The Enticing World of Quantized Wheels

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Photograph by Michael Lisnet

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GEOMETRICAL QUANTIZATION : CARBON NANOTUBES

8th International Conference on the Science and Application of Nanotubes,
Ouro Preto, MG, 24-29 de junho, 2007

INCT
Nanomateriais
de carbono
(CNPq,Fapemig)

PREVIOUS APPROACHES

1) J. R. Regester , “ A Long and bumpy road,” Phys. Teach. 35, 232–233 (1997).

3) E. C. Valadares , Physics, Fun, and Beyond (Prentice Hall, Upper Saddle River, NJ, 2006), pp. 45–48.

4) L. Hall and S. Wagon , “ Roads and wheels,” Math. Mag. 65(5), 283–301 (1992).

5) N. H. Klein “ Square wheel,” Am. J. Phys. 61, 893–896 (1993).

6) L. Takacs “ The square wheel revisited,” Am. J. Phys. 62, 1141–1144 (1994).

Physics on sale – Brazilian style

Some years ago I visited the Cavendish Laboratory in Cambridge and was astonished by the simplicity of the devices that Maxwell and Thomson had used to make their fundamental discoveries. Back in Brazil, I decided to start a project to present physics to laypeople through demonstrations using simple materials that had been thrown away as refuse. I also encouraged the undergraduates on my introductory university physics courses – the maths, physics and chemistry teachers of the future – to produce simple demonstrations to explain the physics of everyday life.

The impact on the students was almost immediately visible. Initially, they were rather afraid of the new challenge, but after their first success, they refused to learn physics in any other way. Unfortunately, the Brazilian secondary-school system rarely focuses on the practical side of physics, and most students end up with a very poor understanding of how things work. Many of them therefore see no reason at all to learn science. In particular, physics – with its abstract formulae – is abhorred.

After two years I had gathered several gadgets produced by my students that used recycled and cheap materials, such as party balloons, rubber bands and marbles. The first test of whether the ideas would interest the wider world came when I had to “sell” my approach to schoolteachers and to colleagues from other university departments through a seminar entitled “The world as a demonstration room”. However, my audience was delighted to see how physics can be fun, and to discover that it is part of all our lives.

To convey my enthusiasm for physics I introduced fun concepts like the “square wheel” – two squares of wood, connected by an axle, that “rolled” smoothly when placed on a surface made from hyperbolic cosine sections. There was also a “magic can”, which was a specially designed cylinder that stored kinetic energy in an elastic band when rolled and reversed its motion after stopping, converting elastic energy into kinetic energy. And just to please soap-opera buffs, I placed an extractor fan in front of a TV monitor, the light from which acted as a stroboscope and caused the fan’s blades to appear to stop when they were rotated at just the right speed.

With the help of my students, I then felt encouraged to show these demonstrations to the general public in a local park in Belo Horizonte. Local newspapers announced the forthcoming event, which attracted the attention of both the public and the national media. A crew from a major TV station popped up and we became instant celebrities. Then, to our surprise, the marketing director of a major shopping mall in the city invited us to host a five-day exhibition in a temporarily vacant shop.

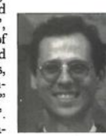
A shopping mall might appear at first sight to be an unusual place to demonstrate physics, but with the professional touch of the mall’s marketing department, we produced some highly creative and colourful publicity material. To entice the public to our exhibits, we used banners with titles like “How the weak becomes strong”, “The secret of the ballet dancer” and “The astronaut of the elevator”. In short, we renamed topics that would have otherwise come under rather mundane headings, such as “Structuring materials to improve their mechanical properties”, “Conservation of angular momentum” and the “Principle of equivalence of general relativity”.

We also started putting together related demonstrations to help the public’s understanding of particular topics. For example, we created a railway track from the strips of magnetized rubber gasket that normally line the inside of fridge doors. A wooden wagon with additional strips attached to its underside was able to levitate almost miraculously above the track. The principle of magnetic repulsion was then reinforced by displaying this exhibit next to an electric motor containing a small pair of magnets. And to illustrate vibration, we took a guitar made from an empty plastic chocolate box with a rubber band stretched across it and placed it next to a microphone connected to an old oscilloscope rescued from left-over equipment in our department.

It was like playing a magic flute, mesmerizing children and adults alike

When the first day of the exhibition arrived, we decided to place the “square wheel” in front of the shop to entice passers-by. Curious members of the public, who gathered perplexed in front of the shop, were then lured in by tempting invitations to “touch the phantom of a pig” – an illusion created using parabolic mirrors – and to “experience how a fly sees the shopping mall”. It was like playing a magic flute, mesmerizing children and adults alike. Even other shop-owners and the mall’s security guards took an interest in our displays. There was a good deal of panic in the first few days whenever something was about to break, but we quickly learned how to improvize and sort out apparently intractable difficulties. From naive amateurs we soon became almost professional dealers.

However, money was a major problem. It was not easy to squeeze even a few hundred dollars from the mall’s marketing department to help pay for the exhibits, as they wanted us to generate publicity for them for nothing. But these struggles behind the scenes are part of the game, and it was tremendous to see people from all walks of life enjoying physics, and laughing and smiling as they did so. The marketing department has even invited us back to organize future physics exhibitions under better conditions. Back at the Cavendish, Maxwell and Thomson would, I am sure, have been proud of us.



