# Tracking shifts in students' understanding: Forces, acceleration, and graphs

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## Background

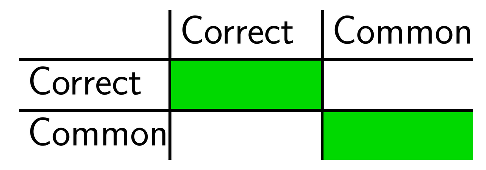
- Student learning differs on various clusters of the FMCE [1, 2]
- Model analysis shows how a class's ideas change over time [3]
- **How do individual students' responses change?**
- **Do individuals answer isomorphic questions coherently?**

Identifying isomorphic questions from the Force Sled (FS), Force Graphs (FG) and Acceleration Graphs (AG) question clusters

| Case | Described Motion | Question FS | FG | AG |
|---|---|---|---|---|
| 1 | moving right, speeding up | 1 | 16 | 22 |
| 2 | moving right, steady speed | 2 | 14 | 26 |
| 3 | moving right, slowing down | 3 | 18 | 23 |
| 4 | moving left, speeding up | 4 | 19 | 25 |

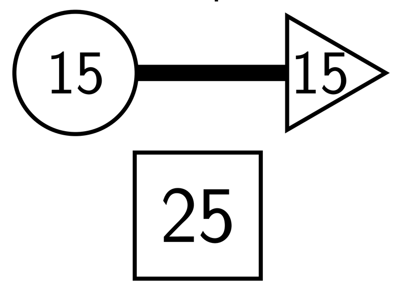
## Contingency Tables

- Compare Force Graphs to Force Sled or Force Graphs to Accel. Graphs
- Table shows number of students who gave each response pair
- Diagonal cells show within-student coherent responses
- Large numbers show between-students consistent responses
- Ignore answer choices with fewer than 5% of responses on pre- and post-test

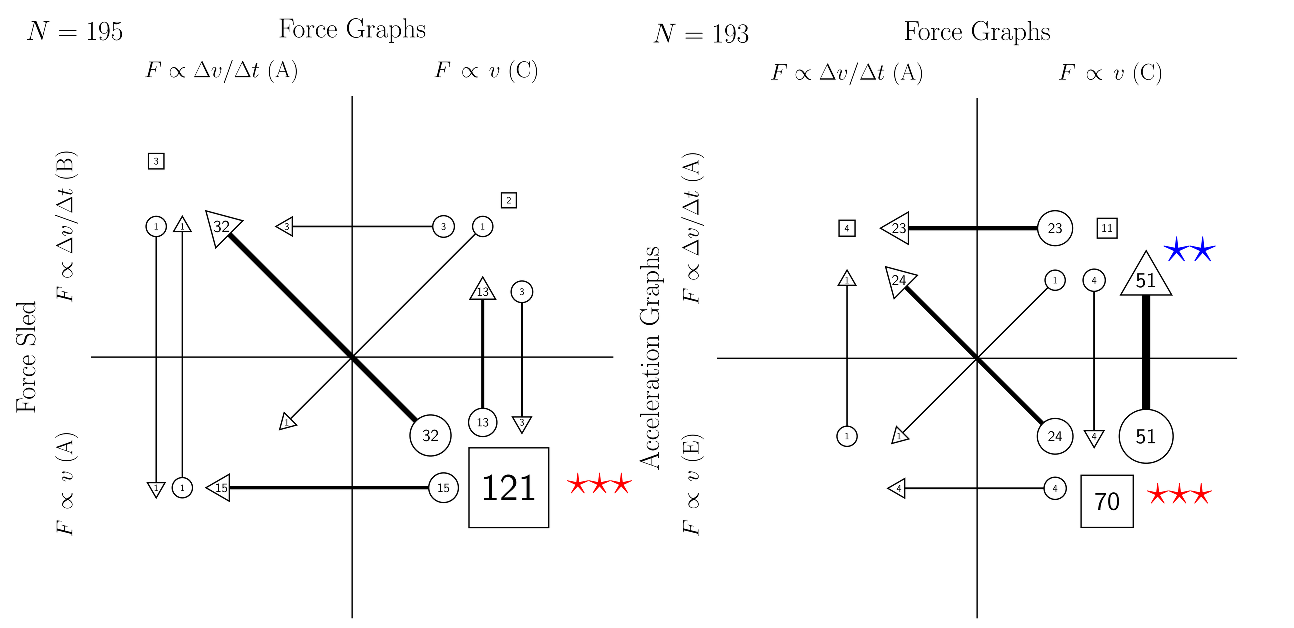



## Consistency Plots

- Visualizing student transitions between table cells [6]
- "Arrows" show the number of students who went from one pair of pretest responses to a different pair
  - Start in circles (pretest)
  - End in triangles (post-test)
- Squares show students who did not change their answers




