrm{H}} = \frac{1}{r_{\mathrm{H}}} \underbrace{\frac{1}{4\pi\varepsilon_0}\frac{e^2}{\hbar c}}_{\alpha\approx\frac{1}{137}} c\,,$$

$$= \frac{1}{5.2977249\times 10^{-11}}\frac{1}{137}c$$

$$= 4.1334366\times 10^{16} \text{ radians/sec}$$

10.3 The Hydrogen Proton Angular Velocity

$$\boxed{\Omega_{\mathrm{H}} = 2.705755062\times 10^{17} \text{ radians/sec}}$$

Proof: By [Dan4, p.22], the Hydrogen Proton Angular Velocity

$$\Omega_{\mathrm{H}} \approx \omega_{\mathrm{H}} \sqrt[4]{\frac{M_{\mathrm{p}}}{m_{\mathrm{e}}}}.$$

$$\sim 4.1334366\times 10^{16} \times \sqrt[4]{1836.152701}$$

$$= 4.1334366\times 10^{16} \times 6.546018057$$

$$= 2.705755062\times 10^{17}.\square$$

10.4 The Neutron's Proton Frequency

$$\boxed{\frac{\Omega_{\mathrm{p}}}{2\pi} = 5.69422884\times 10^{20}}$$

Proof: $\dfrac{\Omega_{\mathrm{p}}}{2\pi} = \dfrac{3.577789504\times 10^{21} \text{ radians/sec}}{2\pi}$

$$= 5.69422884 \times 10^{20}. \square$$

10.5 The Hydrogen Proton Frequency

$$\boxed{\frac{\Omega_{\mathrm{H}}}{2\pi} = 4.30634292 \times 10^{16} \ \text{cycles/sec}}$$

Proof: $$\frac{\Omega_{\mathrm{H}}}{2\pi} = \frac{2.705755062 \times 10^{17} \ \text{cycles/sec}}{2\pi}$$

$$= 4.30634292 \times 10^{16}. \square$$

10.6 The Neutron's Proton Frequency is $\sim$13,223$\times$(Hydrogen Proton Frequency)

Proof: $$\frac{\frac{\Omega_{\mathrm{N}}}{2\pi}}{\frac{\Omega_{\mathrm{H}}}{2\pi}} = \frac{\Omega_{\mathrm{p\ in\ N}}}{\Omega_{\mathrm{p\ in\ H}}}$$

$$= \frac{3.577789504 \times 10^{21}}{2.705755062 \times 10^{17}} \sim 13{,}223. \square$$

11.

The Electron's Speed

11.1 $$\boxed{\beta_{\mathrm{e}}{}^{8} + \left(\frac{\beta_{\mathrm{p}}{}^{2}}{\rho_{\mathrm{p}}}\frac{e^{2}}{10^{7}}\frac{1}{m_{\mathrm{e}}}\right)^{2}\beta_{\mathrm{e}}{}^{2} - \left(\frac{\beta_{\mathrm{p}}{}^{2}}{\rho_{\mathrm{p}}}\frac{e^{2}}{10^{7}}\frac{1}{m_{\mathrm{e}}}\right)^{2} \approx 0}$$

Proof: From the Force balance for the electron, 4.1,

$$\frac{m_{\mathrm{e}}}{\sqrt{1-\beta_{\mathrm{e}}{}^{2}}}\beta_{\mathrm{e}}{}^{2}r_{\mathrm{N}} = \frac{e^{2}}{10^{7}},$$

$$\beta_{\mathrm{e}}{}^{2}r_{\mathrm{N}} = \frac{e^{2}}{10^{7}}\frac{1}{m_{\mathrm{e}}}\sqrt{1-\beta_{\mathrm{e}}{}^{2}},$$

Substituting from 4.2, $r_{\mathrm{N}} \approx \frac{\rho_{\mathrm{p}}}{\beta_{\mathrm{p}}{}^{2}}\beta_{\mathrm{e}}{}^{2}$,

$$\beta_{\mathrm{e}}{}^{2}\frac{\rho_{\mathrm{p}}}{\beta_{\mathrm{p}}{}^{2}}\beta_{\mathrm{e}}{}^{2} \approx \frac{e^{2}}{10^{7}}\frac{1}{m_{\mathrm{e}}}\sqrt{1-\beta_{\mathrm{e}}{}^{2}},$$

Squaring both sides,

$$\beta_{\mathrm{e}}{}^{8} + \left(\frac{\beta_{\mathrm{p}}{}^{2}}{\rho_{\mathrm{p}}}\frac{e^{2}}{10^{7}}\frac{1}{m_{\mathrm{e}}}\right)^{2}\beta_{\mathrm{e}}{}^{2} - \left(\frac{\beta_{\mathrm{p}}{}^{2}}{\rho_{\mathrm{p}}}\frac{e^{2}}{10^{7}}\frac{1}{m_{\mathrm{e}}}\right)^{2} \approx 0.\Box$$

Substituting

$$\rho_{\mathrm{p}} \approx 2.209505336\times 10^{-15},$$

$$\beta_{\mathrm{p}}{}^{2} = 6.943480038\times 10^{-4},$$

$$\frac{\beta_{\mathrm{p}}{}^2}{\rho_{\mathrm{p}}}\frac{e^2}{10^7}\frac{1}{m_{\mathrm{e}}} \sim \frac{6.943480038\times 10^{-4}}{2.209505336\times 10^{-15}}\frac{(1.60217733)^2\times 10^{-38}}{10^7(9.109\ 389\ 7\times 10^{-31})}$$

$$= 8.85551901\times 10^{-4}.$$

$$\left(\frac{\beta_{\mathrm{p}}{}^2}{\rho_{\mathrm{p}}}\frac{e^2}{10^7}\frac{1}{m_{\mathrm{e}}}\right)^2 \sim 7.84202169\times 10^{-7}.$$

$$\beta_{\mathrm{e}}{}^8 + (7.84202169\times 10^{-7})\beta_{\mathrm{e}}{}^2 - (7.84202169\times 10^{-7}) \approx 0.$$

11.2 $$\boxed{\beta_{\mathrm{e}}{}^8 + (7.84202169\times 10^{-7})\beta_{\mathrm{e}}{}^2 - (7.84202169\times 10^{-7}) \sim 0}$$

Denoting $\beta_{\mathrm{e}}{}^2 \equiv x$, we seek the zeros of the polynomial

$$f(x) = x^4 + (7.84202169\times 10^{-7})x - (7.84202169\times 10^{-7})$$

between $x = 0$, and $x = 1$.

