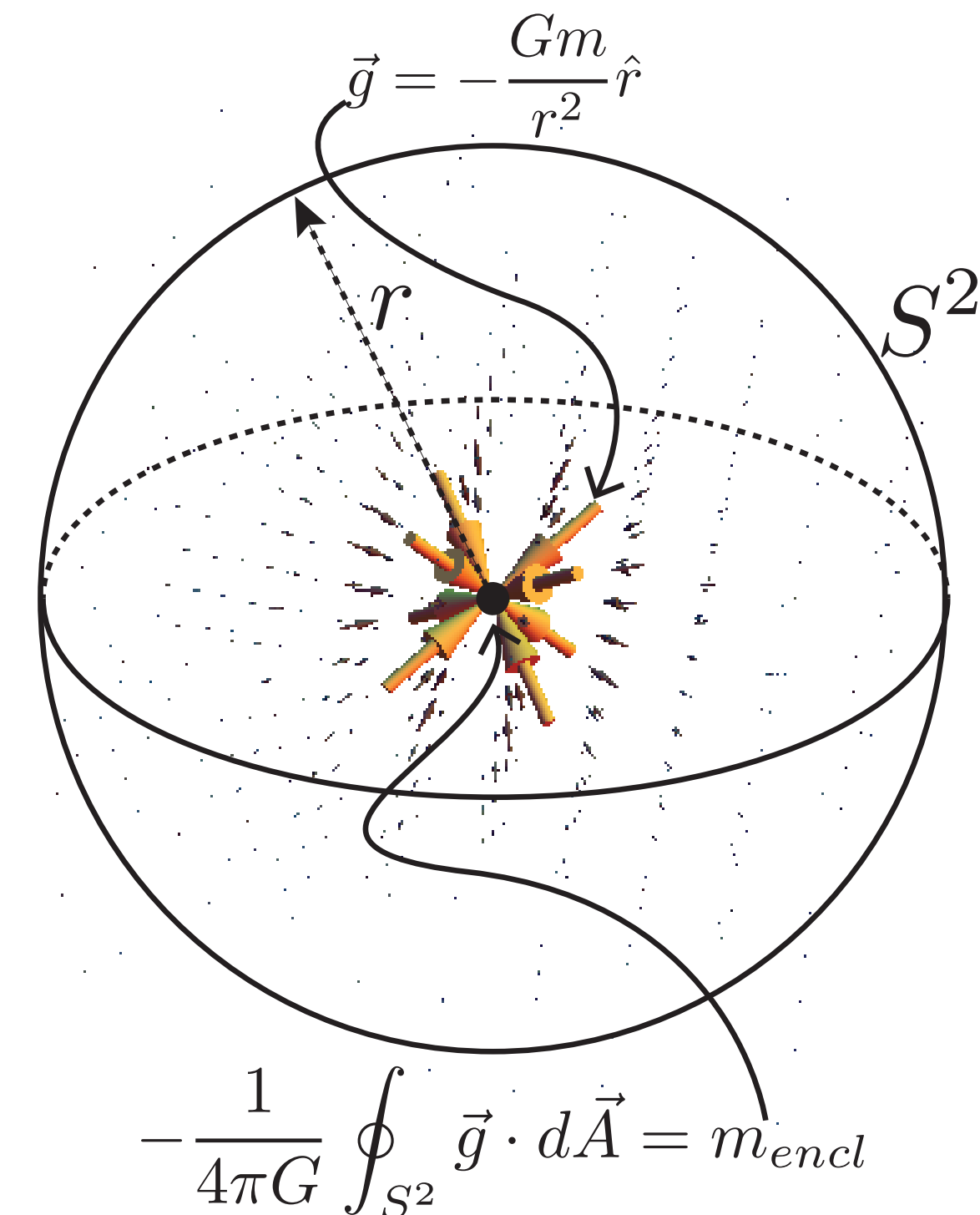