## Case 1: School 1




Force Sled vs. Force Graphs

| Pretest | $F \propto \frac{\Delta v}{\Delta t}$ | $F \propto v$ |
|---|---|---|
| $F \propto \frac{\Delta v}{\Delta t}$ | 4 | 9 |
| $F \propto v$ | 1 | 181 |

$w = 0.48$

| Post-test | $F \propto \frac{\Delta v}{\Delta t}$ | $F \propto v$ |
|---|---|---|
| $F \propto \frac{\Delta v}{\Delta t}$ | 39 | 15 |
| $F \propto v$ | 17 | 124 |

$w = 0.60$ ★★★★

Force Sled vs. Acceleration Graphs

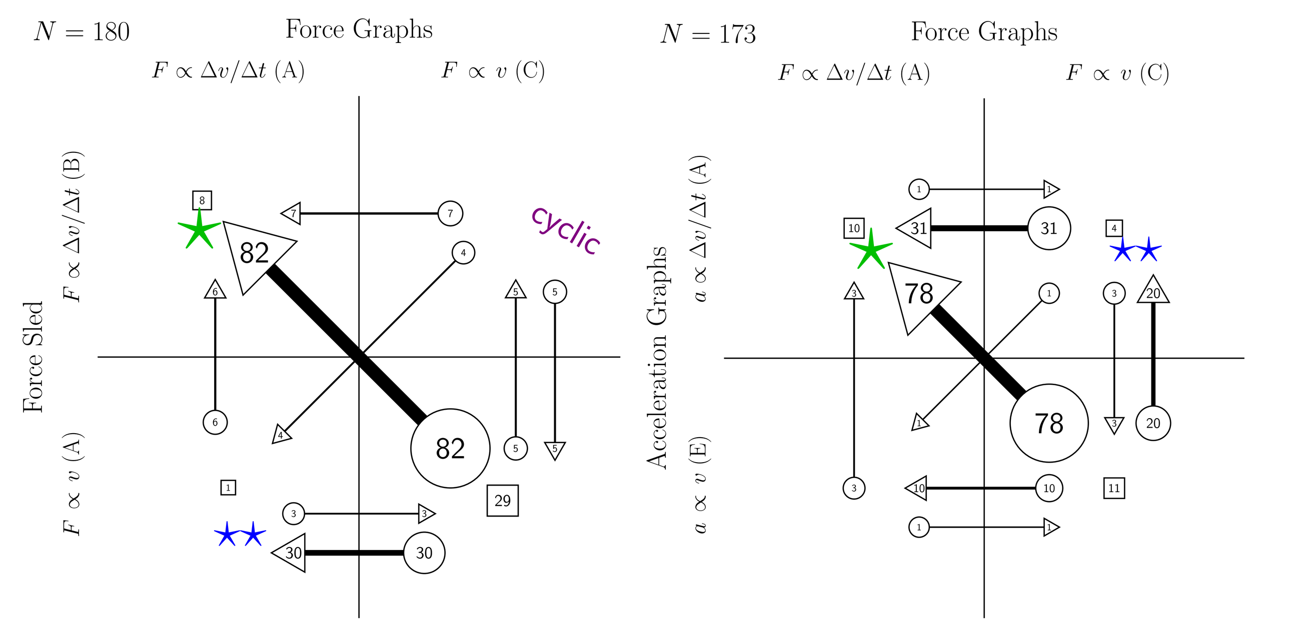
| Pretest | $F \propto \frac{\Delta v}{\Delta t}$ | $F \propto v$ |
|---|---|---|
| $F \propto \frac{\Delta v}{\Delta t}$ | 4 | 39 |
| $F \propto v$ | 1 | 149 |