Using Newton's Iterations

$$x_{j+1} = x_j - \frac{f(x_j)}{f'(x_j)}$$

$$x_{j+1} = x_j + \frac{(7.84202169\times 10^{-7}) - x_j{}^4 - (7.84202169\times 10^{-7})x_j}{4x_j{}^3 + 7.84202169\times 10^{-7}}$$

We observe that

If $x_j > 0.05$, then $x_{j+1} < x_j$. That is, the iterations decrease.

Starting with $x_1 = 0.05$, and obtained with a calculator

$$x_2 \approx 0.04$$

$$x_3 \approx 0.03336$$

$$x_4 \approx 0.03108$$

$$x_5 \approx 0.03108$$

Maple Root-Finding Program requires x_1 to be to the right of the zero. We took $x_1 = 0.01$

Maple Input and Output follow:

```
> with(RootFinding):
```

$> \; r0 := NextZero\left(x \to x^4 + 7.84202169 \cdot 10^{-7} x - 7.84202169 \cdot 10^{-7}, 0.01\right);$

$$r0 := 0.02953601268$$

$> \; \mathrm{sqrt}(r0)$

$$0.1718604454$$

11.3 $$\boxed{\beta_e^2 \sim 0.02953601268}$$

11.4 $$\boxed{\beta_e \sim 0.1718604454}$$

11.5 The Neutron's Electron Speed

$$\boxed{v_{\mathrm{e}} \sim 51,558,134 \ \mathrm{m/sec}}.$$

<u>*Proof*</u>: $\beta_{\mathrm{e}} c \sim 0.1718604454c$

$$= 51,558,134 \ \mathrm{m/sec}. \square$$

11.6 The Hydrogen Electron Speed

$$\boxed{v_{\mathrm{B}} \sim \underbrace{\alpha}_{\approx \frac{1}{137}} c = 2,189,781 \ \mathrm{m/sec}}.$$

<u>*Proof*</u>: In the Hydrogen Atom,

$$m_{\mathrm{e}} \frac{v_{\mathrm{H}}^2}{r_{\mathrm{H}}} = \frac{1}{4\pi\varepsilon_0} \frac{e^2}{r_{\mathrm{H}}^2}.$$

$$\underbrace{m_{\mathrm{e}} v_{\mathrm{H}} r_{\mathrm{H}}}_{\hbar} v_{\mathrm{H}} = \frac{1}{4\pi\varepsilon_0} e^2,$$

$$v_{\mathrm{H}} = \underbrace{\frac{1}{4\pi\varepsilon_0} \frac{e^2}{\hbar c}}_{\alpha \approx \frac{1}{137}} c,$$

$$= 2,189,781 \ \mathrm{m/sec}. \square$$

11.7 The Neutron's Electron Speed is

$\sim$23.5$\times$(Hydrogen Electron Speed)

<u>*Proof*</u>: $$\frac{51,558,134}{2,189,781} \approx 23.5. \square$$

12.

The Neutron's Radius

12.1 $$\boxed{r_{\mathrm{N}} \approx \rho_{\mathrm{p}} \frac{\beta_{\mathrm{e}}^2}{\beta_{\mathrm{p}}^2} \sim 9.398741807 \times 10^{-14}}$$

<u>*Proof*</u>: Substituting

$$\rho_{\mathrm{p}} \sim 2.209505336 \times 10^{-15},$$

$$\beta_{\mathrm{p}}{}^2 \sim 6.943480038 \times 10^{-4},$$

$$\beta_{\mathrm{e}}^2 \sim 0.02953601268. \square$$

12.2 The Hydrogen Radius is ~563×(Neutron Radius)

<u>*Proof*</u>:. $$\frac{5.29277249 \times 10^{-11}}{9.398741807 \times 10^{-14}} \sim 563 \ .\square$$

12.3 $$\boxed{\frac{r_{\mathrm{N}}}{\rho_{\mathrm{p}}} \sim 42.53776567}$$

<u>*Proof*</u>: $$\frac{r_{\mathrm{N}}}{\rho_{\mathrm{p}}} \approx \sqrt{\frac{M_{\mathrm{p}}}{m_{\mathrm{e}}}} \sqrt[4]{\frac{1-\beta_{\mathrm{e}}^2}{1-\beta_{\mathrm{p}}^2}} \approx \sqrt{\frac{M_{\mathrm{p}}}{m_{\mathrm{e}}}},$$

because $$\sqrt[4]{\frac{1-\beta_{\mathrm{e}}^2}{1-\beta_{\mathrm{p}}^2}} \approx 0.9925315278. \square$$

13.

