# Holographic Scaling in Newtonian Gravity

## Mass and Black Hole Physics

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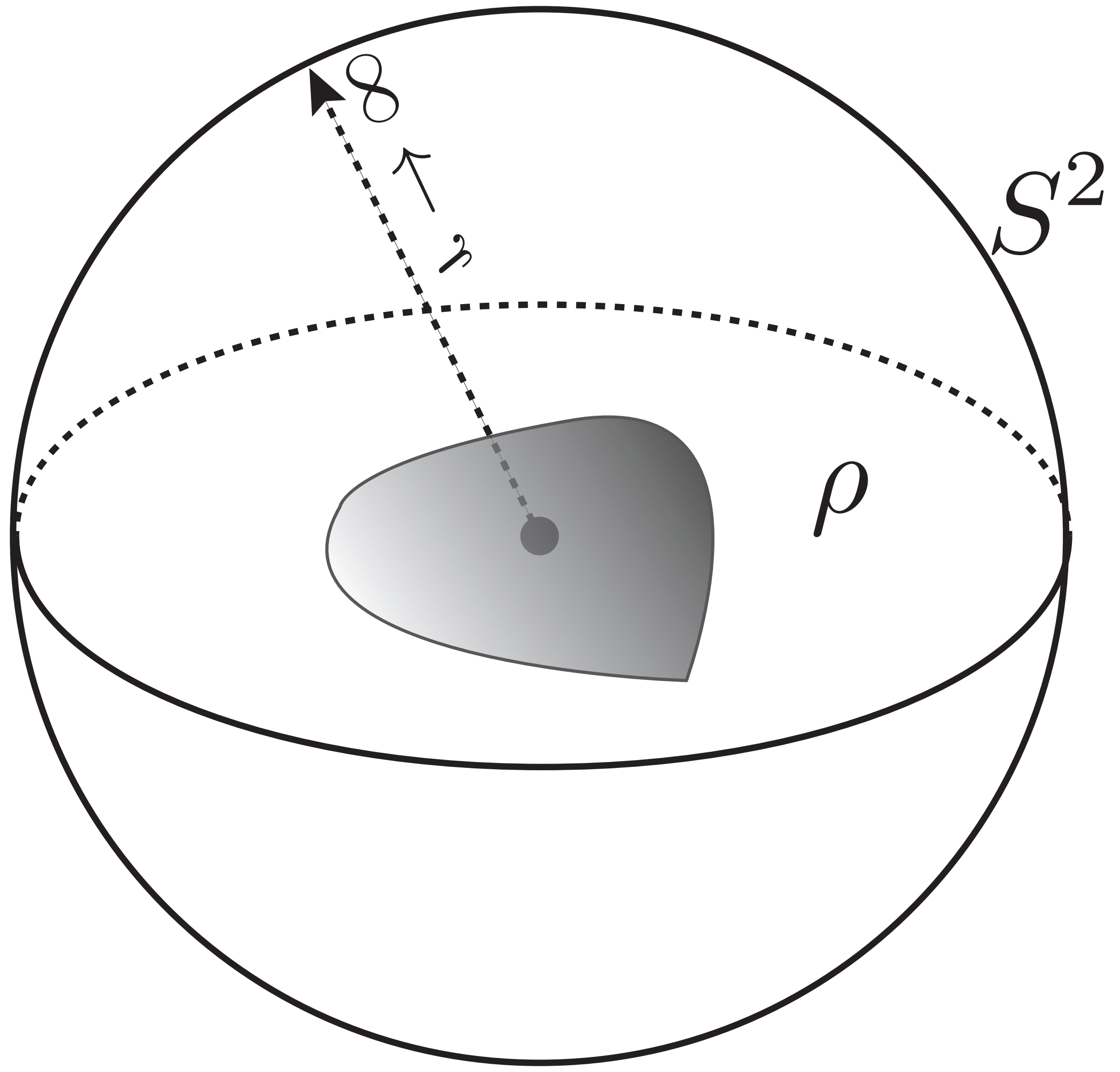
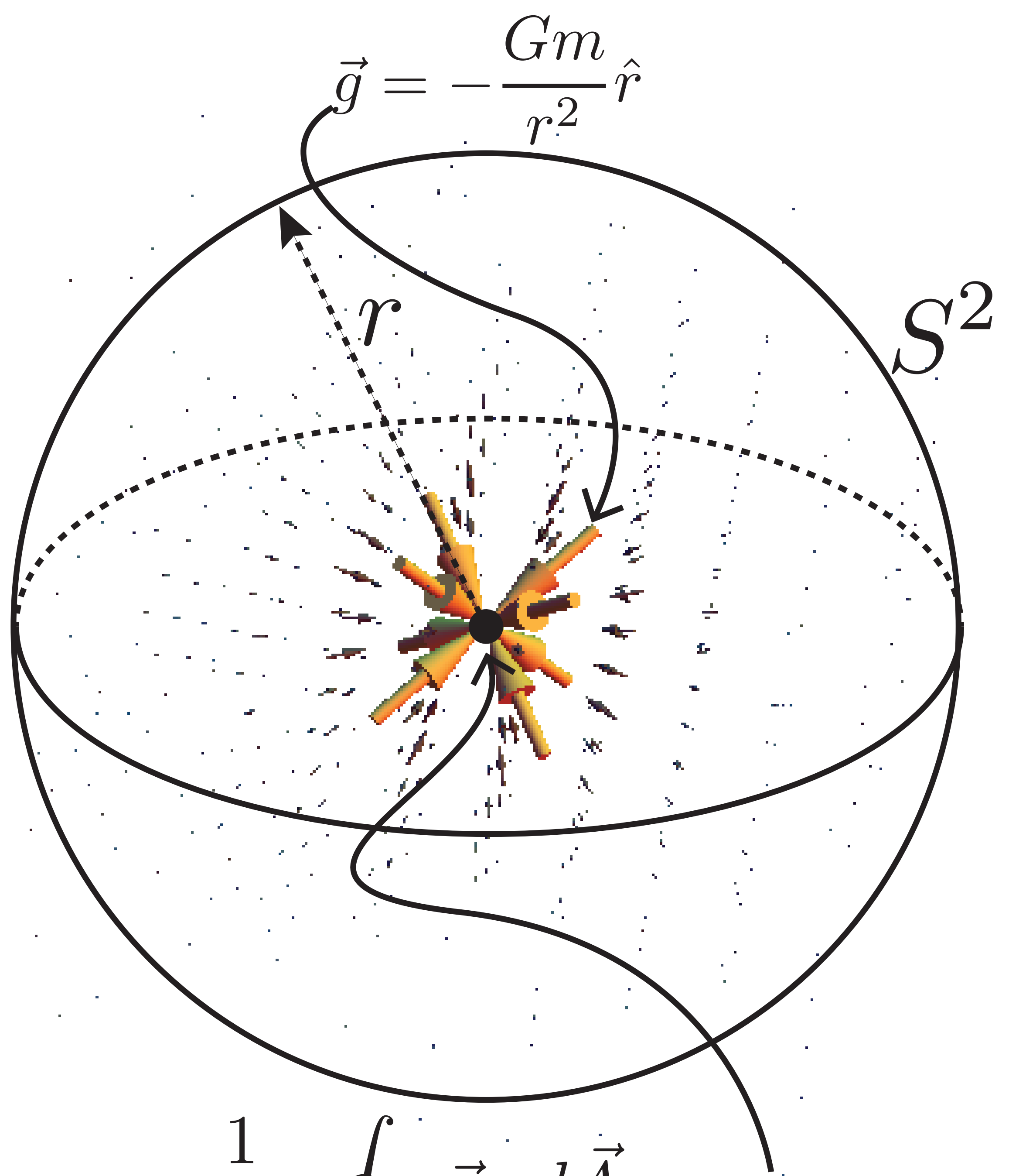



