Empowering Undergraduates with General Relativity

Stepping stones to build research skills

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While many calculations in general relativity require intimate knowledge of the theory, there are a large number of problems which can be reduced to calculations which exploit physics and skills that undergraduates possess or are in the process of developing. These types of problems provide excellent opportunities to reinforce early lessons in fundamental physics by giving students an opportunity to apply their knowledge and practice their skills against the exciting backdrop of a modern and evolving branch of physics. They also provide well defined problems for exercising and developing skills useful in later research endeavors (e.g., numerical programming). This poster describes the elements of undergraduate training which we have found can be tapped and applied to good effect in undergraduate GR research projects. Several real life examples which have led to publications in peer reviewed research journals are described to illustrate the basic philosophy we advocate.

What can undergraduates do in GR?

While at its most fundamental level, General Relativity can be a complex theory to describe mathematically or physically, the underlying precepts and building blocks are standard concepts that are taught to young physicists when they walk into their first freshman classes: *energy, momentum, particle trajectories, conservation laws, etc.*

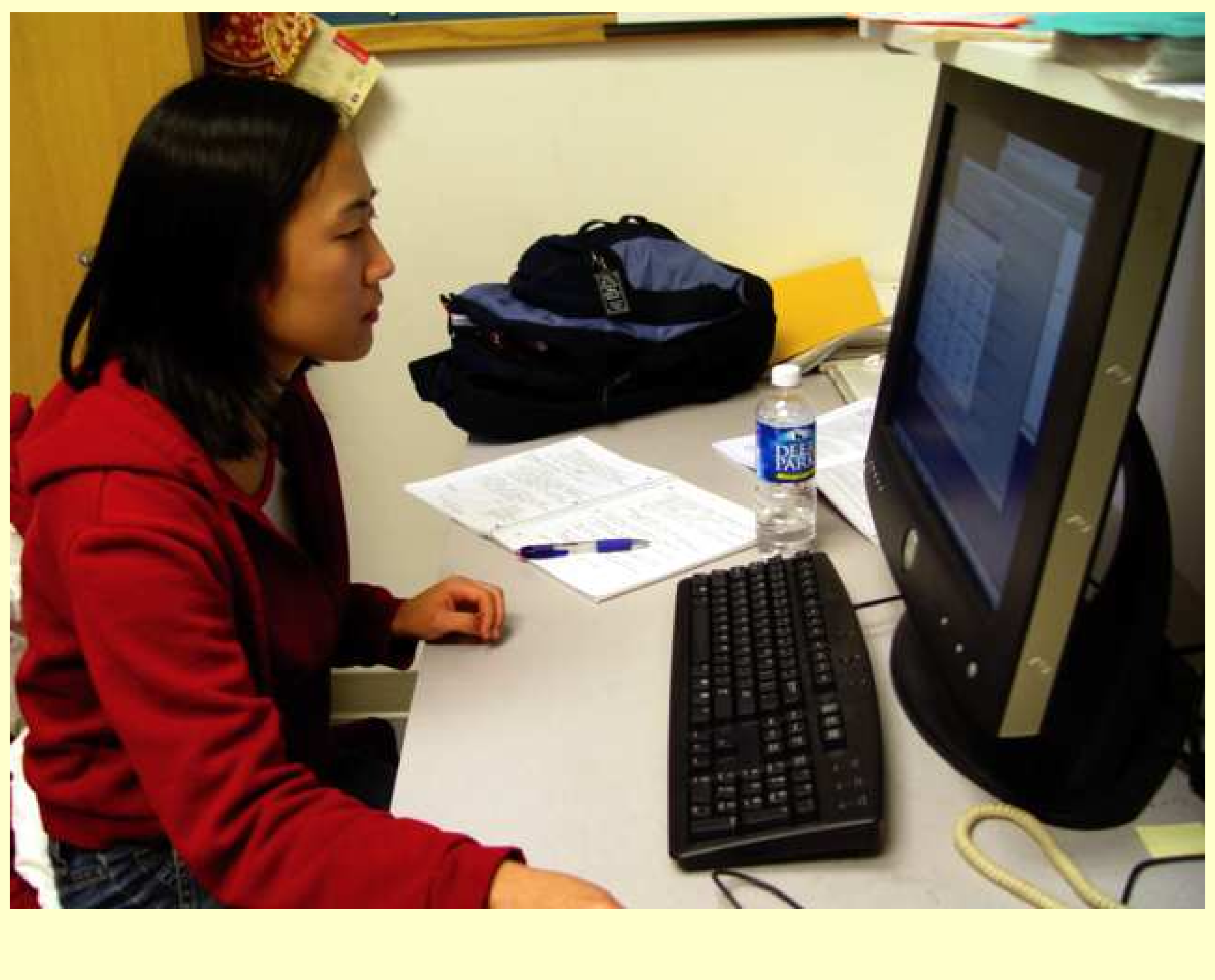
What do we look for when designing a research problem in general relativity for undergraduates?