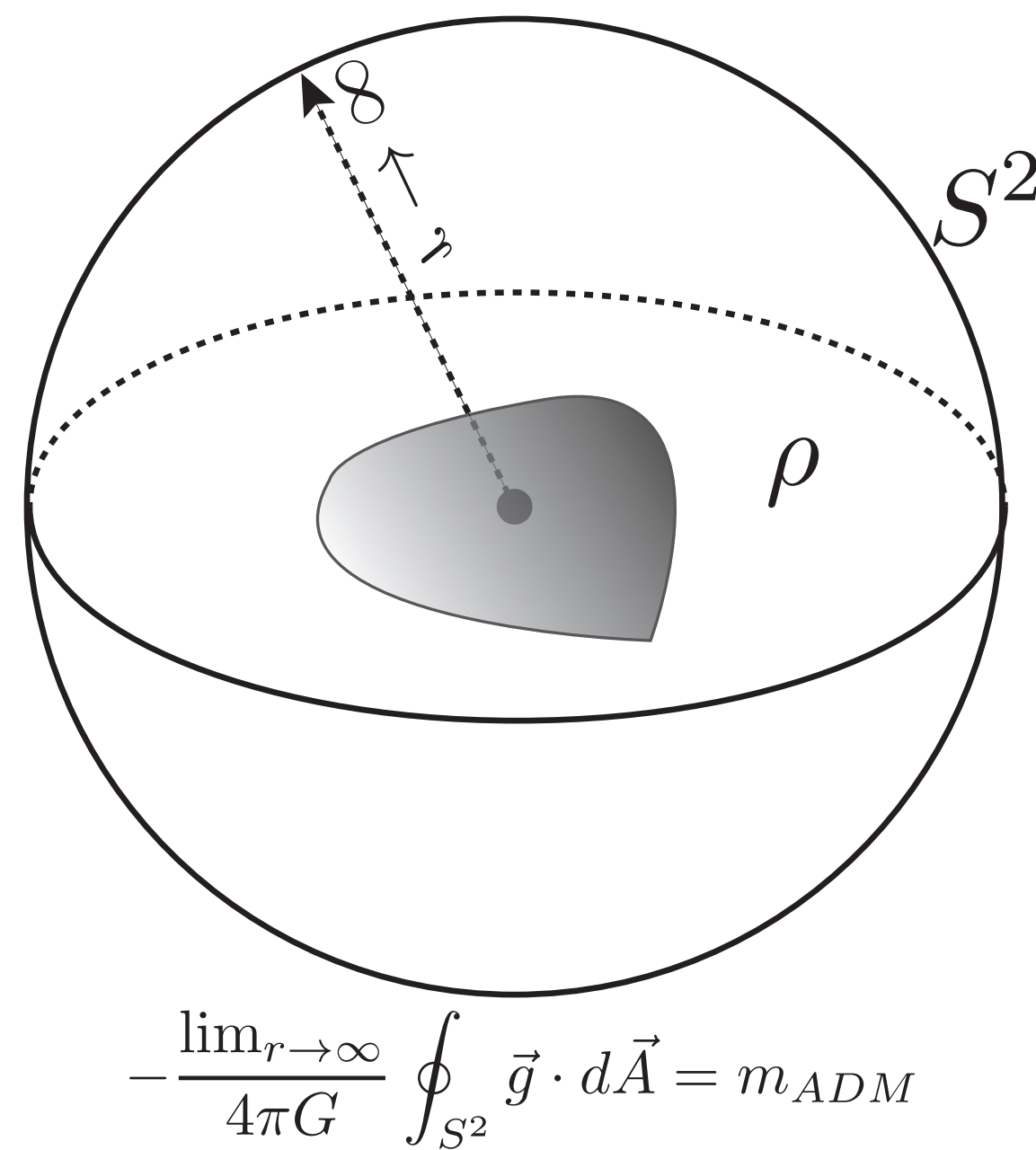