$$-\frac{\lim_{r\to\infty}}{4\pi G}\oint_{S^2}\vec{g}\cdot d\vec{A} = m_{ADM}$$




$$-\frac{1}{4\pi G}\oint_{S^2}\vec{g}\cdot d\vec{A} = m_{encl}$$

# A Question For Gen. Phys. Students

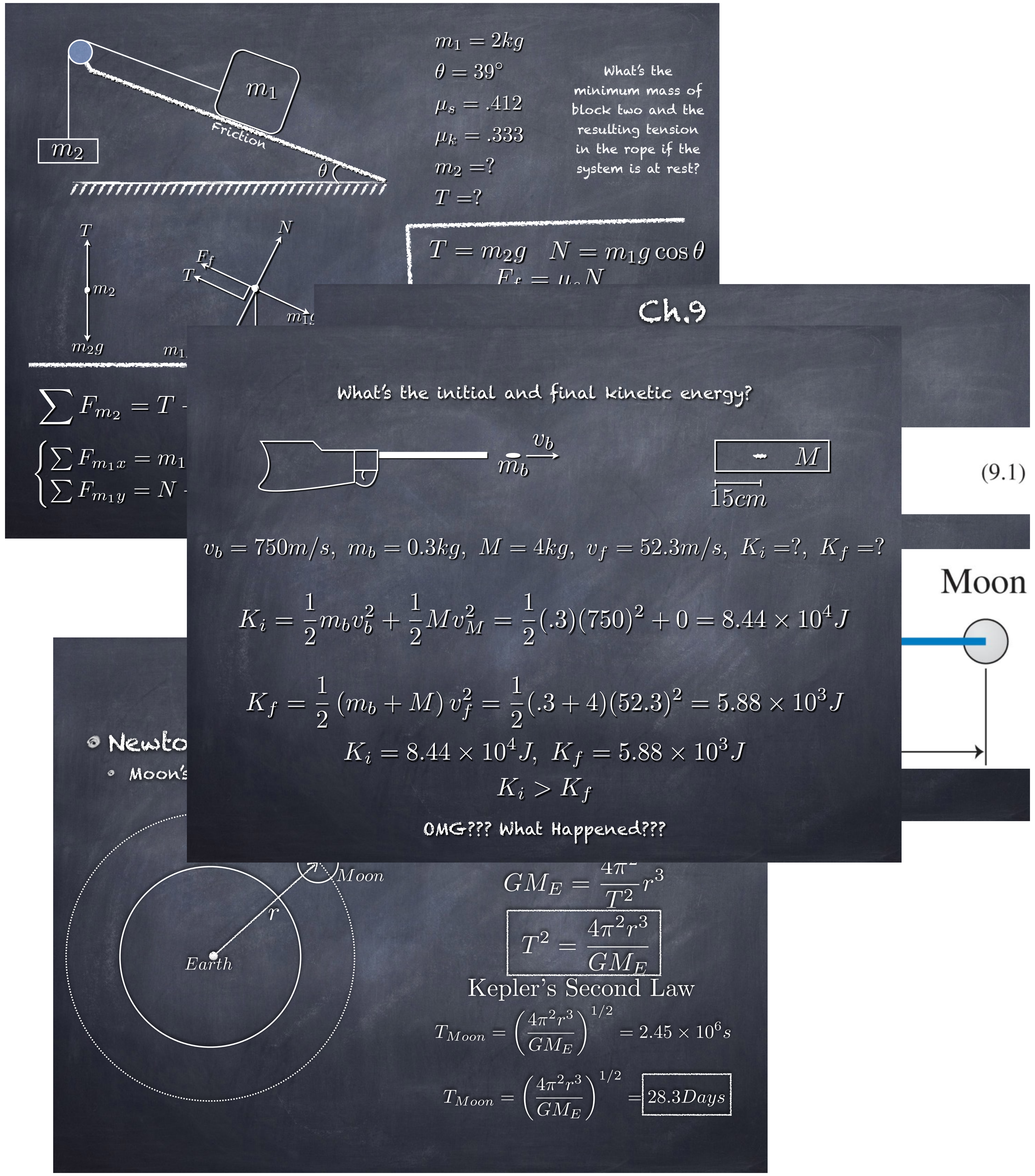
## What is Mass?

…All the "Stuff" inside an object…

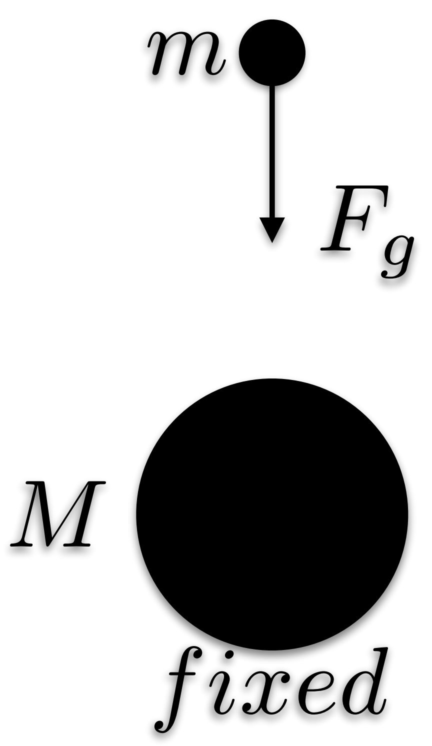
## What's All The Stuff?