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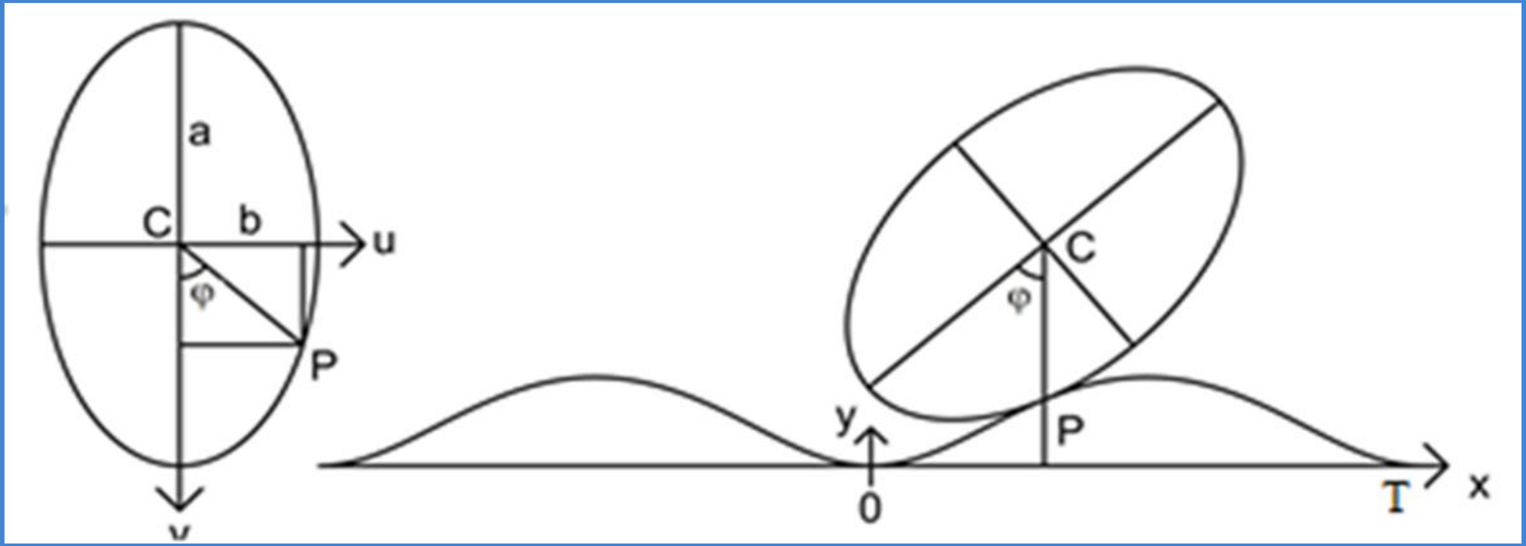
• Readers are invited to submit their own lateral thoughts for possible publication. Articles should be between 850 and 950 words long

E.C. Valadares, Phys. World 12, 64 (1999).

Basics:

- Periodic road: **y(x+T) = y(x)**

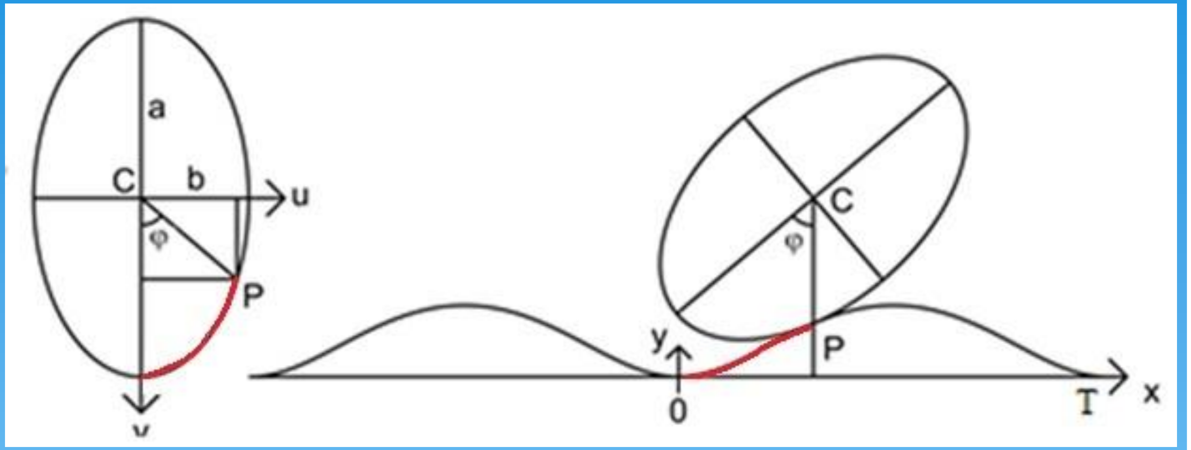
The center of mass of the wheel (C) must move along a horizontal trajectory and must remain directly above the contact point P (CP=R(x)), hence: **y(x)+ R(x) = c ≡ constant.**



Periodic roads and quantized wheels, Am.J.P. 84, 581(2016)

Coordinates of the wheel (P)

$$u(x)=R(x)\sin\varphi$$
$$v(x)=R(x)\cos\varphi$$



In the case of pure rotation (no slippage):

