Newton Iteration Based π Calculation

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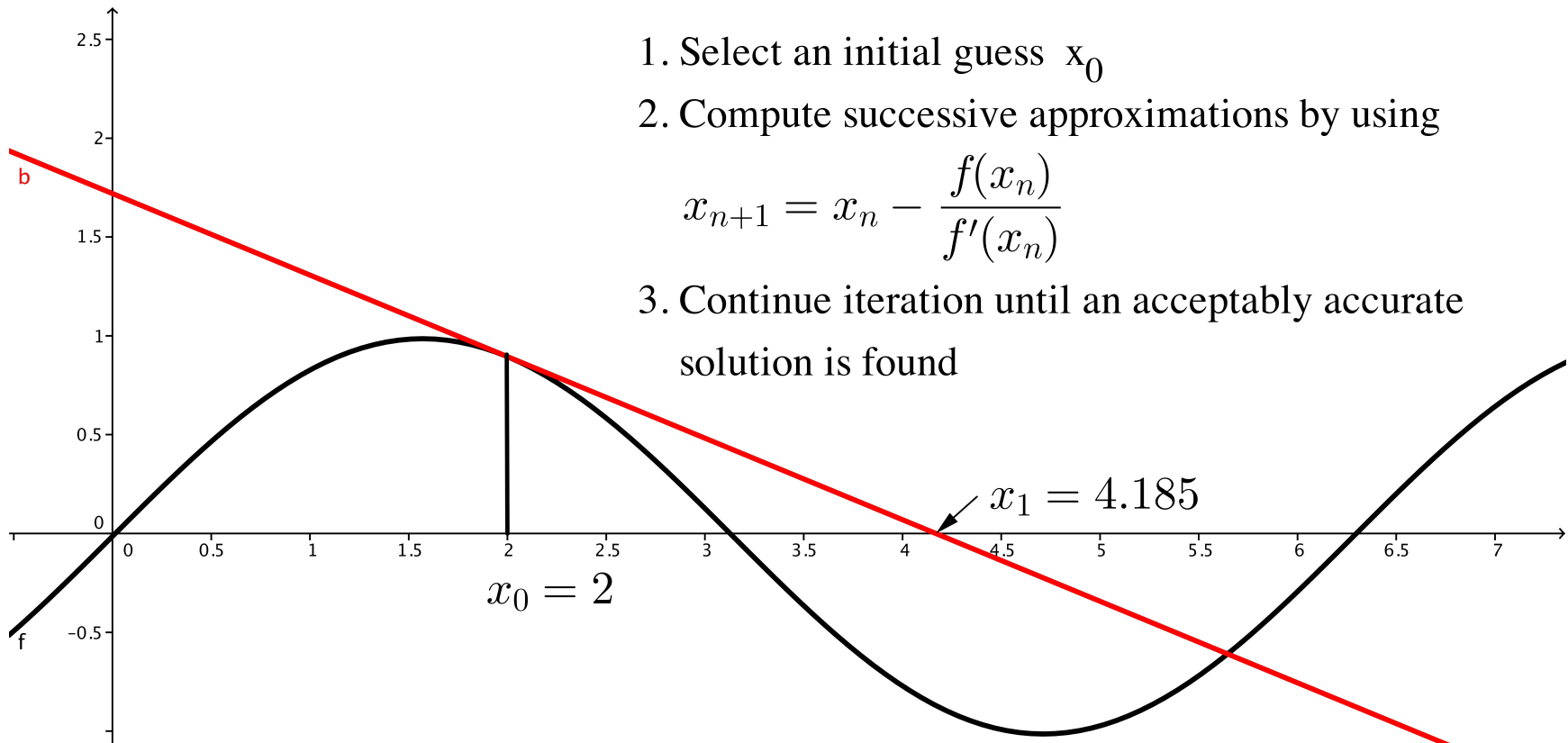