…Well, It's All the "Matter"…

## What's All The Matter?



# Let's Answer the Question




$$\sum F = -F_g = ma$$

$$-\frac{GMm}{r^2} = ma$$

$$-\frac{GM_g m_g}{r^2} = m_I a$$

$$-\frac{GM_g \not{m}_g}{r^2} = \not{m}_I a$$

$$g = -\frac{GM_g}{r^2}$$

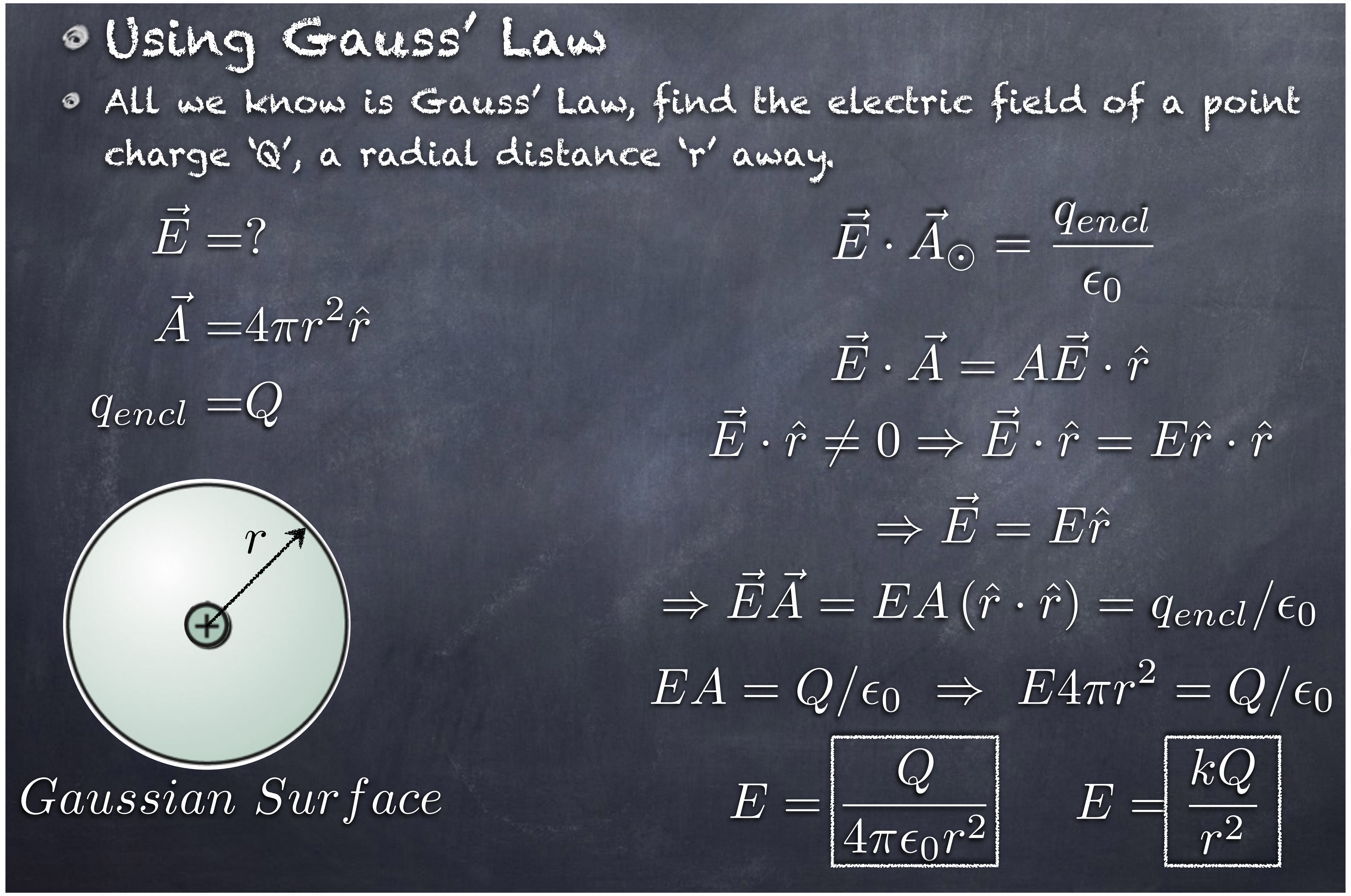
$$m_g \stackrel{?}{=} m_I$$

Seems to be true classically.

- Gravitational Mass
  - Nöther's Theorem: $\int T_{00} dV = m_{ADM}$
  - ADM Procedure: $M_{ADM} = \frac{1}{16\pi G} \lim_{r\to\infty} \oint \delta^{ij} \left(\partial_i g_{jk} - \partial_k g_{ij}\right) n^k dS$
- Inertial Mass
  - Objects' property to resist change in its state of motion
  - Weinberg Salam: $\mathcal{L}_h = |D_\mu h|^2 - \lambda \left(|h|^2 - \frac{v^2}{2}\right)^2$
- Way to difficult for General Physics!