Holographic Scaling in Newtonian Gravity

Mass and Black Hole Physics

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A Question For Gen. Phys. Students

What is Mass?

...All the "Stuff" inside an object...

What's All The Stuff?

...Well, It's All the "Matter" ...

What's All The Matter?



$m_1 = 2kg$
 $\theta = 39^\circ$
 $\mu_s = .412$
 $\mu_k = .333$
 $m_2 = ?$
 $T = ?$

What's the minimum mass of block two and the resulting tension in the rope if the system is at rest?

$T = m_2g$ $N = m_1g \cos \theta$
 $F_f = \mu_s N$

Ch.9

What's the initial and final kinetic energy?

$\sum F_{m_2} = T$
 $\begin{cases} \sum F_{m_1x} = m_1g \sin \theta \\ \sum F_{m_1y} = N - m_1g \cos \theta \end{cases}$

$v_b = 750m/s, m_b = 0.3kg, M = 4kg, v_f = 52.3m/s, K_i = ?, K_f = ?$

$K_i = \frac{1}{2}m_b v_b^2 + \frac{1}{2}M v_M^2 = \frac{1}{2}(.3)(750)^2 + 0 = 8.44 \times 10^4 J$

$K_f = \frac{1}{2}(m_b + M) v_f^2 = \frac{1}{2}(.3 + 4)(52.3)^2 = 5.88 \times 10^3 J$

$K_i = 8.44 \times 10^4 J, K_f = 5.88 \times 10^3 J$
 $K_i > K_f$

OMG??? What Happened???

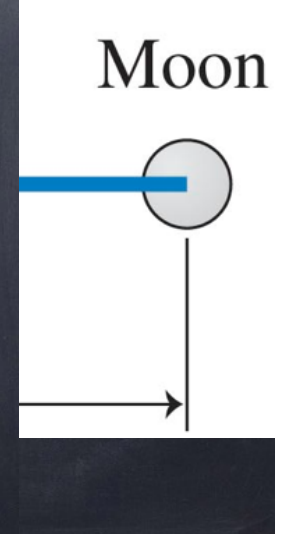
Newton's Law of Universal Gravitation

Moon's Orbit

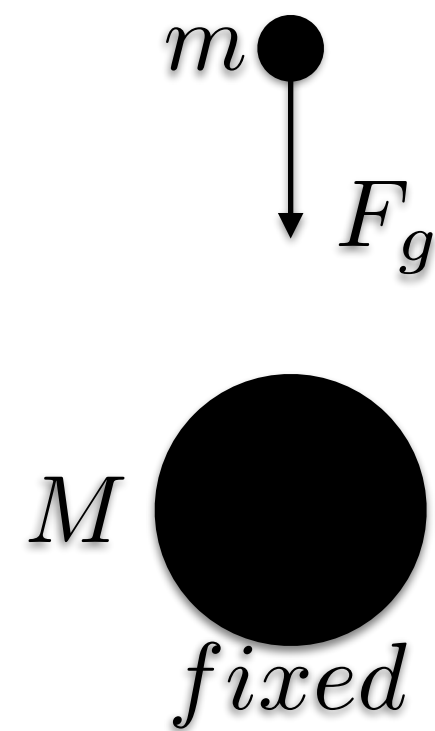
$GM_E = \frac{4\pi^2}{T^2} r^3$
 $T^2 = \frac{4\pi^2 r^3}{GM_E}$

Kepler's Second Law

$T_{Moon} = \left(\frac{4\pi^2 r^3}{GM_E} \right)^{1/2} = 2.45 \times 10^6 s$
 $T_{Moon} = \left(\frac{4\pi^2 r^3}{GM_E} \right)^{1/2} = 28.3 Days$



Let's Answer the Question



$$\begin{aligned} \sum F &= -F_g = ma \\ -\frac{GMm}{r^2} &= ma \\ -\frac{GM_g m_g}{r^2} &= m_I a \\ -\frac{GM_g \cancel{m_g}}{r^2} &= \cancel{m} a \\ g &= -\frac{GM_g}{r^2} \\ m_g &\stackrel{?}{=} m_I \end{aligned}$$

Seems to be true classically.

- Gravitational Mass

- Nöther's Theorem:

$$\int T_{00} dV = m_{ADM}$$

- ADM Procedure:

$$M_{ADM} = \frac{1}{16\pi G} \lim_{r \rightarrow \infty} \oint \delta^{ij} (\partial_i g_{jk} - \partial_k g_{ij}) n^k dS$$

- Inertial Mass

- Objects' property to resist change in its state of motion

- Weinberg Salam:

$$\mathcal{L}_h = |D_\mu h|^2 - \lambda \left(|h|^2 - \frac{v^2}{2} \right)^2$$

- Way to difficult for General Physics!