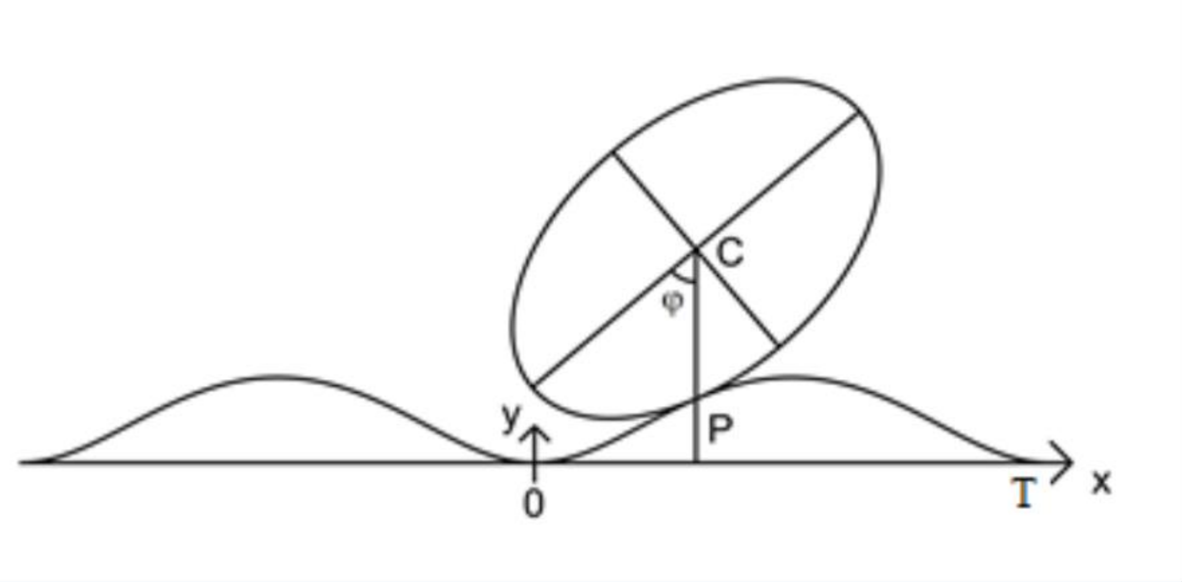
$$(ds)^2=(dx)^2+(dy)^2=(du)^2+(dv)^2$$

$$\frac{d\varphi}{dx}=\frac{1}{R(x)}=\frac{1}{c-y(x)}$$

$$\omega=\frac{d\varphi}{dx}\cdot\frac{dx}{dt}=\frac{v_{CM}}{R(x)}$$

Periodic roads and quantized wheels, Am.J.P. 84, 581(2016)

Translational symmetry must match rotational symmetry:



$$\Delta\varphi = |\varphi(T) - \varphi(0)| = \frac{2\pi}{\ell}$$

$\ell = 1, 2 \ldots$

$$\varphi = \int \frac{dx}{R(x)} = \int \frac{dx}{c - y(x)}$$

$c \longrightarrow c_\ell$ (quantization of the maximum radius of the wheel)

HOW DO YOU GET THE WHEELS IF YOU KNOW THE ROAD PROFILE y(x)?

The radius of the wheels: $R(x) = c_l - y(x)$ $(l=1,2,3\ldots)$

$$\Delta\varphi = |\varphi(T) - \varphi(0)| = \frac{2\pi}{\ell} = \int_0^T \frac{dx}{c_\ell - y(x)}$$

$\ell = 1,2\ldots$

$$\varphi(x) = \int_0^x \frac{dx}{R(x)}$$

With $R(x)$ and $\varphi(x)$ you determine all the wheels!

HOW DO YOU GET THE ROAD PROFILE y(x) FOR A GIVEN WHEEL $u(\varphi)=R(x)\sin\varphi$ and $v(\varphi)=R(x)\cos\varphi$?

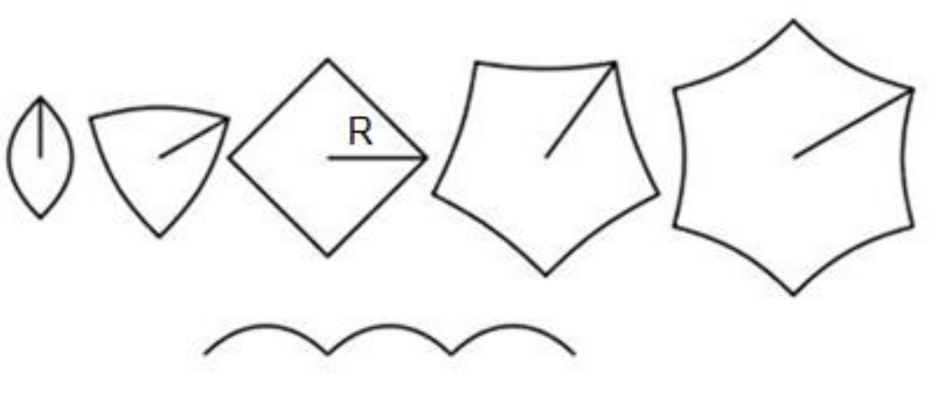
1) FIRST DETERMINE THE CENTER OF MASS OF THE WHEEL;