The Electron's Frequency

13.1 The Electron's Angular Velocity

$$\boxed{\omega_e = \frac{v_e}{r_N} = 5.485642074 \times 10^{20} \text{ radians/sec}}$$

Proof: $\omega_e = \frac{v_e}{r_N}$

$$\approx \frac{51,558,134 \text{ m/sec}}{9.398741807 \times 10^{-14}\,\text{m}}$$

$$= 5.485642074 \times 10^{20} \text{ radians/sec}. \square$$

13.2 The Electron's Frequency

$$\boxed{\frac{\omega_e}{2\pi} = 8.73067052 \times 10^{19}}$$

Proof: $\nu_e = \frac{\omega_e}{2\pi}$

$$= \frac{5.485642074 \times 10^{20} \text{ radians/sec}}{2\pi}$$

$$= 8.73067052 \times 10^{19}. \square$$

13.3 The Hydrogen Electron Angular Velocity

$$\boxed{\omega_{\mathrm{H}} = \frac{v_{\mathrm{H}}}{r_{\mathrm{H}}} = 4.1334366 \times 10^{16} \text{ radians/sec}}$$

<u>*Proof*</u>: $\omega_{\mathrm{H}} = \frac{v_{\mathrm{H}}}{r_{\mathrm{H}}}$

$$\approx \frac{2{,}189{,}781 \text{ m/sec}}{5.2977249 \times 10^{-11}\text{m}}$$

$$= 4.1334366 \times 10^{16} \text{ radians/sec}.\square$$

13.4 The Hydrogen Electron Frequency

$$\boxed{\frac{\omega_{\mathrm{H}}}{2\pi} = 6.58472424 \times 10^{15} \text{ cycles/sec}}$$

<u>*Proof*</u>: $\frac{\omega_{\mathrm{H}}}{2\pi} = \frac{4.137304269 \times 10^{16}}{2\pi}$

$$= 6.58472424 \times 10^{15} \text{ cycles/sec}.\square$$

13.5 The Neutron's Electron Frequency is $\sim$13,271$\times$(Hydrogen Electron Frequency)

Proof: $$\frac{\frac{\omega_{\mathrm{N}}}{2\pi}}{\frac{\omega_{\mathrm{H}}}{2\pi}} = \frac{\omega_{\mathrm{N}}}{\omega_{\mathrm{H}}}$$

$$= \frac{5.485642074 \times 10^{20}}{4.1334366 \times 10^{16}}$$

$$= 13,271\,.\square$$

14.

The Electron's Quantum of Angular Momentum

The Electron's Quantum of Angular Momentum is the Angular Momentum of the electron's 1st orbit

14.1 The Electron's Quantum of Angular Momentum

$$\boxed{\frac{m_{\mathrm{e}}}{\sqrt{1-\beta_{\mathrm{e}}^{2}}} v_{\mathrm{e}} r_{\mathrm{N}}}$$

By direct computing (with many approximate values), we obtain

14.2 $$(0.042490317)\hbar$$

<u>*Proof*</u>: $$\frac{1}{\sqrt{1-\beta_{\mathrm{e}}^{2}}}=\frac{1}{\sqrt{1-(0.171860446)^{2}}}=1.015103413$$

$$\frac{m_{\mathrm{e}}}{\sqrt{1-\beta_{\mathrm{e}}^{2}}} v_{\mathrm{e}} r_{\mathrm{N}} =$$

$$=\frac{(1.015103413)(9.1093897)10^{-31}(51,558,134)(9.398741807)10^{-14}}{(1.05457266)10^{-34}}\hbar$$

$$=(0.042490317)\hbar\,.\square$$

Alternatively, we have

14.3
$$\frac{m_e}{\sqrt{1-\beta_e{}^2}} v_e r_N = \frac{\alpha}{\beta_e}\hbar,$$
$$\approx 0.042472076$$

where $\alpha = \dfrac{e^2}{4\pi\varepsilon_0\hbar c} \approx \dfrac{1}{137}$ ***is the Fine Structure Constant***

<u>Proof</u>:
$$\frac{m_e}{\sqrt{1-\beta_e{}^2}}\frac{v_e^2}{r_N} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_N{}^2}.$$

$$\underbrace{\frac{m_e}{\sqrt{1-\beta_e{}^2}}\beta_e c r_N}_{\text{Angular Momentum}} = \frac{e^2}{4\pi\varepsilon_0\beta_e c},$$

$$= \underbrace{\frac{e^2}{4\pi\varepsilon_0\hbar c}}_{\alpha=\frac{1}{137}}\frac{\hbar}{\beta_e}$$

$$= \frac{\alpha}{\beta_e}\hbar.\square$$

In the proceedings we used the value

14.4
$$\boxed{(0.043132065)\hbar}$$

15.

The Proton's Quantum of Angular Momentum

The Proton's Quantum of Angular Momentum is the Angular Momentum of the Proton's 1^{st} orbit

15.1 The Proton's Quantum of Angular Momentum

$$\boxed{\frac{M_{\mathrm{p}}}{\sqrt{1-\beta_{\mathrm{p}}^2}} V_{\mathrm{p}} \rho_{\mathrm{p}} \sim (0.277007069)\hbar}$$

15.2
$$\frac{M_{\mathrm{p}}}{\sqrt{1-\beta_{\mathrm{p}}^2}} V_{\mathrm{p}} \rho_{\mathrm{p}} = \frac{\alpha}{\beta_{\mathrm{p}}} \hbar,$$

$$\approx (0.277007069)\hbar$$

where $\alpha = \dfrac{e^2}{4\pi\varepsilon_0 \hbar c} \approx \dfrac{1}{137}$ *is the Fine Structure Constant*

<u>*Proof*</u>:
$$\frac{M_{\mathrm{p}}}{\sqrt{1-\beta_{\mathrm{p}}^2}} \frac{V_{\mathrm{p}}^2}{\rho_{\mathrm{p}}} = \frac{1}{4\pi\varepsilon_0} \frac{e^2}{\rho_{\mathrm{p}}^2}.$$

$$\underbrace{\frac{M_{\mathrm{p}}}{\sqrt{1-\beta_{\mathrm{p}}^2}}\beta_{\mathrm{p}} c\rho_{\mathrm{p}}}_{\text{Angular Momentum}} = \frac{e^2}{4\pi\varepsilon_0 \beta_{\mathrm{p}} c},$$

$$= \frac{e^2}{\underbrace{4\pi\varepsilon_0 \hbar c}_{\alpha=\frac{1}{137}}}\frac{\hbar}{\beta_{\mathrm{p}}}$$

$$= \frac{\alpha}{\beta_{\mathrm{p}}}\hbar . \square$$

16.

Neutron's Zero Point Energy

The electrical binding energy will exist at temperature zero, where all thermal motions cease, and is called Zero Point Energy.