Gravielectric Duality

$$\vec{E} \cdot \vec{A}_{\odot} = \frac{q_{encl}}{\epsilon_0}$$

Gauss' Law

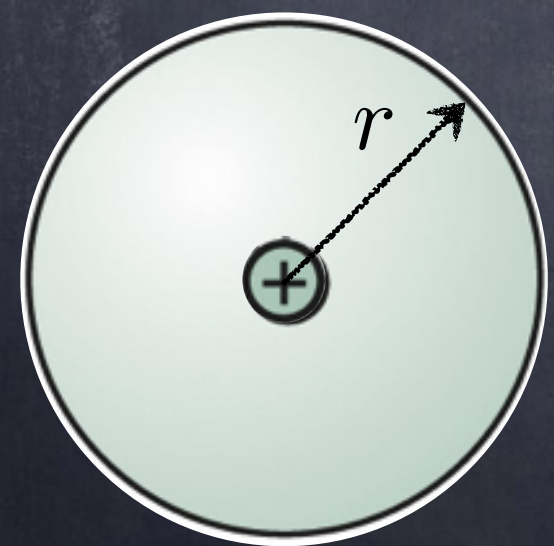
Using Gauss' Law

- All we know is Gauss' Law, find the electric field of a point charge 'Q', a radial distance 'r' away.

$$\vec{E} = ?$$

$$\vec{A} = 4\pi r^2 \hat{r}$$

$$q_{encl} = Q$$



Gaussian Surface

$$\vec{E} \cdot \vec{A}_{\odot} = \frac{q_{encl}}{\epsilon_0}$$

$$\vec{E} \cdot \vec{A} = A \vec{E} \cdot \hat{r}$$

$$\vec{E} \cdot \hat{r} \neq 0 \Rightarrow \vec{E} \cdot \hat{r} = E \hat{r} \cdot \hat{r}$$

$$\Rightarrow \vec{E} = E \hat{r}$$

$$\Rightarrow \vec{E} \vec{A} = EA (\hat{r} \cdot \hat{r}) = q_{encl} / \epsilon_0$$