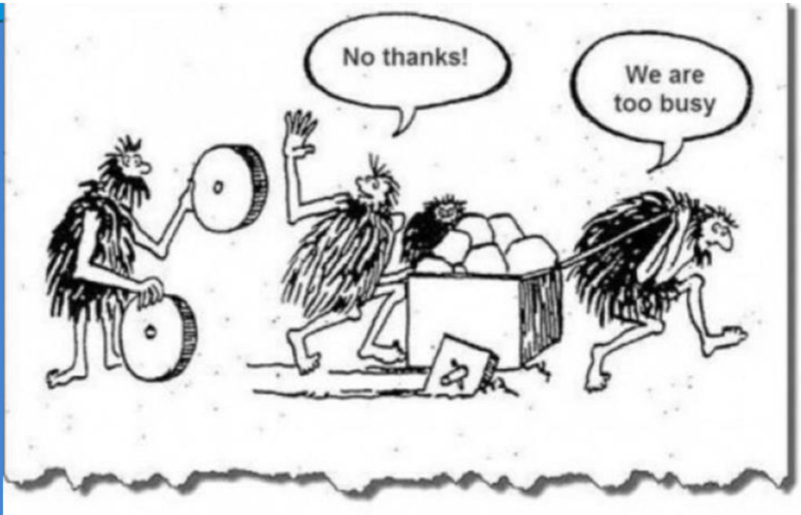
2) NEXT CALCULATE THE MAXIMUM RADIUS (THIS WILL BE THE CONSTANT C);

$$y(\varphi) = c - (u(\varphi)^2 + v(\varphi)^2)^{1/2}$$

$$x(\varphi) = \int_0^{\varphi} \left(\left(\frac{du}{d\varphi}\right)^2 + \left(\frac{dv}{d\varphi}\right)^2 - \left(\frac{dy}{d\varphi}\right)^2 \right)^{1/2} d\varphi$$

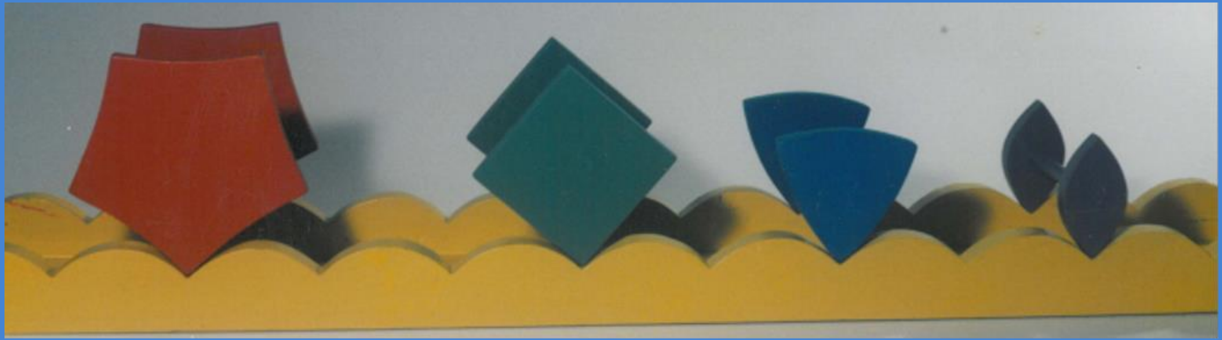
Am. J. Phys. 84 ,581 (2016)





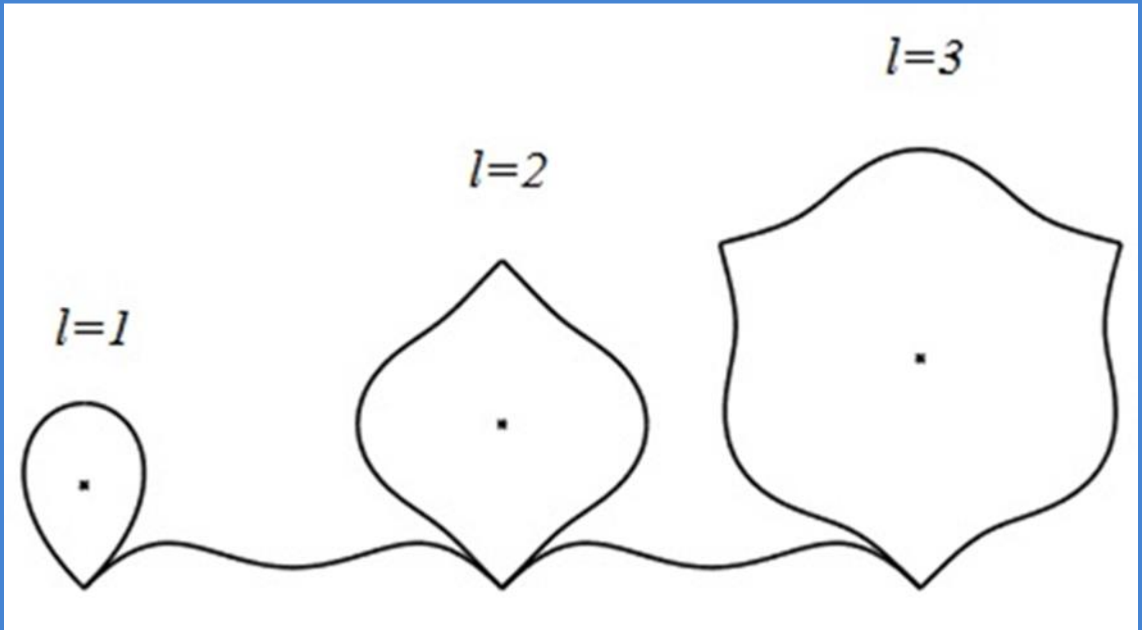
$$y(x) = R - R g_n \cosh\left(k_n - \frac{x}{R g_n}\right),\ n=4 \quad \text{(CATENARY)}$$