$w = 0.22$

| Post-test | $F \propto \frac{\Delta v}{\Delta t}$ | $F \propto v$ |
|---|---|---|
| $F \propto \frac{\Delta v}{\Delta t}$ | 52 | 62 |
| $F \propto v$ | 5 | 74 |

$w = 0.42$ ★★★★

## Case 1: School 2




Force Sled vs. Force Graphs

| Pretest | $F \propto \frac{\Delta v}{\Delta t}$ | $F \propto v$ |
|---|---|---|
| $F \propto \frac{\Delta v}{\Delta t}$ | 8 | 16 |
| $F \propto v$ | 10 | 146 |

$w = 0.31$

| Post-test | $F \propto \frac{\Delta v}{\Delta t}$ | $F \propto v$ |
|---|---|---|
| $F \propto \frac{\Delta v}{\Delta t}$ | 103 | 5 |
| $F \propto v$ | 35 | 37 |

$w = 0.54$ ★★★★

Force Sled vs. Acceleration Graphs

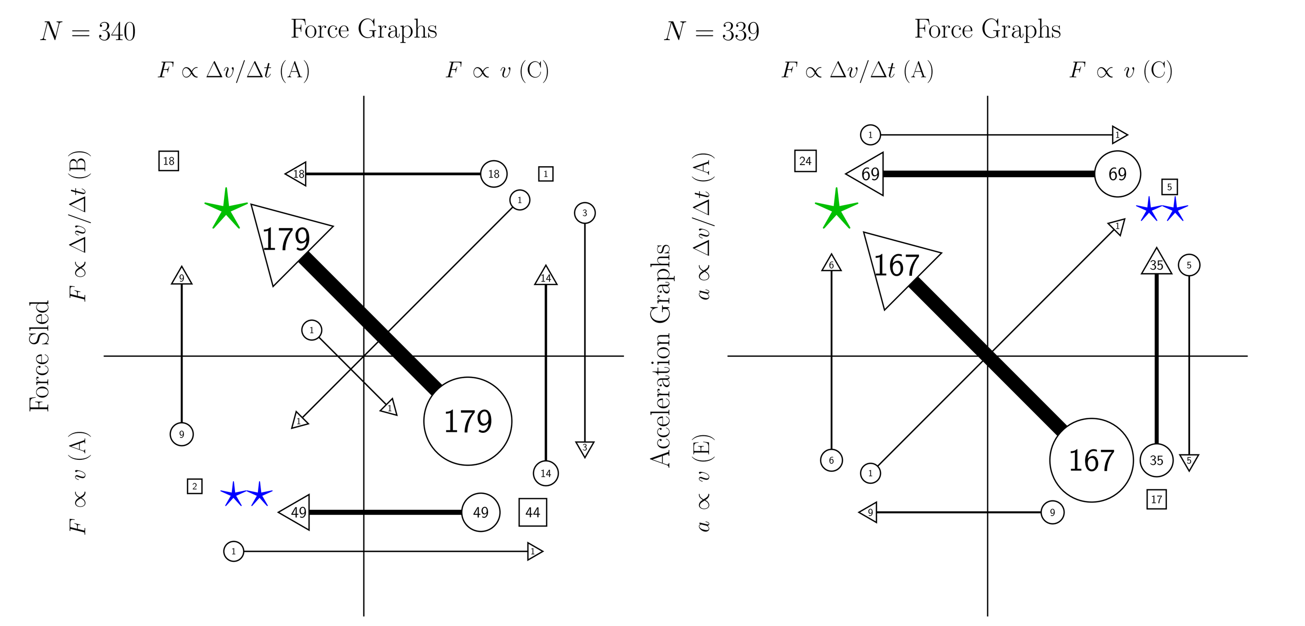
| Pretest | $F \propto \frac{\Delta v}{\Delta t}$ | $F \propto v$ |
|---|---|---|
| $F \propto \frac{\Delta v}{\Delta t}$ | 11 | 39 |
| $F \propto v$ | 4 | 119 |