- Problems with obvious milestones that build on one another. Individual milestones are less daunting targets than the ultimate finish line, and provide convenient "stopping points" when the inevitable tasks of undergraduate life divert your student's attention.
- Problems which can be characterized in terms of concepts the student is already familiar with from classic physics courses.
- Problems which are amenable to building skills that the student will be able to utilize in future research, regardless of the field. Good examples of skills to tap and develop include computer programming, applied mathematics (*e.g.* solving ODEs which characterize physical systems), and statistical analysis.

Working with undergrads in GR

There are many reasons why we like working with undergraduates:

- *Undergraduates are eager to be involved in research!* It's more exciting than the treatment of physics in textbooks, puts newfound knowledge to practical use, and helps them get a flavor of what "the research life" will be like if they continue on in their education.
- Undergraduates can make meaningful contributions in research, even in a field like general relativity! Problems which graduate students or senior researchers might not approach because they aren't "exciting" enough are often perfect for undergraduates to cut their teeth on.
- The advent of widespread computing resources (both in hardware and software) has made it possible for students to approach serious computations, and more often than not, emerge from the research project with a definite concrete result.



Some difficulties we have encountered:

- Computer programming is not always a standard part of an undergraduate curriculum. For some problems using algebraic systems such as *Mathematica* or *Matlab* will fill this gap. In other cases, programming languages must be learned as part of the research exercise.
- Some undergraduates flounder and don't tell you. Regular meetings can help prevent this, as can partners or junior researchers (grad students or postdocs) who aren't as intimidating when inevitable questions arise (e.g. *"I'm confused about these $G = c = 1$ units…"*).
- Some undergraduates don't have as much time to commit to research as they think! Intermediate milestones in research projects help provide short term goals which boost confidence when they are reached, and provide natural stopping points as exams approach.