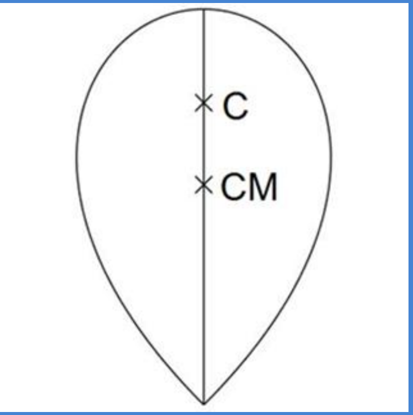
where $g_n = \cos(\frac{\pi}{n})$ e $k_n = \ln(\frac{1+\sin(\pi/n)}{g_n})$



Acknowledgements: Prof. Carlos Escobar (Fermilab)

What about l=1 (the ground state)?



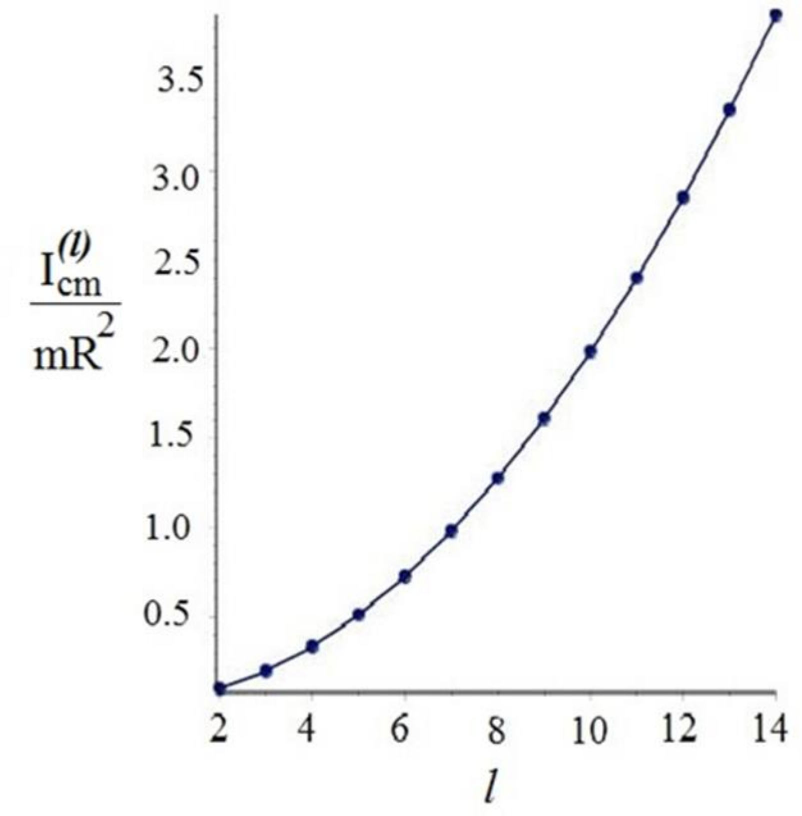


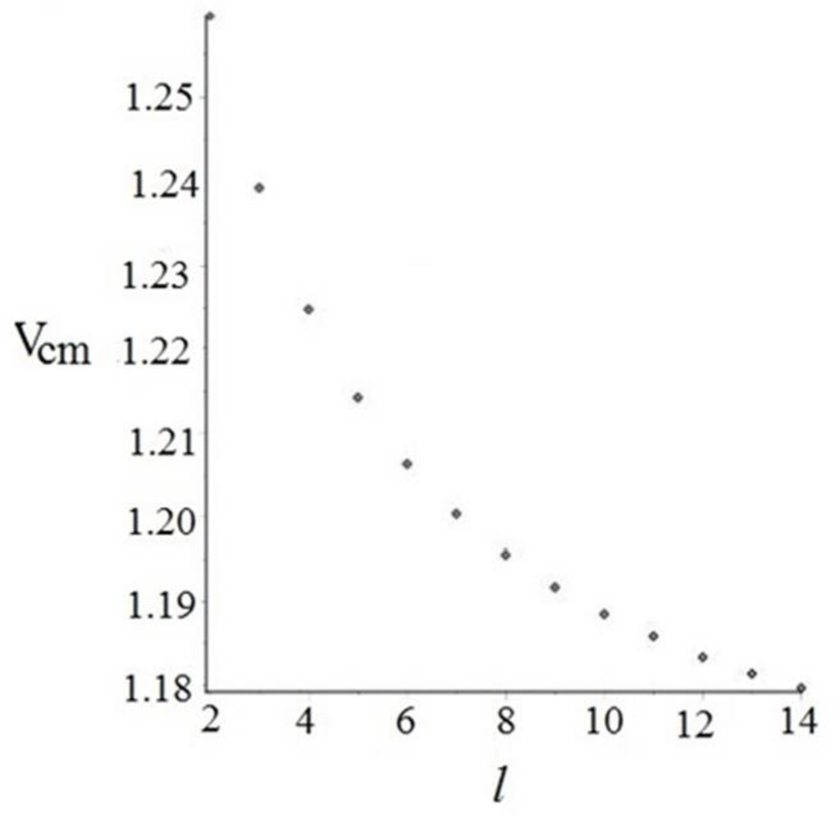
l=1 (“teardrop”), l=2 (“onion”), l=3 (“shield”)

