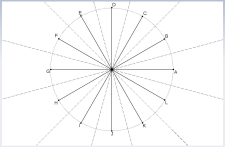

Study Of Normal Modes and Symmetry Breaking in a Two-Dimensional Pendulum

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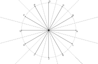
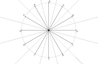


Study of Normal Modes and Symmetry Breaking in a Two-Dimensional Pendulum

Concepts introduced through this experiment

- Demonstration of normal modes in a single oscillator.
- Concept of symmetry breaking.
- Foucault's pendulum suspension design consideration.

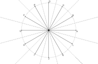


Study of Normal Modes and Symmetry Breaking in a Two-Dimensional Pendulum

Cylindrical symmetric oscillator

$$H_{(}p_x, p_y, x, y) = \frac{1}{2m}(p_x^2 + p_y^2) + \frac{1}{2}k(x^2 + y^2)$$

- Single frequency $\omega_o = \sqrt{\frac{k}{m}}$.
- Time invariant motion pattern (In an inertial frame).
- Degenerate modes (Modes language).



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Broken cylindrical symmetry

$$H(p_x, p_y, x, y) = \frac{1}{2m}(p_x^2 + p_y^2) + \frac{1}{2}(k_x x^2 + k_y y^2 + 2kxy)$$

Can be visualized as

- A system of two springs with force constants k_1 and k_2 attached to the mass m along x and y directions.
- The angular frequencies are given by $\omega_1 = \sqrt{\frac{k_1}{m}}$ and $\omega_2 = \sqrt{\frac{k_2}{m}}$.
- In a system of n different springs the end result is the same!
- The problem can be diagonalized by choosing appropriate coordinates. Eigen modes and eigen frequencies.

Realization of the normal modes and symmetry breaking in a single oscillator

- The oscillator is a two-dimensional pendulum oscillating under gravity.
- The symmetry is broken by attaching a spring toward one side of the suspension.
- The linearity is achieved by restricting the amplitude of oscillations to a small value.

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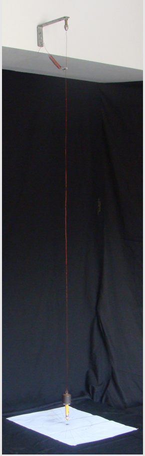

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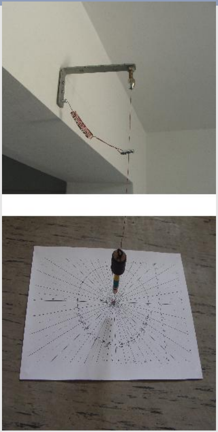
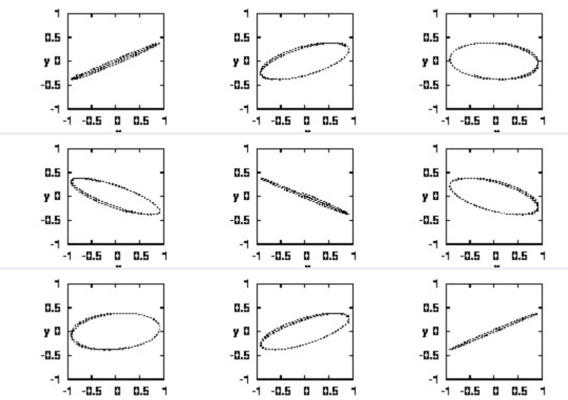
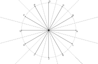


Motion after Symmetry Breaking

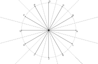




Study of Normal Modes and Symmetry Breaking in a Two-Dimensional Pendulum

Results and Discussion

- Uniformity in time period measured after releasing the pendulum in different directions represents cylindrical symmetry.
- After breaking the symmetry the time periods of two modes are measured.
- Return time: The time taken by the pendulum to come back to its original plane is measured and it is related to the strength of symmetry breaking.



Study of Normal Modes and Symmetry Breaking in a Two-Dimensional Pendulum

First Spring

- $T = \frac{2\pi}{\Delta\omega}$ measured from $\omega_1 - \omega_2$ is $196s$.
- T measured directly from return time measurements $206 - 191s$ for 15^o to 75^o.

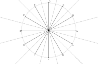
Second Spring

- $T = \frac{2\pi}{\Delta\omega}$ measured from $\omega_1 - \omega_2$ is $330s$.
- T measured directly from return time measurements $387 - 374s$ for 15^o to 75^o.

Study of Normal Modes and Symmetry Breaking in a Two-Dimensional Pendulum

Relation to Foucault's pendulum suspension

- One observes that the symmetry breaking leads to rotation of the plane of the oscillator over a time scale related to the strength of symmetry breaking.
- What if we want to build a Foucault's pendulum? In that case the oscillator plane should not shift (due to this effect) over one day.
- $\Delta\omega \to 0$
- Frequencies of the two modes should be same up to parts per million, in order to observe the effects of Coriolis force.



Central points

Demonstration of normal modes in a single oscillator. Usually two oscillators are used to demonstrate normal modes.

Concept of symmetry breaking. Demonstration of experimentally calculating eigen values of a general quadratic Hamiltonian in two dimensions.

Connection with Foucault's pendulum. How it is important to build a Foucault's pendulum in such a way so that cylindrical symmetry is not broken up to one part in one million.