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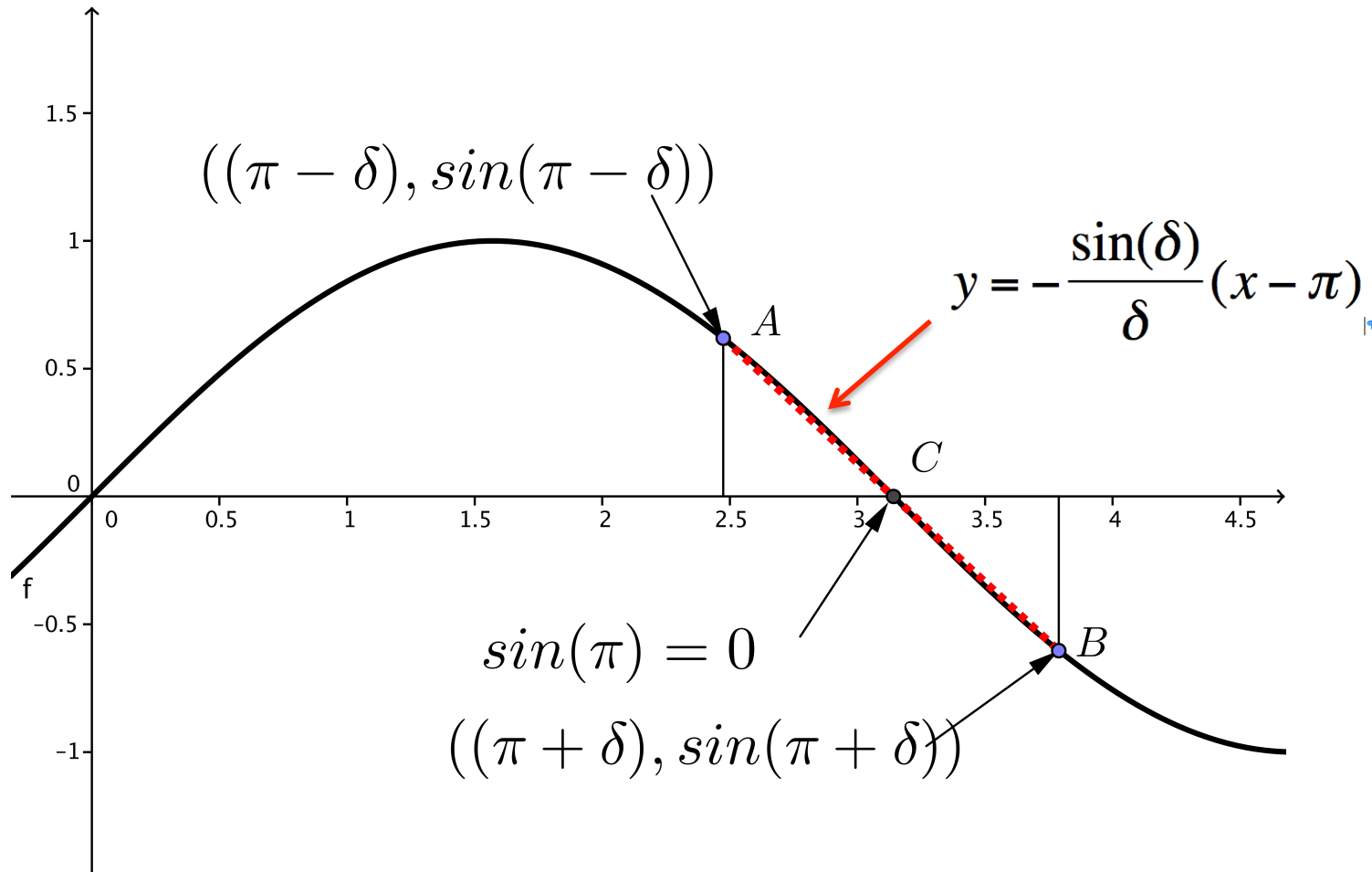www.socprep.org