# Gravielectric Duality

$$\vec{E} \cdot \vec{A}_{\odot} = \frac{q_{encl}}{\epsilon_0}$$ Gauss' Law

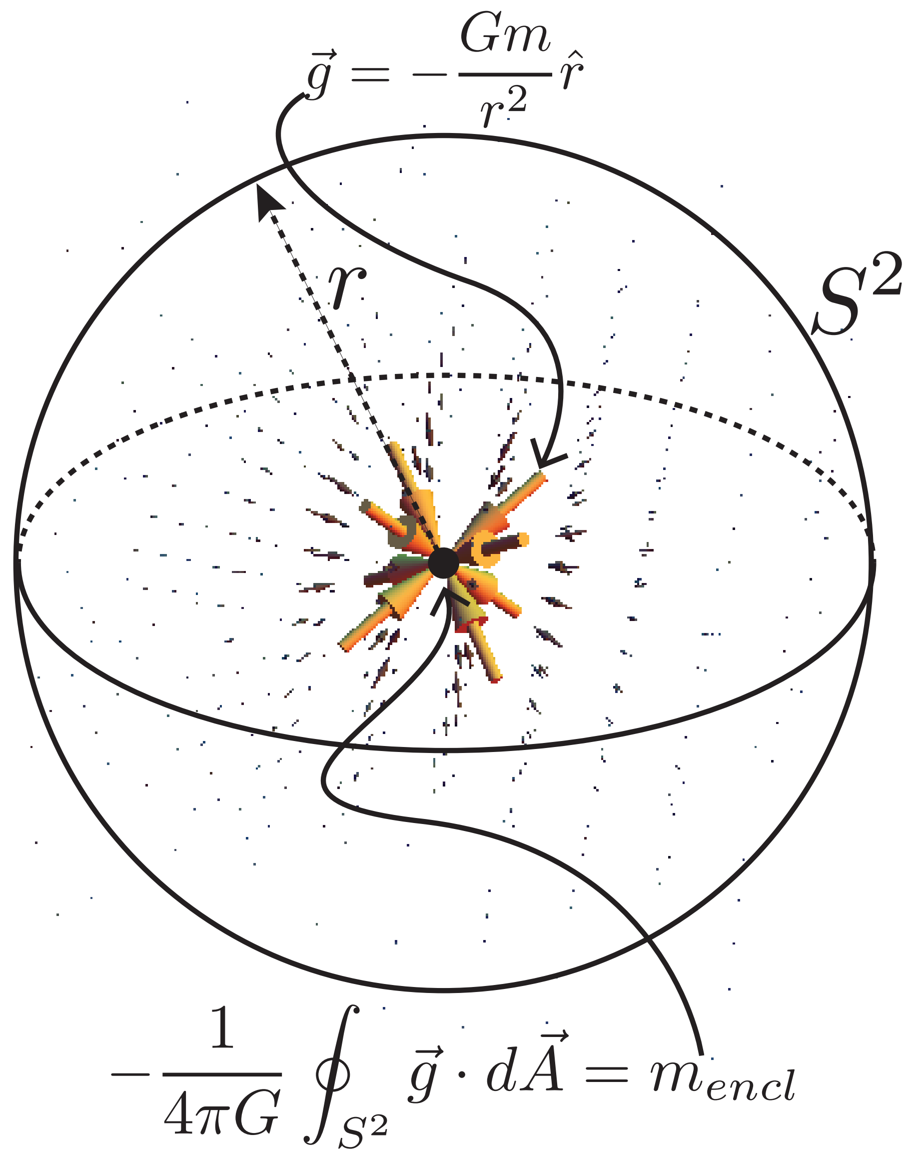



$q \to m$ charge-mass

$\frac{1}{4\pi\epsilon_0} \to -4\pi G$ Coulomb-Newton Constant

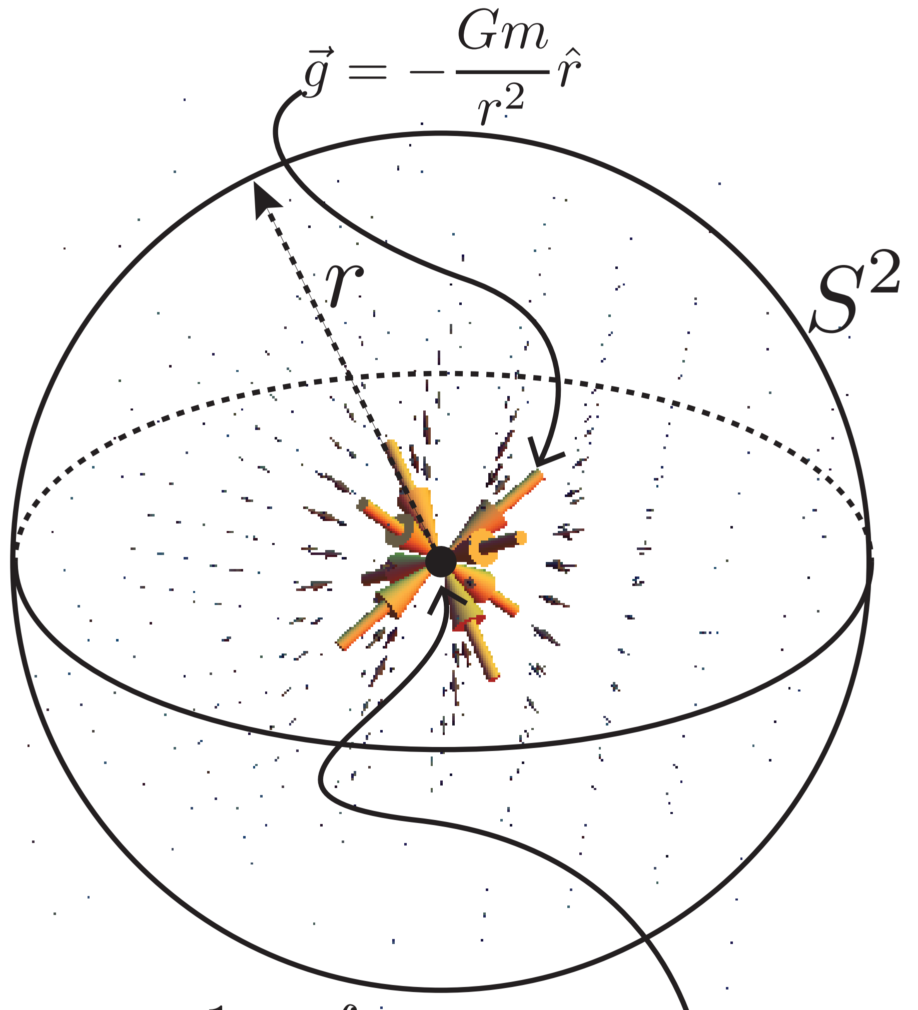
$\vec{E} \to \vec{g}$ Electric-Gravitational Field