$$EA = Q / \epsilon_0 \Rightarrow E4\pi r^2 = Q / \epsilon_0$$

$$E = \frac{Q}{4\pi\epsilon_0 r^2} \quad E = \frac{kQ}{r^2}$$

$$q \rightarrow m$$

$$\frac{1}{4\pi\epsilon_0} \rightarrow -4\pi G$$

$$\vec{E} \rightarrow \vec{g}$$

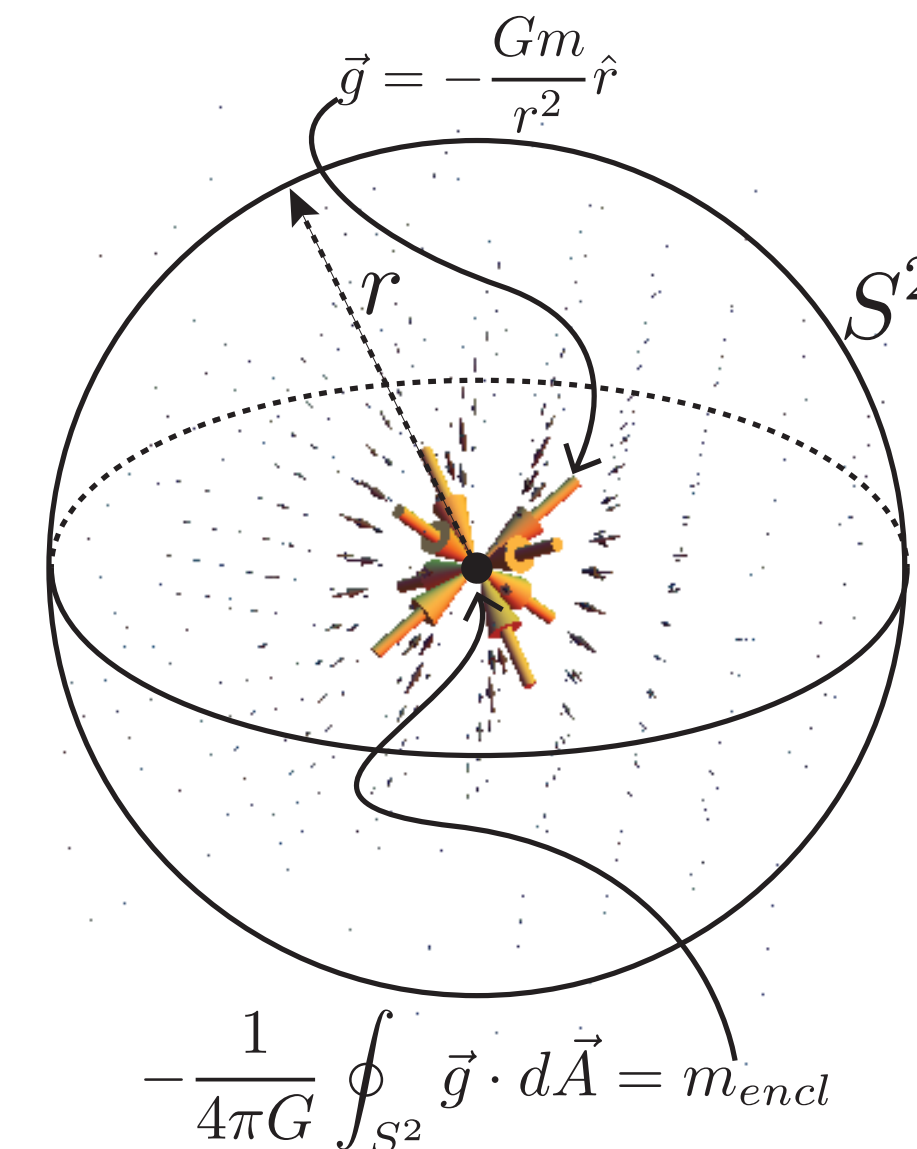
charge-mass

Coulomb-Newton Constant