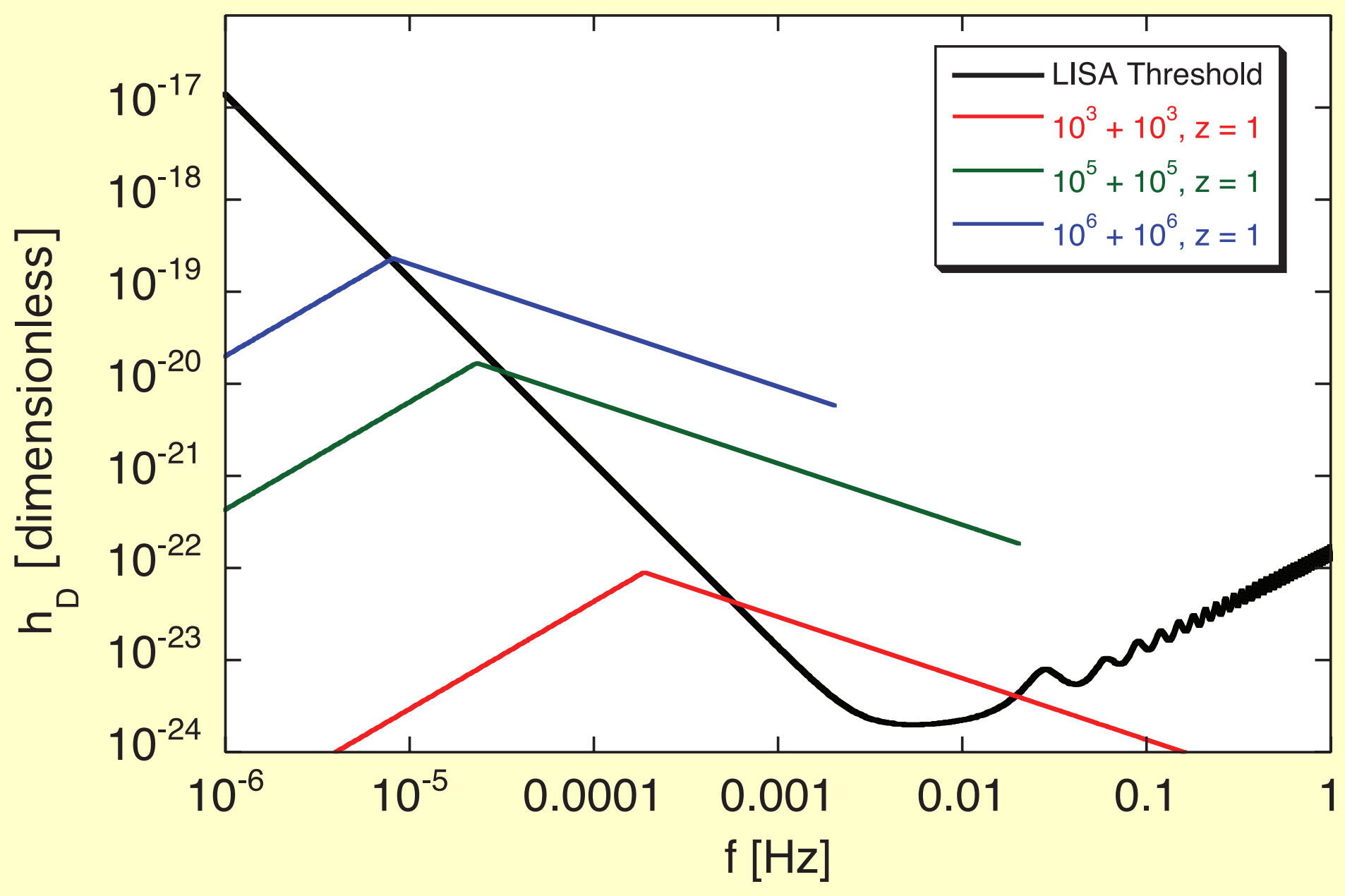
Low frequency gravitational waves from binary black hole populations

⟨Leah Liu, Shane L. Larson & Deirdre M. Shoemaker⟩
It is expected that LISA will observe between 10 and 1000 mergers of supermassive binary black holes. Prior to the merger, the binaries will emit gravitational waves at much lower frequencies in the LISA band.

Project: Build a set of modular computer codes which can be used to study the population of binary black holes across the LISA band. The codes should be generic enough that different possible models of binary black hole populations can be run through the pipeline.

Milestones:

- A code which computes the time a binary spends at a given frequency, and the expected strength of the emitted gravitational waves as a function of frequency (illustrated below).



- A code which generates a random sample of binary black holes with realistic astrophysical parameters (drawn from a set of distribution functions for the mass, mass ratio and redshift).
- A code which utilizes the previous two codes to evaluate the contribution of an entire population of binary black holes to the gravitational wave signal LISA will observe.

Project Status: The second milestone was completed prior to the end of spring semester 2006. Leah is currently on a summer internship at Oak Ridge National Laboratory, and is expecting to return to work with us at the start of her sophomore year, fall 2006.