$w = 0.30$

| Post-test | $F \propto \frac{\Delta v}{\Delta t}$ | $F \propto v$ |
|---|---|---|
| $F \propto \frac{\Delta v}{\Delta t}$ | 122 | 25 |
| $F \propto v$ | 11 | 15 |

$w = 0.34$ ★★★★

## Case 1: School 3




Force Sled vs. Force Graphs

| Pretest | $F \propto \frac{\Delta v}{\Delta t}$ | $F \propto v$ |
|---|---|---|
| $F \propto \frac{\Delta v}{\Delta t}$ | 19 | 23 |
| $F \propto v$ | 12 | 286 |

$w = 0.47$

| Post-test | $F \propto \frac{\Delta v}{\Delta t}$ | $F \propto v$ |
|---|---|---|
| $F \propto \frac{\Delta v}{\Delta t}$ | 224 | 15 |
| $F \propto v$ | 52 | 49 |

$w = 0.49$ ★★★★

Force Sled vs. Acceleration Graphs

| Pretest | $F \propto \frac{\Delta v}{\Delta t}$ | $F \propto v$ |
|---|---|---|
| $F \propto \frac{\Delta v}{\Delta t}$ | 25 | 79 |
| $F \propto v$ | 7 | 228 |

$w = 0.33$

| Post-test | $F \propto \frac{\Delta v}{\Delta t}$ | $F \propto v$ |
|---|---|---|
| $F \propto \frac{\Delta v}{\Delta t}$ | 267 | 42 |
| $F \propto v$ | 9 | 22 |

$w = 0.42$ ★★★★

## Case 4: Moving left, Speeding up

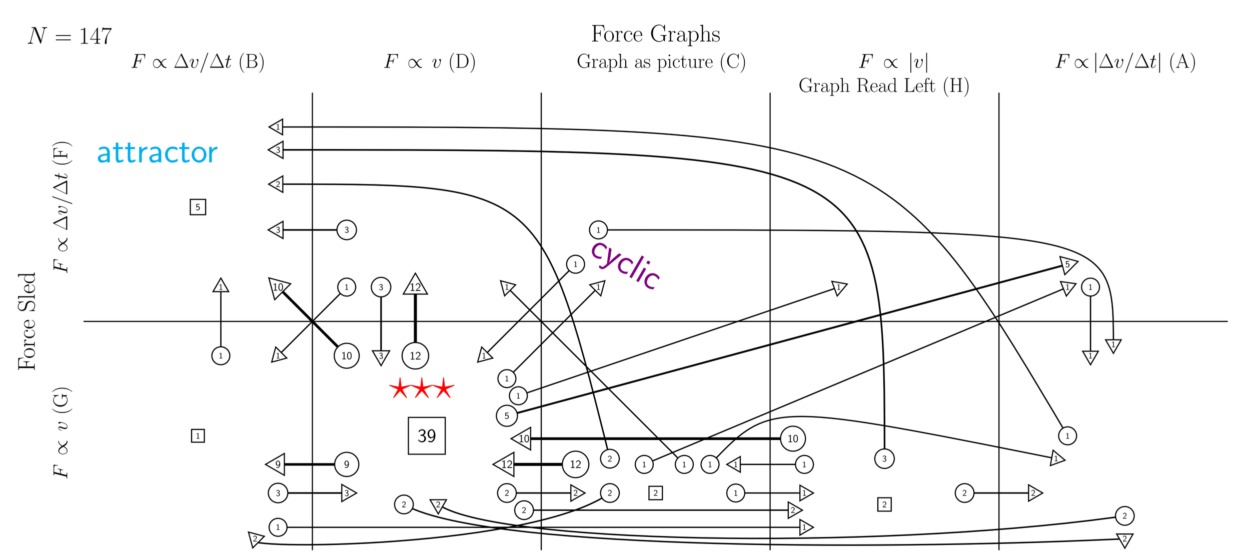
### School 1

| Pretest | $F \propto \frac{\Delta v}{\Delta t}$ | $F \propto v$ | Picture | $F \propto \lvert v \rvert$; left | $F \propto \left\lvert \frac{\Delta v}{\Delta t} \right\rvert$ |
|---|---|---|---|---|---|
| $F \propto \frac{\Delta v}{\Delta t}$ | 5 | 7 | 2 | 0 | 1 |
| $F \propto v$ | 6 | 83 | 22 | 18 | 3 |