If the proton's motion is neglected, the Zero Point Energy is the electron's total binding energy.

16.1

$$\boxed{\underbrace{\frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_N}}_{-U_{\text{Electron Binding}}} = \frac{1}{2}h\left(\frac{\alpha}{\beta_e}\frac{\omega_e}{2\pi}\right)}$$

<u>*Proof*</u>:

$$\frac{1}{2}\frac{e^2}{4\pi\varepsilon_0}\frac{1}{r_N} = \frac{1}{2}\underbrace{\frac{e^2}{4\pi\varepsilon_0 c\hbar}}_{\alpha}\underbrace{\frac{c}{\omega_e r_N}}_{1/\beta_e}\hbar\omega_e$$

$$= \frac{1}{2}\frac{\alpha}{\beta_e}\hbar\omega_e$$

$$= \frac{1}{2}h\left(\frac{\alpha}{\beta_e}\frac{\omega_e}{2\pi}\right).\square$$

16.2 ***The Neutron's 1st electron orbit has photon's energy***

$$-\frac{1}{2}h\left(\frac{\alpha}{\beta_e}\frac{\omega_e}{2\pi}\right).$$

16.3 *The Neutron Ground state Energy is Zero Point Energy*

$$\frac{1}{2}h\left(\frac{\alpha}{\beta_e}\frac{\omega_e}{2\pi}\right).$$

16.4 Neutron's Zero Point Energy

$$\boxed{-\frac{1}{2}\frac{\alpha}{\beta_e}\hbar\omega_e = -7668 \text{ eV}}$$

$$\boxed{-\frac{1}{8\pi\varepsilon_0}\frac{e^2}{r_N} \sim -7671 \text{ eV}}$$

<u>*Proof*</u>:

$$\frac{1}{2}\frac{\alpha}{\beta_e}\hbar\omega_e = \frac{1}{2}\frac{\frac{1}{137}}{(0.171860446)}\left[(6.5821220)10^{-16}\text{eV}\right](5.485642074)10^{20}$$

$$= 7667.731517 \text{ eV}$$

$$\frac{1}{8\pi\varepsilon_0}\frac{e^2}{r_N} = \frac{1}{2}\frac{c^2}{10^7}\frac{(1.60217733)^2 10^{-38}}{(9.398741807)10^{-14}}\text{Joul}\frac{\text{eV}}{(1.60217733)10^{-19}\text{Joul}}$$

$$= 7671.02462 \text{ eV}$$

16.5 The Zero Point Energy Frequency

$$\boxed{2\frac{7671\text{eV}}{h} \sim 3.709677747 \times 10^{18}\text{cycles/second}}$$

$$\boxed{\frac{\alpha}{\beta_e}\frac{\omega_e}{2\pi} \sim 3.708097094 \times 10^{18}\,\text{cycles/second}}$$

Proof: $2\frac{7671\text{eV}}{h} = 2\frac{7671\text{eV}}{4.1356692 \times 10^{-15}\,\text{eV}}$

$$\sim 3.709677747 \times 10^{18}\,\text{cycles/second}$$

$$\frac{\alpha}{\beta_e}\frac{\omega_e}{2\pi} = \frac{\frac{1}{137}}{(0.171860446)} 8.73067052 \times 10^{19}$$

$$\sim 3.708097094 \times 10^{18}\,\text{cycles/second}$$

16.6 Neutron's Zero Point Energy is $\sim$564$\times$(Hydrogen's Zero Point Energy)

Proof: $\frac{7671}{13.6} \sim 564\,.\square$

17.

Neutron's Nuclear Energy Binding

Most of binding is due to the orbiting proton's, and the Proton's binding Energy is the actual Zero Point Energy.

17.1 Neutron's Nuclear Energy Binding

$$\boxed{\frac{1}{2}\frac{e^2}{4\pi\varepsilon_0}\frac{1}{\rho_{\rm p}} = \frac{1}{2}\frac{c^2}{10^7}\frac{e^2}{\rho_{\rm p}} = \frac{1}{2}\frac{\alpha}{\beta_{\rm p}}\hbar\Omega_{\rm p}}$$

<u>*Proof*</u>:

$$\frac{1}{2}\frac{e^2}{4\pi\varepsilon_0}\frac{1}{\rho_{\rm p}} = \frac{1}{2}\underbrace{\frac{e^2}{4\pi\varepsilon_0 c\hbar}}_{\alpha}\underbrace{\frac{1}{\frac{\Omega_{\rm p}\rho_{\rm p}}{c}}}_{\frac{1}{\beta_{\rm p}}}\hbar\Omega_{\rm p} = \frac{1}{2}\frac{\alpha}{\beta_{\rm p}}\hbar\Omega_{\rm p}.\square$$

17.2 Neutron's Nuclear Energy Binding is $\sim$42.5(Neutron's Zero Point Energy Binding)

<u>*Proof*</u>:

$$\frac{\frac{1}{2}\frac{c^2}{10^7}\frac{e^2}{\rho_{\rm p}}}{\frac{1}{2}\frac{c^2}{10^7}\frac{e^2}{r_{\rm N}}} = \frac{r_{\rm N}}{\rho_{\rm p}} \sim 42.5$$

17.3 Neutron's Nuclear Energy Binding is $\sim$0.977$\times$(Neutron's Total Energy Binding)

Proof: $\dfrac{1}{1+\dfrac{1}{42.5}} \sim 0.977\,.\square$

17.4

$$\boxed{-\frac{1}{2}\frac{\alpha}{\beta_{\mathrm{p}}}\hbar\Omega_{\mathrm{p}} = -326{,}168 \text{ eV}}$$

$$\boxed{-\frac{1}{8\pi\varepsilon_0}\frac{e^2}{\rho_{\mathrm{p}}} \sim -326{,}308 \text{ eV}}$$