DYNAMICS OF THE WHEELS (CORRESPONDENCE PRINCIPLE)

$$E_k = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I_{cm} \omega^2$$

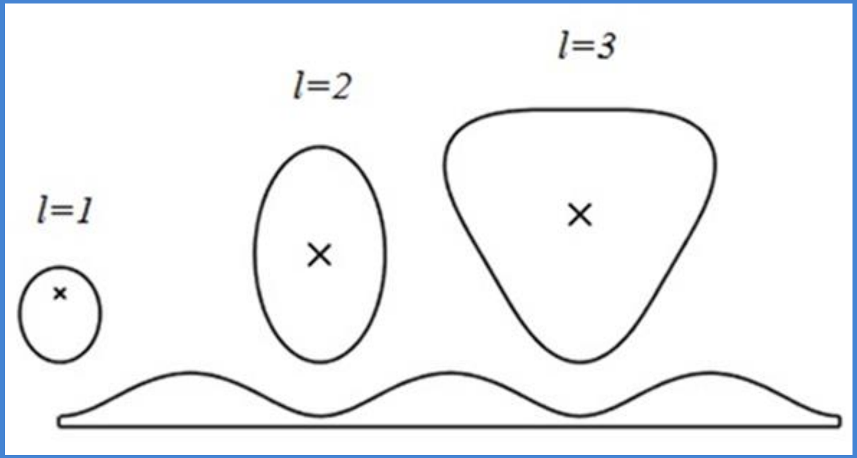
$$\omega = \frac{d\varphi}{dx} \cdot \frac{dx}{dt} = \frac{v_{CM}}{R(x)}$$