$w = 0.36$

| Post-test | $F \propto \frac{\Delta v}{\Delta t}$ | $F \propto v$ | Picture | $F \propto \lvert v \rvert$; left | $F \propto \left\lvert \frac{\Delta v}{\Delta t} \right\rvert$ |
|---|---|---|---|---|---|
| $F \propto \frac{\Delta v}{\Delta t}$ | 25 | 13 | 1 | 1 | 6 |
| $F \propto v$ | 13 | 70 | 5 | 6 | 7 |

$w = 0.48$ ★★★★



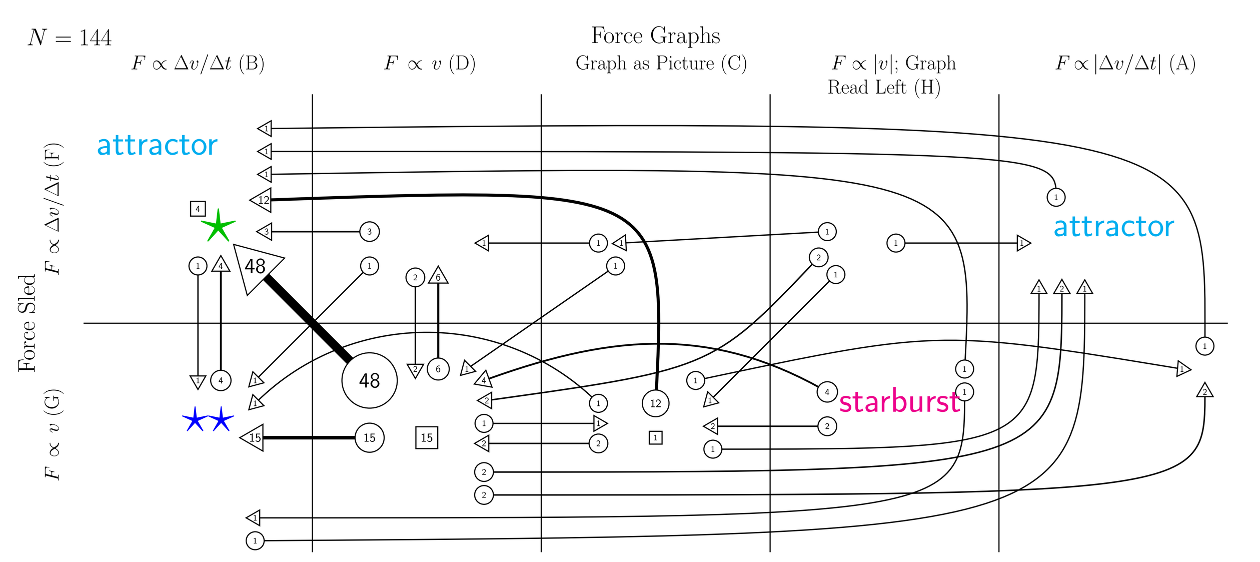

### School 2

| Pretest | $F \propto \frac{\Delta v}{\Delta t}$ | $F \propto v$ | Picture | $F \propto \lvert v \rvert$; left | $F \propto \left\lvert \frac{\Delta v}{\Delta t} \right\rvert$ |
|---|---|---|---|---|---|
| $F \propto \frac{\Delta v}{\Delta t}$ | 5 | 6 | 2 | 5 | 1 |
| $F \propto v$ | 7 | 89 | 18 | 10 | 1 |

$w = 0.38$

| Post-test | $F \propto \frac{\Delta v}{\Delta t}$ | $F \propto v$ | Picture | $F \propto \lvert v \rvert$; left | $F \propto \left\lvert \frac{\Delta v}{\Delta t} \right\rvert$ |
|---|---|---|---|---|---|
| $F \propto \frac{\Delta v}{\Delta t}$ | 76 | 7 | 1 | 0 | 5 |
| $F \propto v$ | 23 | 25 | 4 | 0 | 3 |