Proof:

$$\frac{1}{2}\frac{\alpha}{\beta_{\mathrm{p}}}\hbar\Omega_{\mathrm{p}} = \frac{1}{2}\frac{\frac{1}{137}}{(0.02635048394)}\left[(6.5821220)10^{-16}\text{eV}\right](3.577789504)10^{21}$$

$$= 326{,}168.1650 \text{ eV}$$

$$\frac{1}{8\pi\varepsilon_0}\frac{e^2}{\rho_{\mathrm{p}}} = \frac{1}{2}\frac{c^2}{10^7}\frac{(1.60217733)^2 10^{-38}}{(2.209505336)10^{-15}}\text{Joul}\frac{\text{eV}}{(1.60217733)10^{-19}\text{Joul}}$$

$$= 326{,}308.2405 \text{ eV}$$

17.5 Hydrogen's Nuclear Binding Energy

$$\boxed{\frac{1}{2}\frac{e^2}{4\pi\varepsilon_0}\frac{1}{\rho_{\mathrm{H}}} = \frac{1}{2}\hbar\left(\frac{\alpha}{\beta_{\mathrm{p\ in\ H}}}\Omega_{\mathrm{H}}\right) = \frac{1}{2}\hbar\left(\frac{\alpha c}{\rho_{\mathrm{H}}}\right)}$$

Proof:
$$\frac{1}{2}\frac{e^2}{4\pi\varepsilon_0}\frac{1}{\rho_{\mathrm{H}}} = \frac{1}{2}\underbrace{\frac{e^2}{4\pi\varepsilon_0 c\hbar}}_{\alpha}\underbrace{\frac{c}{\Omega_{\mathrm{H}}\rho_{\mathrm{H}}}}_{1/\beta_{\mathrm{p\ in\ H}}}\hbar\Omega_{\mathrm{H}} = \frac{1}{2}\hbar\left(\frac{\alpha}{\beta_{\mathrm{p\ in\ H}}}\Omega_{\mathrm{H}}\right).\square$$

$$= \frac{1}{2}\hbar\left(\frac{\alpha c}{\rho_{\mathrm{H}}}\right).\square$$

17.6
$$\boxed{\Omega_{\mathrm{H}} = \frac{v_{\mathrm{H}}}{r_H}\left(\frac{M_{\mathrm{p}}}{m_{\mathrm{e}}}\right)^{\frac{1}{4}} \approx (2.708286845)10^{17}\ \text{radians/sec}}$$

Proof:
$$\Omega_{\mathrm{H}} = \underbrace{\omega_{\mathrm{H}}}_{\frac{v_{\mathrm{H}}}{r_H}}\left(\frac{M_{\mathrm{p}}}{m_{\mathrm{e}}}\right)^{\frac{1}{4}}$$

$$= \frac{\frac{1}{137}c}{(5.29277249)10^{-11}}(1836.152701)^{\frac{1}{4}}$$

$$= (2.708286845)10^{17}.\square$$

17.7
$$\boxed{-\frac{1}{2}\hbar\left(\frac{\alpha c}{\rho_{\mathrm{H}}}\right) = -590\ \text{eV}}$$

$$\boxed{-\frac{1}{8\pi\varepsilon_0}\frac{e^2}{\rho_{\mathrm{H}}} \sim -590\ \text{eV}}$$

$$\frac{1}{2}\hbar\frac{\alpha c}{\rho_{\mathrm{H}}} = \frac{1}{2}\Big[(6.5821220)10^{-16}\mathrm{eV}\Big]\frac{\frac{1}{137}c}{(1.221173735)10^{-12}}$$

$$= 590.1455881 \text{ eV}.\square$$

$$\frac{1}{8\pi\varepsilon_0}\frac{e^2}{\rho_{\mathrm{H}}} = \frac{1}{2}\frac{c^2}{10^7}\frac{(1.60217733)^2 10^{-38}}{(1.221173735)10^{-12}}\mathrm{Joul}\frac{\mathrm{eV}}{(1.60217733)10^{-19}\mathrm{Joul}}$$

$$= 590.3990381 \text{ eV}.\square$$

17.8 Neutron's Nuclear Energy Binding is $\sim$553$\times$(Hydrogen's Nuclear Energy Binding)

Proof: $\dfrac{326,308 \text{ eV}}{590 \text{ eV}} \sim 553.\square$

18.

Nuclear Force and Zero Point Energy Force

18.1 Neutron's Nuclear Force

$$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{\rho_{\mathrm{p}}^2}$$

18.2 Hydrogen's Nuclear Force

$$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{\rho_{\mathrm{H}}^2}$$

18.3 Neutron's Nuclear Force is $\sim$305,467(Hydrogen's Nuclear Force)

Proof: $$\frac{\dfrac{1}{4\pi\varepsilon_0}\dfrac{e^2}{\rho_{\mathrm{p}}^2}}{\dfrac{1}{4\pi\varepsilon_0}\dfrac{e^2}{\rho_{\mathrm{H}}^2}} = \left(\frac{\rho_{\mathrm{H}}}{\rho_{\mathrm{p}}}\right)^2.$$

By [Dan4, 4.4], $\rho_{\mathrm{H}} \approx 1.221173735 \times 10^{-12}$

$$= \left(\frac{1.221173735 \times 10^{-12}}{2.209505336 \times 10^{-15}} \right)^2$$

$$= 305,467.3522\,.\square$$

18.4 Neutron's Nuclear Force is $\sim$1836(Neutron's Zero Point Energy Force)

Proof: $$\frac{\dfrac{1}{4\pi\varepsilon_0}\dfrac{e^2}{\rho_{\mathrm{p}}^2}}{\dfrac{1}{4\pi\varepsilon_0}\dfrac{e^2}{r_{\mathrm{N}}^2}} = \frac{r_{\mathrm{N}}^2}{\rho_{\mathrm{p}}^2} \approx \frac{M_{\mathrm{p}}}{m_{\mathrm{e}}} \sim 1836\,.\square$$

19.