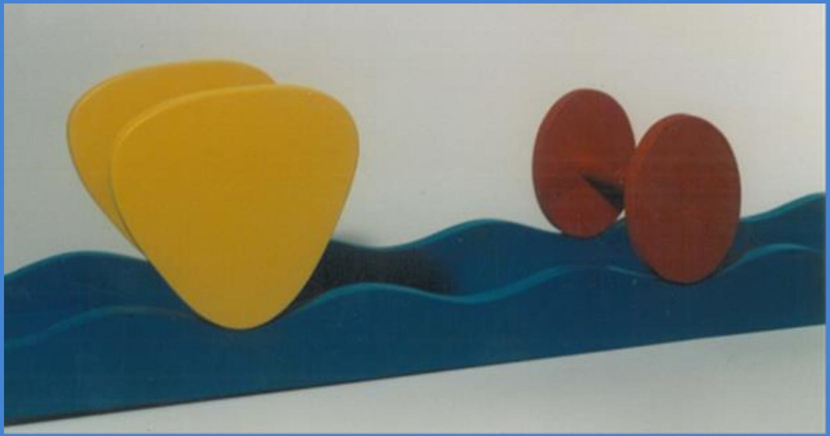




OTHER ROADS AND WHEELS

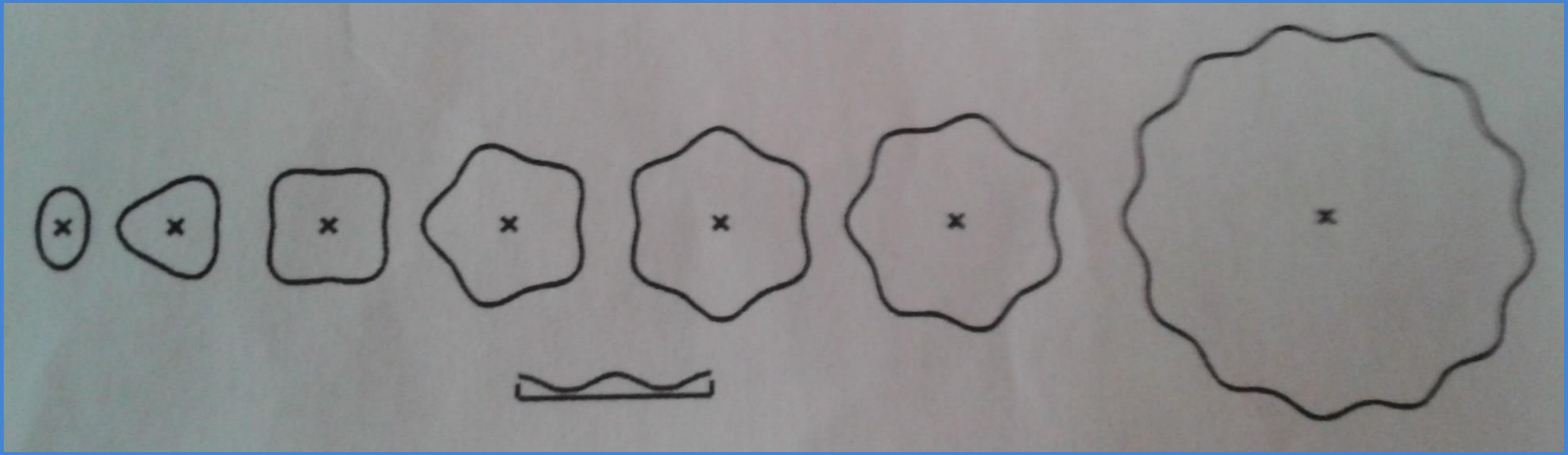
I- Elliptical wheels and beyond





Design

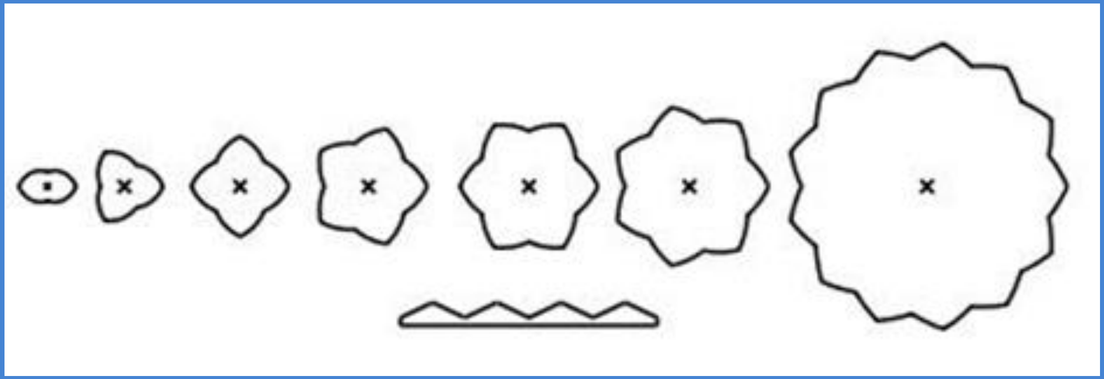
II. SINUSOIDAL ROADS



Generalization: roadbeds described in terms of a discrete Fourier series.