$$\vec{g} \cdot \vec{A}_{\odot} = -4\pi G m_{encl}$$ Gravitational Gauss' Law



# Holography and ADM Mass

$\vec{g} \cdot \vec{A}_{\odot} = -4\pi G m_{encl}$ Gravitational Gauss' Law



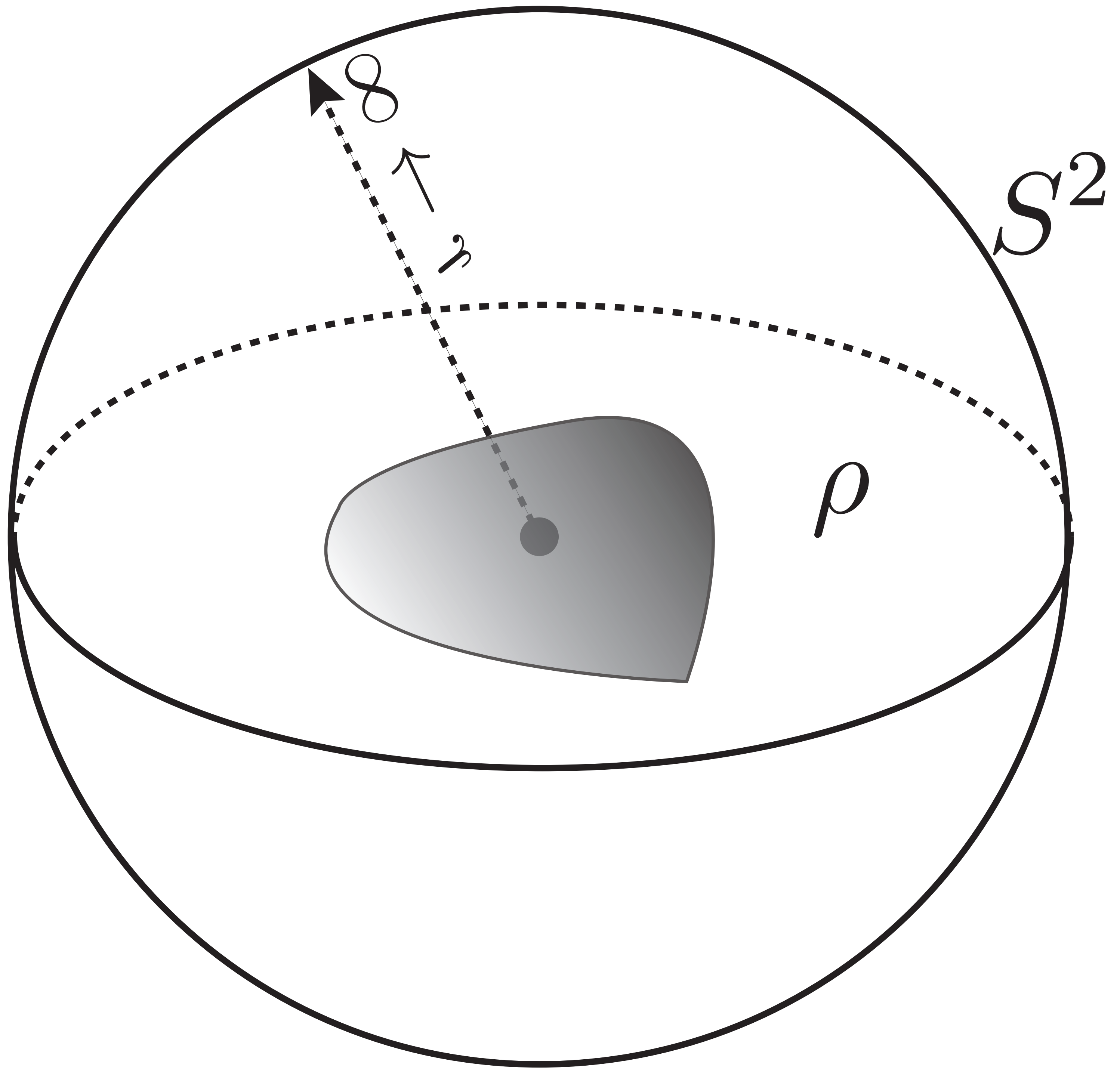

$$-\frac{1}{4\pi G}\oint_{S^2} \vec{g} \cdot d\vec{A} = m_{encl}$$