Gamma Rays Origin is Neutron's Proton

Soft X Rays are photons at frequencies

$$10^{18}\,\text{cycles/sec}\,.$$

Hard X Rays start at

$$10^{19}\,\text{cycles/sec}$$

The Neutron's Electron Frequency

$$\frac{\omega_{\text{ne}}}{2\pi} = 8.73067052 \times 10^{19}\,\text{cycles/sec}\,,$$

is in the range of Hard X Rays.

Gamma Rays start at

$$10^{20}\,\text{cycles/sec}$$

The Neutron's Proton Frequency

$$\frac{\Omega_{\text{np}}}{2\pi} \sim 5.69422884 \times 10^{20}\,\text{cycles/second}\,,$$

is in the range of Gamma Rays.

Thus, the existence of Gamma Rays Radiation proves that the Neutron is a condensed Hydrogen Atom, composed of an electron, and a proton.

That is,

19.1 ***a Neutron's Proton excited from its orbit into a higher orbit, returns to a lower Neutron's Orbit, and emits a Gamma Ray Photon***

20.

Kepler's 3rd Law for the Neutron, is Consistent with the Neutron's Model as a Collapsed-Hydrogen Atom

Newton's Gravitational force on a planet orbiting the sun at radius r, with period $T = \frac{2\pi}{\omega}$ is

$$m_{\text{Planet}}\omega_{\text{Planet}}^2 r = G\frac{m_{\text{Planet}}M_{\text{Sun}}}{r^2}.$$

Therefore,

$$\omega_{\text{Planet}}^2 r^3 = GM_{\text{Sun}}.$$

Or,

$$T^2 = \frac{4\pi^2}{\underbrace{GM}_{\text{Constant}}} r^3,$$

which is Kepler's 3rd Law for Gravitation.

Since the Sun too, orbits the system's center of Gravitation with a small radius ρ_{Sun},

$$m_{\text{Planet}}\omega_{\text{Planet}}^2 r_{\text{Planet}}{}^3 = GM_{\text{Sun}}m_{\text{Planet}} = M_{\text{Sun}}\Omega_{\text{Sun}}^2\rho_{\text{Sun}}{}^3.$$

Or,

$$\left(\frac{T_{\text{Planet}}}{T_{\text{Sun}}}\right)^2 = \frac{m_{\text{Planet}}}{M_{\text{Sun}}}\left(\frac{r_{\text{Planet}}}{\rho_{\text{Sun}}}\right)^3.$$

For the Electron and Proton that compose the Neutron,

20.1 $$\boxed{\frac{m_{\text{e}}}{\sqrt{1-\beta_{\text{e}}^2}}\omega_{\text{e}}^2 {r_{\text{N}}}^3 = \frac{e^2}{4\pi\varepsilon_0} = \frac{M_{\text{p}}}{\sqrt{1-\beta_{\text{p}}^2}}\Omega_{\text{p}}^2 {\rho_{\text{p}}}^3}$$

Proof: because

$$\frac{m_{\text{e}}}{\sqrt{1-\beta_{\text{e}}^2}}\omega_{\text{e}}^2 r_{\text{N}} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{{r_{\text{N}}}^2},$$

$$\frac{M_{\text{p}}}{\sqrt{1-\beta_{\text{p}}^2}}\Omega_{\text{p}}^2 \rho_{\text{p}} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{\rho_{\text{p}}^2}.\square$$

20.2 Kepler's 3rd Law for the Neutron is

$$\boxed{\left(\frac{T_{\text{e}}}{T_{\text{p}}}\right)^2 \frac{\sqrt{1-\beta_{\text{e}}^2}}{\sqrt{1-\beta_{\text{p}}^2}} = \frac{m_{\text{e}}}{M_{\text{p}}}\left(\frac{r_{\text{N}}}{\rho_{\text{p}}}\right)^3}$$

Proof: Substitute in 20.1 $\omega_{\text{e}} = \frac{2\pi}{T_{\text{e}}}$; $\Omega_{\text{p}} = \frac{2\pi}{T_{\text{p}}}$.$\square$

20.3 $$\boxed{\frac{\Omega_{\mathrm{p}}}{\omega_{\mathrm{e}}} = \frac{T_{\mathrm{e}}}{T_{\mathrm{p}}} \approx \sqrt[4]{\frac{M_{\mathrm{p}}}{m_{\mathrm{e}}}}\sqrt[8]{\frac{1-\beta_{\mathrm{e}}^2}{1-\beta_{\mathrm{p}}^2}}}$$

Proof: By 20.2,

$$\left(\frac{T_{\mathrm{e}}}{T_{\mathrm{p}}}\right)^2 = \frac{\dfrac{m_{\mathrm{e}}}{\sqrt{1-\beta_{\mathrm{e}}^2}} {r_{\mathrm{N}}}^2}{\dfrac{M_{\mathrm{p}}}{\sqrt{1-\beta_{\mathrm{p}}^2}} {\rho_{\mathrm{p}}}^2} \frac{r_{\mathrm{N}}}{\rho_{\mathrm{p}}}.$$

Substituting from 3.3, $\dfrac{m_{\mathrm{e}}}{\sqrt{1-\beta_{\mathrm{e}}^2}} {r_{\mathrm{N}}}^2 \approx \dfrac{M_{\mathrm{p}}}{\sqrt{1-\beta_{\mathrm{p}}^2}} {\rho_{\mathrm{p}}}^2$,

$$\left(\frac{T_e}{T_p}\right)^2 \approx \frac{r_{\mathrm{N}}}{\rho_{\mathrm{p}}}$$

$$\approx \sqrt{\frac{M_{\mathrm{p}}}{m_{\mathrm{e}}}}\sqrt[4]{\frac{1-\beta_{\mathrm{e}}^2}{1-\beta_{\mathrm{p}}^2}}.\square$$

20.4 The Proton's Angular Velocity

$$\boxed{\Omega_{\mathrm{p}} \approx \omega_{\mathrm{e}}\sqrt[4]{\frac{M_{\mathrm{p}}}{m_{\mathrm{e}}}}\sqrt[8]{\frac{1-\beta_{\mathrm{e}}^2}{1-\beta_{\mathrm{p}}^2}} = 3.577789633 \times 10^{21}}$$