$w = 0.49$ ★★★★



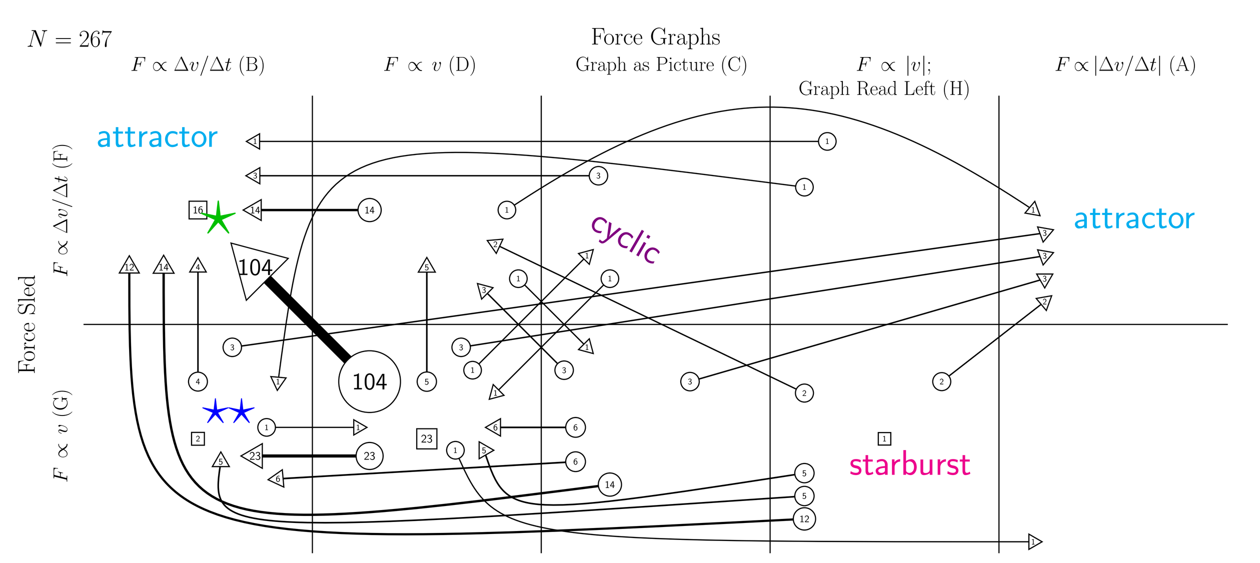

### School 3

| Pretest | $F \propto \frac{\Delta v}{\Delta t}$ | $F \propto v$ | Picture | $F \propto \lvert v \rvert$; left | $F \propto \left\lvert \frac{\Delta v}{\Delta t} \right\rvert$ |
|---|---|---|---|---|---|
| $F \propto \frac{\Delta v}{\Delta t}$ | 16 | 16 | 4 | 2 | 0 |
| $F \propto v$ | 10 | 160 | 32 | 27 | 0 |

$w = 0.45$

| Post-test | $F \propto \frac{\Delta v}{\Delta t}$ | $F \propto v$ | Picture | $F \propto \lvert v \rvert$; left | $F \propto \left\lvert \frac{\Delta v}{\Delta t} \right\rvert$ |
|---|---|---|---|---|---|
| $F \propto \frac{\Delta v}{\Delta t}$ | 168 | 10 | 1 | 0 | 12 |
| $F \propto v$ | 37 | 36 | 1 | 1 | 1 |

$w = 0.52$ ★★★★




## Models of thinking

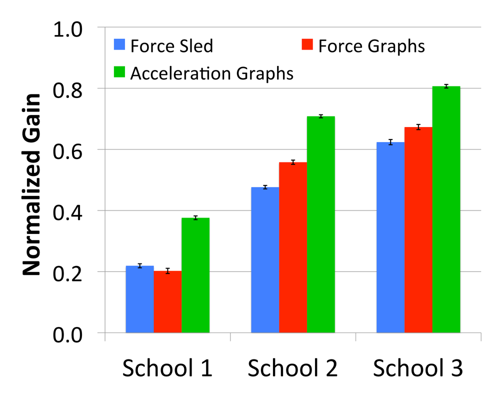
| Model Name | Model Description | Cases |
|---|---|---|
| Correct, $F \propto dv/dt$ | Consistent with Newton's second law: net force is proportional to the rate of change of velocity. | 1–4 |
| Common, $F \propto v$ | Net force is proportional to velocity. | 1–4 |
| Graph as Picture | Graphs can be interpreted as literal pictures of the situation. | 1–4 |
| $F \propto \lvert dv/dt \rvert$ | Similar to the correct model, but ignoring sign/direction. | 3,4 |
| $F \propto \lvert v \rvert$; Graph Read Left | Net force is proportional to speed; reading the graph in the direction of motion. | 4 |