$$m_{encl} = -\frac{1}{4\pi G}\vec{g} \cdot \vec{A}_{\odot}$$

Need to make sure $S^2$ encloses all the charge:

$r \to \infty$

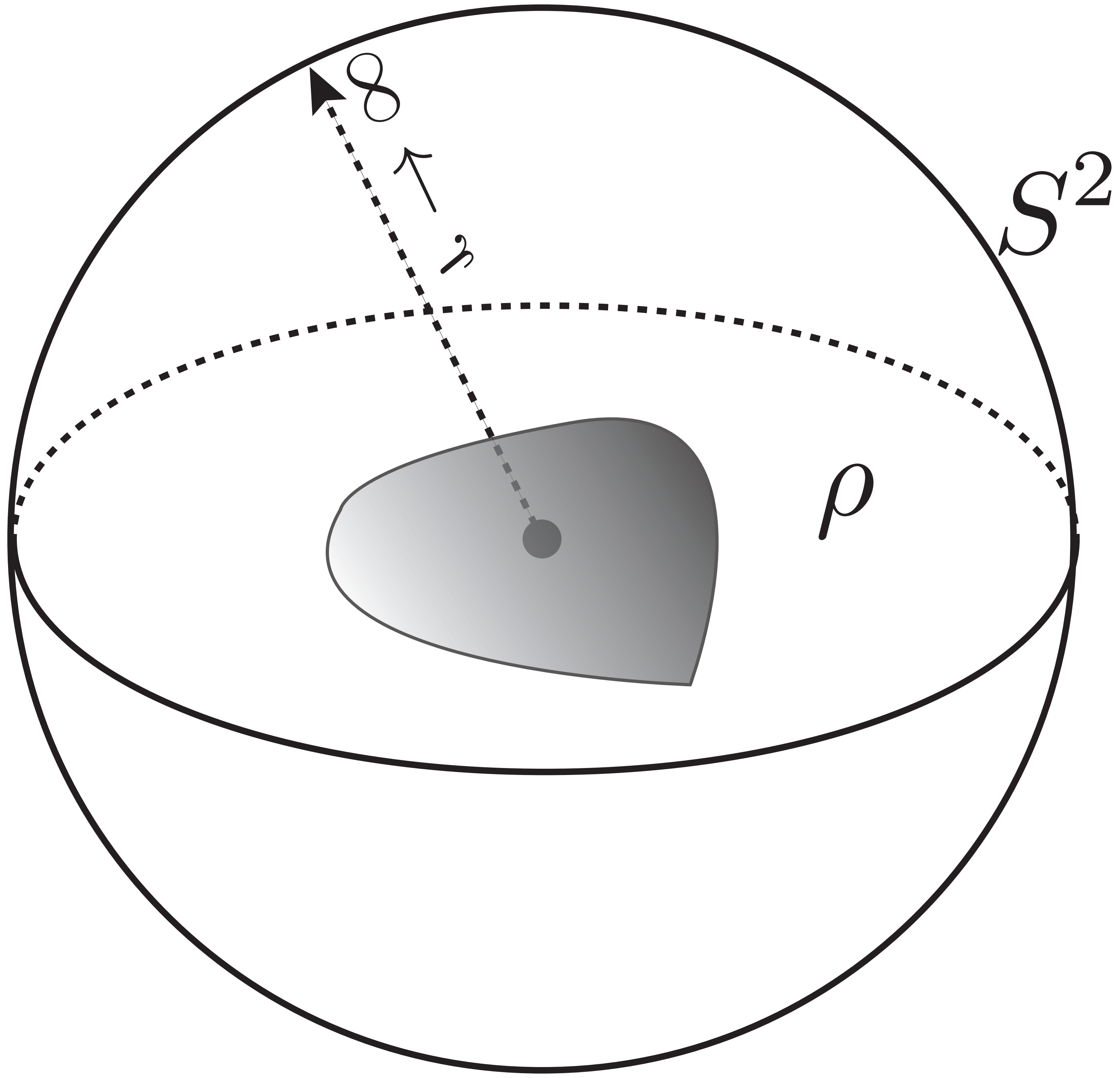
$$m = \lim_{r\to\infty} -\frac{1}{4\pi G}\vec{g} \cdot \vec{A}_{\odot}$$