Electric-Gravitational Field

$$\vec{g} \cdot \vec{A}_{\odot} = -4\pi G m_{encl}$$

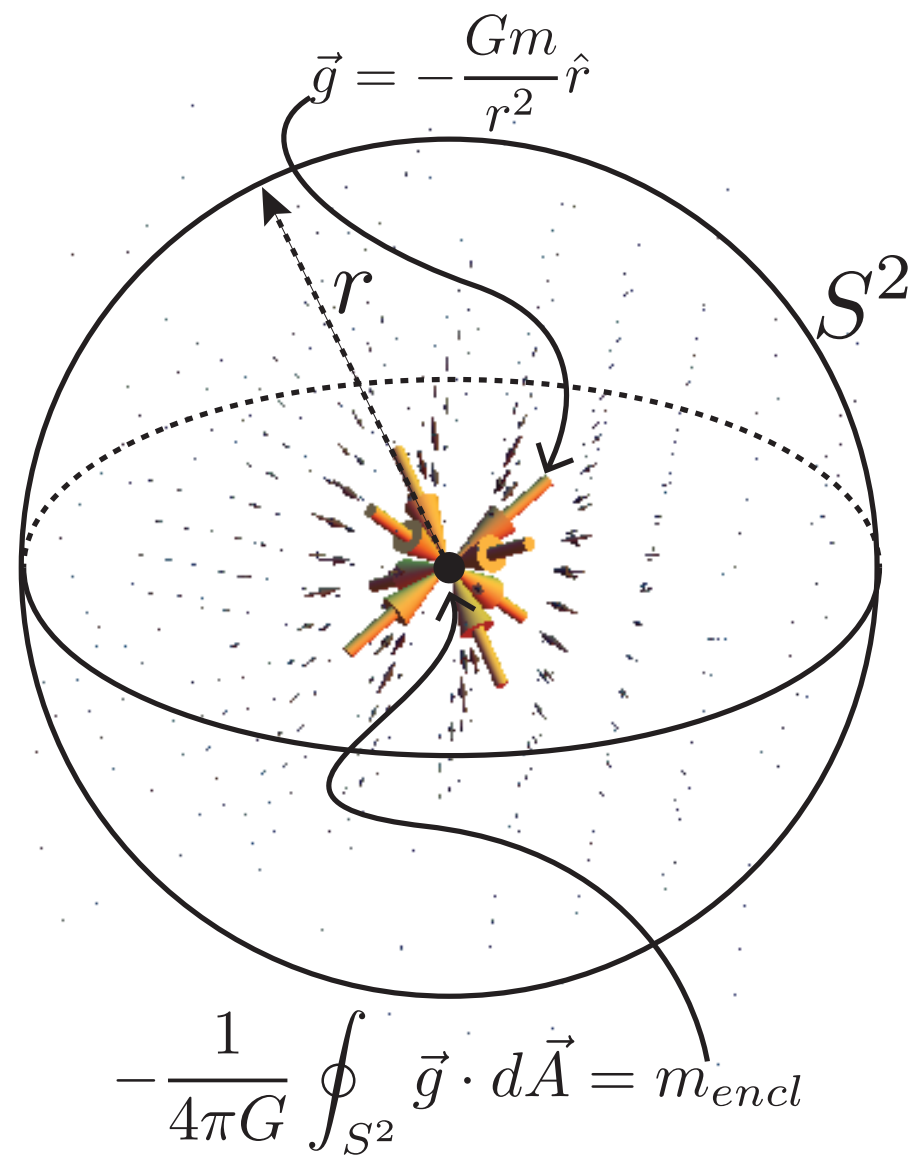
Gravitational Gauss' Law



Holography and ADM Mass

$$\vec{g} \cdot \vec{A}_\odot = -4\pi G m_{encl}$$

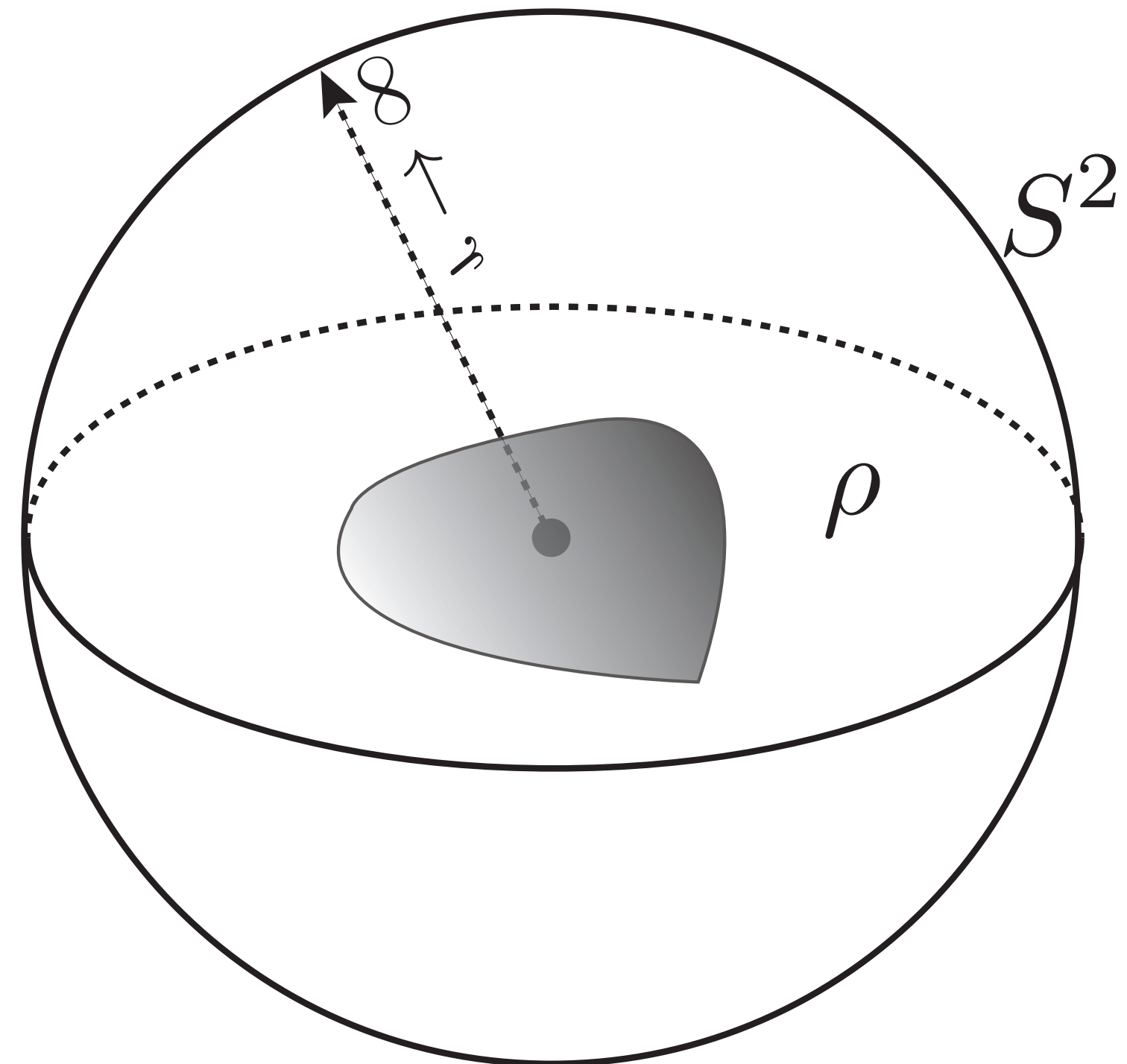
Gravitational Gauss' Law



$$m_{encl} = -\frac{1}{4\pi G} \vec{g} \cdot \vec{A}_\odot$$