Compare with 10.1 $3.577789504 \times 10^{21}$

The discrepancy of $\dfrac{129}{3577789504} = 3.6 \times 10^{-8}$

is due to our rounding errors, and the calculator

Proof:

$$\Omega_{\mathrm{p}} \approx \omega_{\mathrm{e}} \sqrt[4]{\frac{M_{\mathrm{p}}}{m_{\mathrm{e}}}} \sqrt[8]{\frac{1-\beta_{\mathrm{e}}^2}{1-\beta_{\mathrm{p}}^2}}$$

$$\omega_{\mathrm{e}} = 5.485642074 \times 10^{20} \text{ radians/sec}$$

$$\sqrt[4]{\frac{M_p}{m_e}} \approx (1836.152701)^{\frac{1}{4}} \approx 6.546018057$$

$$\sqrt[8]{\frac{1-\beta_{\mathrm{e}}^2}{1-\beta_{\mathrm{p}}^2}} \approx \sqrt[8]{\frac{1-0.02953601268}{1-6.943480038 \times 10^{-4}}} \approx 0.996345893$$

$$\Omega_{\mathrm{p}} \approx 3.577789633 \times 10^{21} . \square$$

Note that Ω_{p} in 10.1 was obtained from our Neutron's Mass-Energy Equation, 7.1. That Mass-Energy equation yielded ρ_{p}, with which we obtained β_{p}, $V_{\mathrm{p}} = \beta_{\mathrm{p}} c$, and $\Omega_{\mathrm{p}} = V_{\mathrm{p}} / \rho_{\mathrm{p}}$.

Then, ρ_{p}, and β_{p} were used to obtain β_{e}, r_{N}, $v_{\mathrm{e}} = \beta_{\mathrm{e}} c$, and $\omega_{\mathrm{e}} = v_{\mathrm{e}} / r_{\mathrm{N}}$.

Thus, Ω_{p} in 16.4 depends on the Mass-Energy Equation.

Nevertheless, the close results of 10.1, and 20.4, establish

20.5 *Kepler's 3rd Law for the Neutron is consistent with the Neutron's Mass-Energy Equation, and with the Model of the Neutron as a Collapsed-Hydrogen Atom*

21.

Neutron Spin Angular Momentum

21.1 Neutron Spin is the Sum of the Electron and the Proton Angular Momentums

$$\frac{m_{\mathrm{e}}}{\sqrt{1-\beta_{\mathrm{e}}^{2}}}c\beta_{\mathrm{e}}r_{\mathrm{N}}+\frac{M_{\mathrm{p}}}{\sqrt{1-\beta_{\mathrm{p}}^{2}}}c\beta_{\mathrm{p}}\rho_{\mathrm{p}}\approx\frac{\hbar}{137}\left(\frac{1}{\beta_{\mathrm{e}}}+\frac{1}{\beta_{\mathrm{p}}}\right)$$

$$=(0.319479146)\hbar$$

<u>Proof</u>: The Force on the Neutron's electron is

$$\frac{m_{\mathrm{e}}}{\sqrt{1-\beta_{\mathrm{e}}^{2}}}\frac{c^{2}\beta_{\mathrm{e}}^{2}}{r_{\mathrm{N}}}=\frac{e^{2}}{4\pi\varepsilon_{0}r_{\mathrm{N}}^{2}}.$$

The Angular Momentum of the Electron generates the Spin

$$\frac{m_{\mathrm{e}}}{\sqrt{1-\beta_{\mathrm{e}}^{2}}}c\beta_{\mathrm{e}}r_{\mathrm{N}}=\frac{e^{2}}{4\pi\varepsilon_{0}c\beta_{\mathrm{e}}},$$

$$=\frac{\mu_{0}c^{2}}{4\pi}\frac{e^{2}}{c\beta_{\mathrm{e}}}$$

$$=\frac{ce^{2}}{10^{7}}\frac{1}{\beta_{\mathrm{e}}}$$

Substituting $e^{2}\mu_{0}c=2h\alpha$, where $\alpha\approx\frac{1}{137}$

$$=\alpha\hbar\frac{1}{\beta_{\mathrm{e}}}.$$

Similarly, the Angular Momentum of the Proton in its Neutron Orbit, generates the Spin

$$\frac{M_{\mathrm{p}}}{\sqrt{1-\beta_{\mathrm{p}}{}^2}} c\beta_{\mathrm{p}}\rho_{\mathrm{p}} = \alpha\hbar\frac{1}{\beta_{\mathrm{p}}}$$

The Neutron Spin is the Sum

$$\frac{m_{\mathrm{e}}}{\sqrt{1-\beta_{\mathrm{e}}{}^2}} c\beta_{\mathrm{e}} r_{\mathrm{N}} + \frac{M_{\mathrm{p}}}{\sqrt{1-\beta_{\mathrm{p}}{}^2}} c\beta_{\mathrm{p}}\rho_{\mathrm{p}} = \alpha\hbar\left(\frac{1}{\beta_{\mathrm{e}}} + \frac{1}{\beta_{\mathrm{p}}}\right)$$

$$\frac{\alpha}{\beta_{\mathrm{e}}} + \frac{\alpha}{\beta_{\mathrm{p}}} \approx \underbrace{\frac{1/137}{0.1718604454}}_{0.0422472077} + \underbrace{\frac{1/137}{0.02635048394}}_{0.277007069} \approx 0.319479146 . \square$$

Consequently,

21.2 ***The Assumption that the Neutron's Spin is*** $(0.5)\hbar$ ***is off by 57%***

22.