## Statistical Comparisons

- Cohen's $w$ indicates the strength of the correlation between individual students' responses on contingency tables [4, 5] weak: $w < 0.1$; moderate: $w \approx 0.3$; strong: $w > 0.5$
- Consistency plots may be compared using $\chi^2$ test of independence; main effect results are significant at $p < 0.05$, pairwise significant at $p < 0.013$ (Bonferroni correction)
- Table shows $p$-values for all comparisons and all cases; * indicates $p < 0.001$.

| $p$-values | Case 1 FS | Case 1 AG | Case 2 FS | Case 2 AG | Case 3 FS | Case 3 AG | Case 4 FS | Case 4 AG |
|---|---|---|---|---|---|---|---|---|
| Main Eff. | * | * | * | * | * | * | * | * |
| 1v2 | * | * | * | * | * | * | * | 0.004 |
| 1v3 | * | * | * | * | * | * | * | * |
| 2v3 | 0.24 | 0.64 | 0.65 | 0.45 | 0.49 | 0.38 | 0.07 | 0.33 |

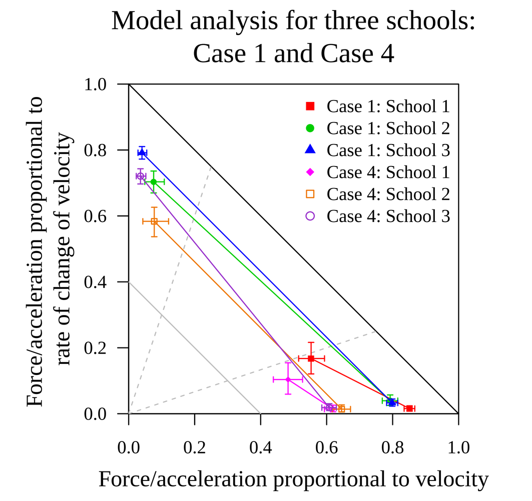
## Normalized Gains and Model Analysis




ANOVA results for individual student gains with Tukey HSD *post hoc* comparisons between schools; * indicates $p < 0.001$.

| | Average $g$ S1 | S2 | S3 | $p$-values Main Eff. | 1v2 | 1v3 | 2v3 |
|---|---|---|---|---|---|---|---|
| Full FMCE | 0.29 | 0.60 | 0.69 | * | * | * | 0.003 |
| Cases 1–4 | 0.23 | 0.61 | 0.71 | * | * | * | 0.02 |
| Case 1 | 0.31 | 0.65 | 0.77 | * | * | * | 0.007 |

Model analysis for three schools: Case 1 and Case 4




## Summary of Results

- Model analysis explicitly treats students as being in a superposition state
- Different approaches reveal discrepant similarities and differences
  - Normalized gains and model analysis show all three schools being different ($p < 0.05$): S3 > S2 > S1
  - Consistency plots show that Schools 2 & 3 are visually and statistically similar ($p > 0.05$); both are much different from School 1
- Most students at Schools 2 and 3 go from common incorrect to correct on all questions (★)
- More students increase on Force Graphs than Force Sled, and more on Acceleration Graphs than Force Graphs (★★)
- A plurality of students at School 1 stay in the common incorrect cell on all questions (★★★)
- Contingency tables with Cohen's $w$ show within-student coherence increasing over time (★★★★)
- Many different transitions for Case 4: "beginning state" + "instruction" ≠ "ending state"
- Possible hierarchy of incorrect responses [7]: starbursts may represent very naïve responses (only pretest); attractors may represent more sophisticated ones (only post-test)
- Cyclic transitions only visible on consistency plots

## Future Directions

- Synthesize results across cases; develop statistic to report between-students consistency
- Conduct interviews to test model definitions
- Closely examine similarities and differences between the instruction at each school

References
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