Need to make sure S^2 encloses all the charge:
 $r \rightarrow \infty$

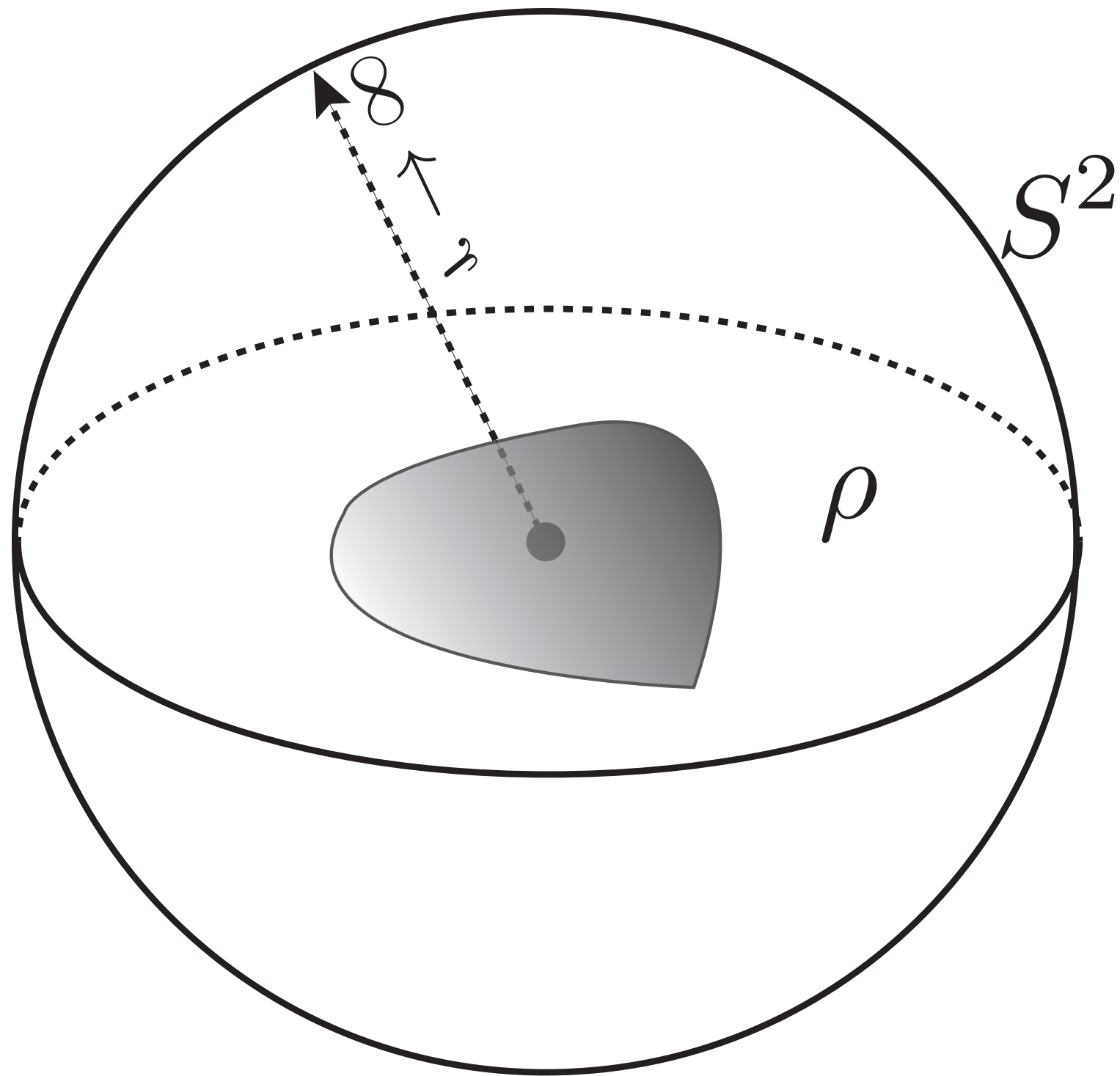
$$m = \lim_{r \rightarrow \infty} -\frac{1}{4\pi G} \vec{g} \cdot \vec{A}_\odot$$



$$-\frac{\lim_{r \rightarrow \infty}}{4\pi G} \oint_{S^2} \vec{g} \cdot d\vec{A} = m_{ADM}$$