Newton's Method

1. Select an initial guess x_0
2. Compute successive approximations by using

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

3. Continue iteration until an acceptably accurate solution is found

$x_1 = 4.185$

$x_0 = 2$

b

f

Find x > 0 where sin(x)=0

$((\pi - \delta), sin(\pi - \delta))$

$y = -\dfrac{\sin(\delta)}{\delta}(x - \pi)$

A

C

$sin(\pi) = 0$

$((\pi + \delta), sin(\pi + \delta))$

B

f

Our Improvement

$slope = f'(x_0) = -cos(\delta)$

$f(x) = sin(x)$

tangent

A

$slope = -\dfrac{sin(\delta)}{\delta}$

$y_0 = sin(\pi - \delta)$
$= sin(\delta)$

$slope = -1$

$x = 4.185$

$x_0 = (\pi - \delta)$

x_1

B

D

C

$sin(\delta)$

δ

$x = \pi$
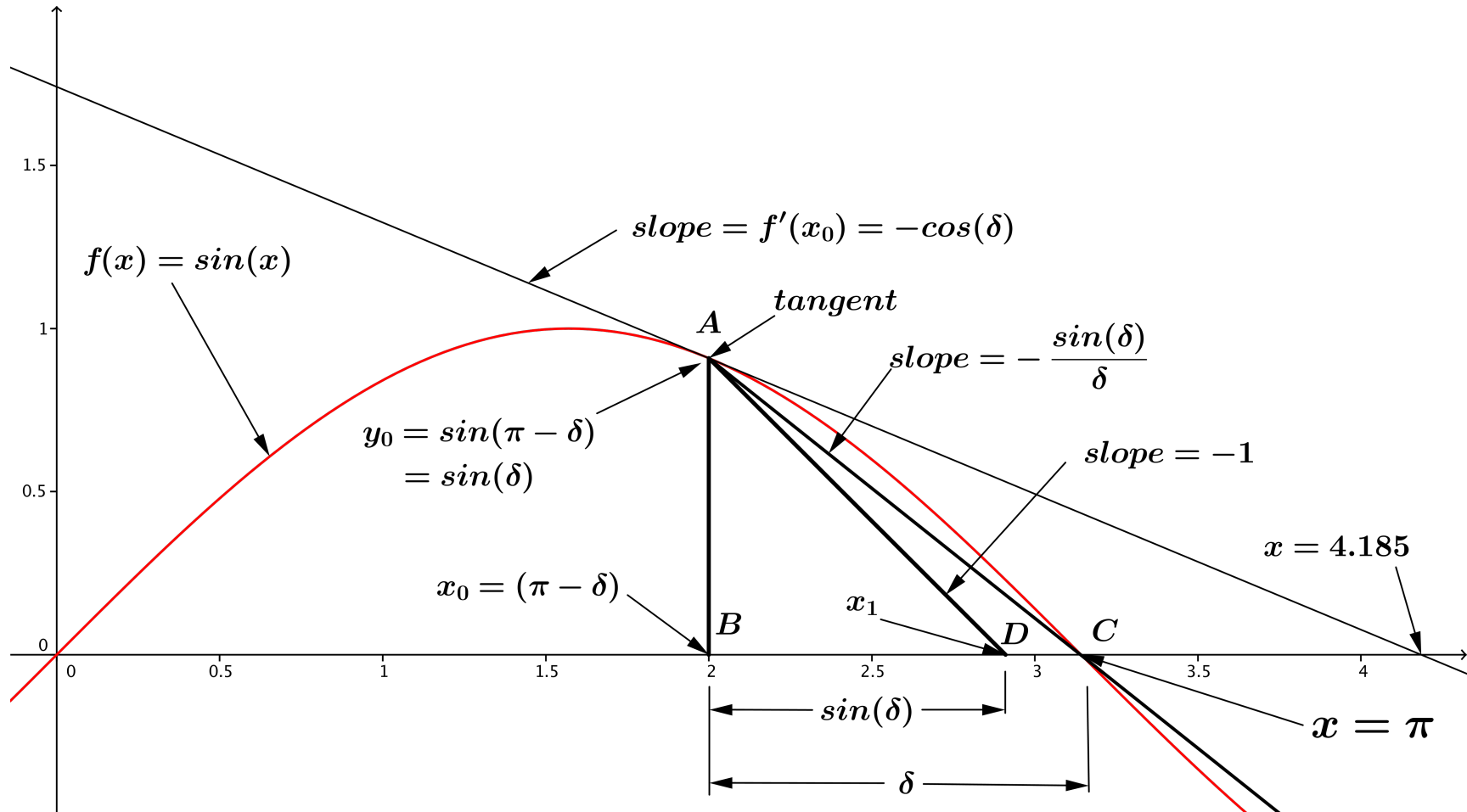
Rapid Convergence

$$x_{n+1} = x_n + \sin(x_n)$$

- $x_0 = 2.0$
- $x_1 = 2.90929743$
- $x_2 = 3.13950913$
- $x_3 = 3.14159265$

3.14159265358979323846264338327950288842

Try This on Your Calculator

For   π/2 ≤ x ≤ 3π/2

π = x + arcsin(sin(x))

choose x = 2	choose x = 3	choose x = 1.765
sin(x) = 0.909297	sin(x) = 0.14112	sin(x) = 0.981202
arcsin(0.909297) = 1.141593	arcsin(0.14112) = 0.141593	arcsin(0.981202) = 1.376593
2 + 1.141593 = π	3 + 0.141593 = π	1.765 + 1.376593 = π

y=arcsin(sin(x))