$$-\frac{\lim_{r\to\infty}}{4\pi G}\oint_{S^2} \vec{g} \cdot d\vec{A} = m_{ADM}$$

# Holography and ADM Mass

$$m_{ADM} = \lim_{r\to\infty} -\frac{1}{4\pi G}\vec{g}\cdot\vec{A}_{\odot}$$




$$-\frac{\lim_{r\to\infty}}{4\pi G}\oint_{S^2}\vec{g}\cdot d\vec{A} = m_{ADM}$$

$$ds_3^2 = g_{ij}dx^i dx^j = g(r)\left(dx^2 + dy^2 + dz^2\right),$$

where $g(r) = 1 - 2\Phi + \mathcal{O}\left(\frac{1}{r^2}\right)$. Using this in $M_{ADM}$, we obtain:

$$\delta^{ij}\left(\partial_i g_{jk} - \partial_k g_{ij}\right)n^k = 4\partial_i \Phi n^i \Rightarrow$$

$$M_{ADM} = \frac{1}{16\pi G}\lim_{r\to\infty}\oint \delta^{ij}\left(\partial_i g_{jk} - \partial_k g_{ij}\right)n^k dS$$

$$= \frac{1}{16\pi G}\lim_{r\to\infty}\oint 4\partial_i \Phi n^i dS$$

$$= \boxed{\frac{-1}{4\pi G}\lim_{r\to\infty}\oint \vec{g}\cdot d\mathbf{S}}$$

- Holographic Definition of Mass
  - Mass is the gravitational flux through the surface of a 2-sphere at radial infinity.

# Black Hole ADM Mass From Holography

$$\Phi = \begin{cases} -\frac{G\left(2Mr-Q^2\right)}{2r^2} & \text{R.N.} \\ -\frac{GMr}{r^2+a^2} & \text{Kerr} \\ -\frac{G\left(2Mr-Q^2\right)}{2(r^2+a^2)} & \text{K.N.} \end{cases}$$

$$\vec{g} = \begin{cases} -\frac{GM}{r^2}\hat{r} + \frac{GQ^2}{r^3}\hat{r} & \text{R.N.} \\ -\frac{GM\left(r^2-a^2\right)}{(r^2+a^2)^2}\hat{r} & \text{Kerr} \\ -\frac{G\left(r\left(Mr-Q^2\right)-Ma^2\right)}{(r^2+a^2)^2}\hat{r} & \text{K.N.} \end{cases}$$

$$m_{ADM} = \lim_{r\to\infty} -\frac{1}{4\pi G}\vec{g}\cdot\vec{A}_{\odot}$$

$$m_{ADM} = \frac{\lim_{r\to\infty}}{4\pi G}\oint_{\partial V}\Gamma^{i}{}_{00}\delta^{ij}dA^{j}$$

$$\Gamma^{r}{}_{tt} = \begin{cases} \frac{GM(r-2GM)}{r^3} & \text{S.s.} \\ \frac{GM\left(r^2+G\left(Q^2-2Mr\right)\right)}{r^4} & \text{R.N.} \\ \frac{GM\left(a^2+r(r-2GM)\right)\left(r^2-a^2\cos^2\theta\right)}{(r^2+a^2\cos^2\theta)^3} & \text{Kerr} \\ \frac{GM\left(a^2+r^2+G\left(Q^2-2GMr\right)\right)\left(r^2-a^2\cos^2\theta\right)}{(r^2+a^2\cos^2\theta)^3} & \text{K.N.} \end{cases}$$

$$m_{ADM} = \frac{\lim_{r\to\infty}}{4\pi G}\int_0^{2\pi}\int_0^{\pi}\Gamma^{r}{}_{tt}r^2\sin^2\theta d\theta d\phi$$

$$= \begin{cases} M & \text{S.s.} \\ M & \text{R.N.} \\ M & \text{Kerr} \\ M & \text{K.N.} \end{cases}$$

# Questions?

Special Thanks to:

- The Audience
- My Coauthors
- AAPT