Electric Collapse Versus Gravitational Collapse

22.1 ***The Gravitational Force between the electron and the Proton in the Hydrogen Atom, or in the Neutron is negligible with respect to the Electrical Force between them***

<u>*Proof*</u>: For the Neutron,

$$\frac{G\dfrac{m_e M_p}{{r_N}^2}}{\dfrac{1}{4\pi\varepsilon_0}\dfrac{e^2}{{r_N}^2}} = G\frac{c^2}{10^7}\frac{m_e M_p}{e^2}$$

$$= 6.67259\cdot 10^{-11}\frac{9\cdot 10^{16}}{10^7}\frac{9.1093897\cdot 10^{-31}1.6726231\cdot 10^{-27}}{(1.602177331)^2 10^{-38}}$$

$$= 3.56453724\times 10^{-20}.\square$$

Similarly, for the Hydrogen,

$$\frac{G\dfrac{m_e M_p}{r_H^2}}{\dfrac{1}{4\pi\varepsilon_0}\dfrac{e^2}{r_H^2}} = G\frac{c^2}{10^7}\frac{m_e M_p}{e^2}$$

$$= 3.56453724\times 10^{-20}.\square$$

Consequently,

22.2 *Neutron Stars are Created by Electric Collapse.*

Gravitational Forces are negligible in the Collapse.

23.

Neutron's Binding Energy

23.1 The Neutron's Total Binding Energy

$$\frac{1}{2}\frac{c^2}{10^7}\frac{e^2}{\rho_{\mathrm{p}}}\left(1+\frac{\rho_{\mathrm{p}}}{r_{\mathrm{N}}}\right) \approx (M_{\mathrm{N}} - M_{\mathrm{p}} - m_{\mathrm{e}})c^2$$

$$= (0.3339861495)\mathrm{MeV}$$

Proof: By 7.1, the Binding Energy is

$$\frac{1}{2}\frac{c^2}{10^7}\frac{e^2}{\rho_{\mathrm{p}}}\left(1+\frac{\rho_{\mathrm{p}}}{r_{\mathrm{N}}}\right) \approx \underbrace{(M_{\mathrm{N}} - M_{\mathrm{p}} - m_{\mathrm{e}})}_{\Delta m}c^2$$

$$= 5.9456103 \times 10^{-31} 9 \times 10^{16}$$

$$= 5.35104927 \times 10^{-14}\,\mathrm{J}$$

Since $\text{Joule} = \dfrac{10^{21}}{60,2177}\mathrm{eV}$,

$$= 0.3339861495\mathrm{MeV}$$

$$= \underbrace{\frac{0.3339861495}{0.51099906}}_{0.653594449}(\mathrm{m_e c^2})\mathrm{MeV}\,.\square$$

23.2 The Hydrogen Atom Total Binding Energy

$$\frac{1}{2}\frac{c^2}{10^7}\frac{e^2}{\rho_{\mathrm{p}}}\left(1+\frac{\rho_{\mathrm{p}}}{r_{\mathrm{H}}}\right) = 604.1773234\mathrm{eV}$$

Proof: Substituting from [Dan4, p.21],

$$\frac{r_{\mathrm{H}}}{\rho_{\mathrm{p}}} = \frac{r}{\rho} = 42.8503524$$

$$\rho_{\mathrm{p}} = 1.221173735 \cdot 10^{-12}$$

$$\frac{1}{2}\frac{c^2}{10^7}\frac{e^2}{\rho_{\mathrm{p}}}\left(1+\frac{\rho_{\mathrm{p}}}{r_{\mathrm{H}}}\right) = \frac{1}{2}\frac{9\cdot 10^{16}}{10^7}\frac{(1.602177331)^2 10^{-38}}{1.221173735\cdot 10^{-12}}\underbrace{(1+\frac{1}{42.8503524})}_{1.02333703}$$

$$= 9.679990114\times 10^{-17}\mathrm{J}$$

$$= 9.679990114\times 10^{-17}\times(6.241507649)10^{18}\mathrm{eV}$$

$$= 604.1773234\mathrm{eV}\,.\square$$

23.3 The Neutron's Total Binding Energy is about $553\times$(Hydrogen Total Binding Energy)

Proof: $$\frac{333986.1495\mathrm{eV}}{604.1773234\mathrm{eV}} \approx 553\,.\square$$

24.

Nuclear Forces, Nuclear Bonding, Nucleus Stability, and Neutron Stars

Over the short distance between the Neutron's electron and Proton, the electric force is enormous compared to the Hydrogen electric Force:

24.1 ***The Proton-Electron Electric Force in the Neutron is 317,000 $\times$ (Proton-Electron Force in the Hydrogen)***

Proof: Substitute

$$r_{\mathrm{H}} = \text{Hydrogen Radius} = 5.29177249 \times 10^{-11}\,\mathrm{m}$$

$$r_{\mathrm{N}} = \text{Neutron Radius} \approx 9.398741807 \times 10^{-14}\,\mathrm{m}$$

$$\frac{\dfrac{1}{4\pi\varepsilon_0}\dfrac{e^2}{{r_{\mathrm{N}}}^2}}{\dfrac{1}{4\pi\varepsilon_0}\dfrac{e^2}{{r_{\mathrm{H}}}^2}} = \left(\frac{r_{\mathrm{H}}}{r_{\mathrm{N}}}\right)^2$$

$$= \left(\frac{5.29177249 \times 10^{-11}}{9.398741807 \times 10^{-14}}\right)^2$$

$$\approx 317{,}000 \,.\square$$

This electric force is the source of the Nuclear Force that binds the protons in the Nucleus.

For instance,

24.2 ***A Nucleus made of a Proton and a Neutron is a Mini One-Electron Molecule*** H_2^+***, with two protons and one electron that orbits the two protons just as it does in the*** H_2^+ ***molecule.***

For such Molecular Bonding see [Gil, p.70].

This sort of Molecular Bonding in the Nucleus ensures the stability of the Nucleus

Namely,

24.3 ***Nuclear Bonding is a Mini Molecular Bonding Through Orbitals of Electrons Supplied by the Neutrons in the Nucleus.***

Hence,

24.4 *In Neutron Stars, the Gravitational Forces are negligible compared to the Neutron's Nuclear Bonding, which keeps the star packed together*

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