Holography and ADM Mass

$$m_{ADM} = \lim_{r \rightarrow \infty} -\frac{1}{4\pi G} \vec{g} \cdot \vec{A}_\odot$$



$$-\frac{\lim_{r \rightarrow \infty}}{4\pi G} \oint_{S^2} \vec{g} \cdot d\vec{A} = m_{ADM}$$

$$ds_3^2 = g_{ij} dx^i dx^j = g(r) (dx^2 + dy^2 + dz^2),$$

where $g(r) = 1 - 2\Phi + \mathcal{O}(\frac{1}{r^2})$. Using this in M_{ADM} , we obtain:

$$\begin{aligned} \delta^{ij} (\partial_i g_{jk} - \partial_k g_{ij}) n^k &= 4\partial_i \Phi n^i \Rightarrow \\ M_{ADM} &= \frac{1}{16\pi G} \lim_{r \rightarrow \infty} \oint \delta^{ij} (\partial_i g_{jk} - \partial_k g_{ij}) n^k dS \\ &= \frac{1}{16\pi G} \lim_{r \rightarrow \infty} \oint 4\partial_i \Phi n^i dS \\ &= \boxed{\frac{-1}{4\pi G} \lim_{r \rightarrow \infty} \oint \vec{g} \cdot d\mathbf{S}} \end{aligned}$$

- Holographic Definition of Mass

- Mass is the gravitational flux through the surface of a 2-sphere at radial infinity.

Black Hole ADM Mass From Holography

$$\Phi = \begin{cases} -\frac{G(2Mr-Q^2)}{2r^2} & \text{R.N.} \\ -\frac{GMr}{r^2+a^2} & \text{Kerr} \\ -\frac{G(2Mr-Q^2)}{2(r^2+a^2)} & \text{K.N.} \end{cases}$$

$$\vec{g} = \begin{cases} -\frac{GM}{r^2}\hat{r} + \frac{GQ^2}{r^3}\hat{r} & \text{R.N.} \\ -\frac{GM(r^2-a^2)}{(r^2+a^2)^2}\hat{r} & \text{Kerr} \\ -\frac{G(r(Mr-Q^2)-Ma^2)}{(r^2+a^2)^2}\hat{r} & \text{K.N.} \end{cases}$$

$$m_{ADM} = \lim_{r \rightarrow \infty} -\frac{1}{4\pi G} \vec{g} \cdot \vec{A}_\odot$$

$$m_{ADM} = \frac{\lim_{r \rightarrow \infty}}{4\pi G} \oint_{\partial V} \Gamma^i_{00} \delta^{ij} dA^j$$

$$\Gamma^r_{tt} = \begin{cases} \frac{GM(r-2GM)}{r^3} & \text{S.s.} \\ \frac{GM(r^2+G(Q^2-2Mr))}{r^4} & \text{R.N.} \\ \frac{GM(a^2+r(r-2GM))(r^2-a^2 \cos^2 \theta)}{(r^2+a^2 \cos^2 \theta)^3} & \text{Kerr} \\ \frac{GM(a^2+r^2+G(Q^2-2GMr))(r^2-a^2 \cos^2 \theta)}{(r^2+a^2 \cos^2 \theta)^3} & \text{K.N.} \end{cases}$$

$$m_{ADM} = \frac{\lim_{r \rightarrow \infty}}{4\pi G} \int_0^{2\pi} \int_0^\pi \Gamma^r_{tt} r^2 \sin^2 \theta d\theta d\phi$$

$$= \begin{cases} M & \text{S.s.} \\ M & \text{R.N.} \\ M & \text{Kerr} \\ M & \text{K.N.} \end{cases}$$

Questions?

Special Thanks to:

- The Audience
- My Coauthors
- AAPT