III. “THORNY” ROADS

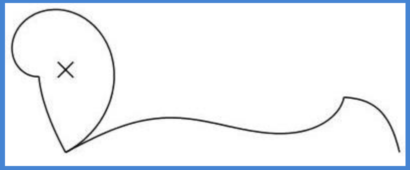
Saw-tooth (symmetric) profile

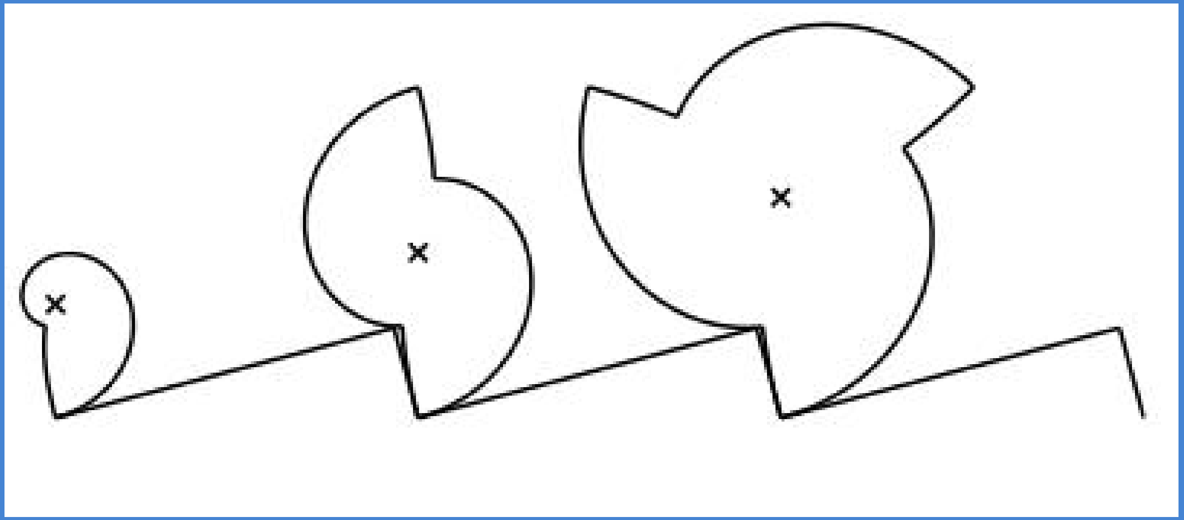


l =2, 3,45, 6 and 13

Tiles

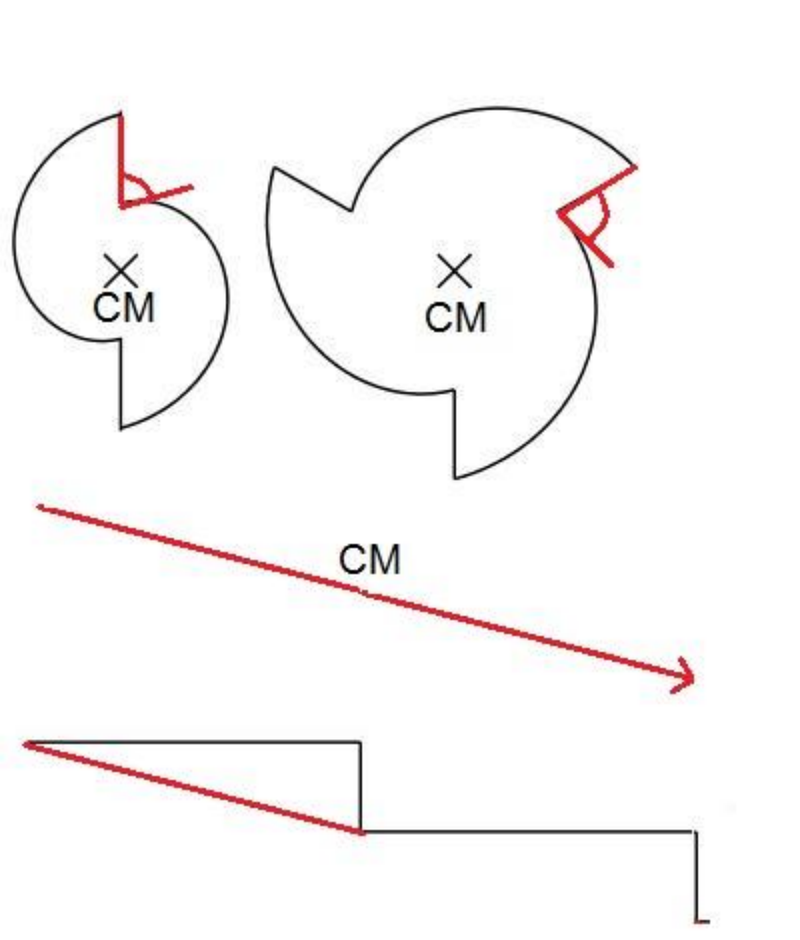
ASSYMETRIC SAW-TOOTH ROAD





Can we use these wheels to roll smoothly on stairs?

THE WHEELS FOR A STAIR DO NOT MATCH IT! THEY WILL CRASH INTO THE ROAD, AS THE VETEX OF A TRIANGULAR WHEEL OR A FIVE-POINTED STAR WILL DO! WHAT WE CAN DO ABOUT THEM?



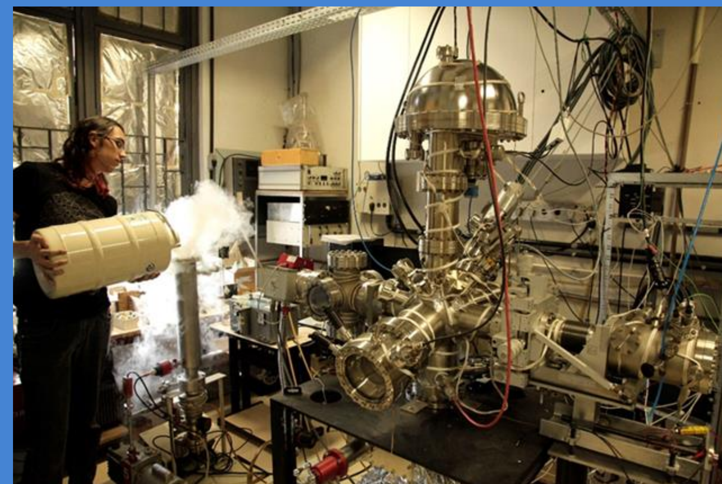
POSSIBLE APPLICATIONS:

- High speed printing presses and cams in machinery
- Use them in a press to create periodic roadbeds pulling a clay slab, for instance.

The wheels can fire imagination!

Can they inspire new approaches to research?

Innovation and Prototyping Lab (DF-UFMG)



Surface Physics Lab (DF-UFMG)

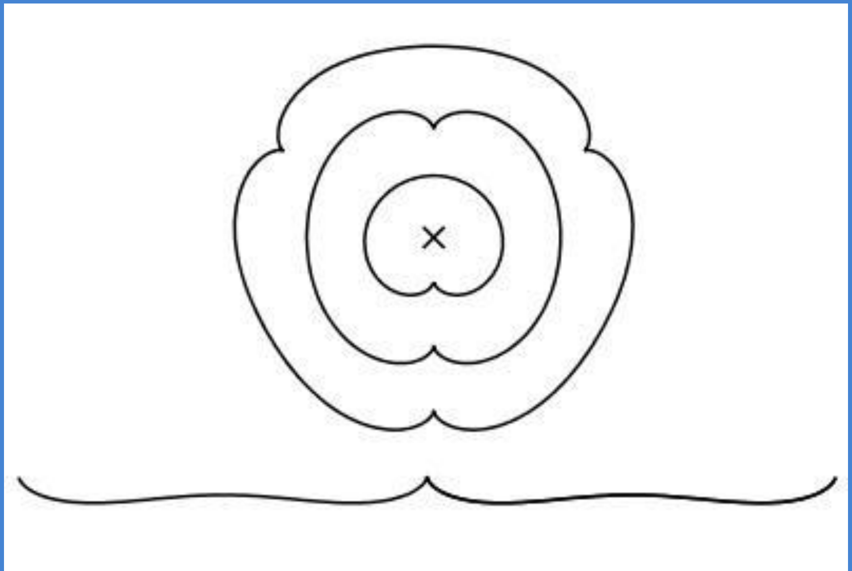
Nanospectroscopy Lab (DF-UFMG)

Biophysics Lab (DF-UFMG)

MY GRAAL: THE ROAD FOR THE HEART!

My starting point: the cardioid

The “excited states” of the road are conjoined hearts!



How about making a huge wheel that embraces all our hearts?

THANK YOU FOR YOUR ATTENTION!