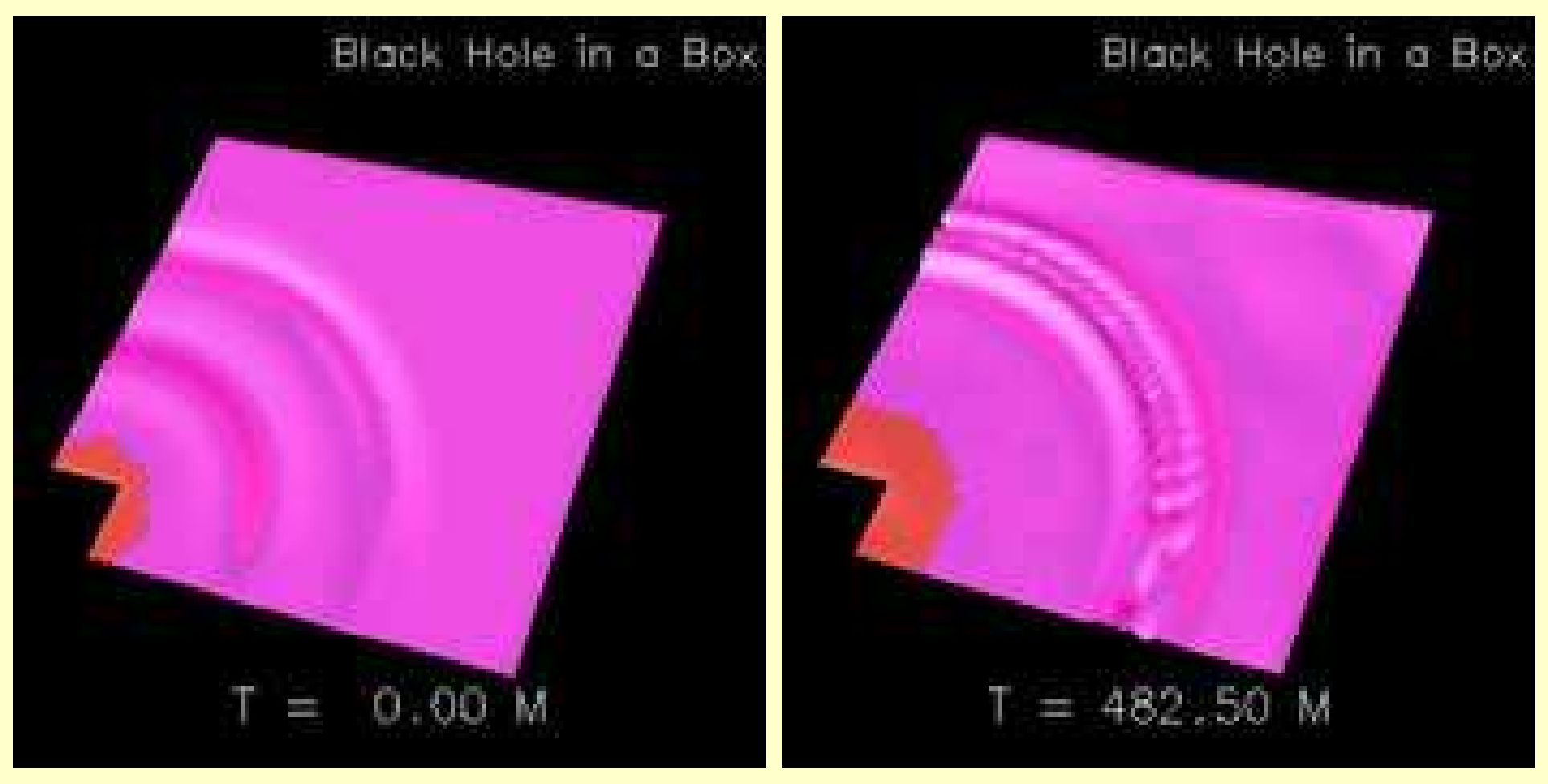
Black Holes and Constraint Violations

⟨Padraic Finnerty & Deirdre M. Shoemaker⟩
The spacetime of an isolated black hole is a solution of the Einstein equation, and is one of the solutions we teach in undergraduate courses on general relativity. A single black hole is also the basic test case for numerical codes designed to evolve binary black hole spacetimes. In most numerical relativity codes, the Einstein equations are split up into the field equations and the constraints. What happens when the field equations are evolved but the constraint equations are not satisfied? This is no longer a solution to the Einstein equation. Is it stable like a black hole with funky matter or doomed to crash?

Project: Use the existing Penn State binary black hole code to evolve a single black hole. By systematically adding initial data that is not constraint preserving, we can characterize the code's sensitivity to constraint violations, and discover whether or not a stable solution results.

Milestones:

- Learn to compile and run Penn State's MAYA code to evolve a single black hole in a square computational domain.
- Edit MAYA to add the initial conditions that violate the constraints, and generate a set of initial data with different levels of violation.
- Run MAYA and understand the output and how to visualize it. The plot below shows the initial data of the pulse around black hole. The black hole's interior is cut out, and is represented in orange.



Project Status: The third milestone is nearly done and the project is almost complete. Padraic has found that, depending on the amplitude of the constraint violation, we can evolve the black hole + constraint violations stably. Padriac graduated this year and will be attending the University of North Carolina in the Fall. He began working in numerical relativity after taking undergraduate general relativity in spring 2005.

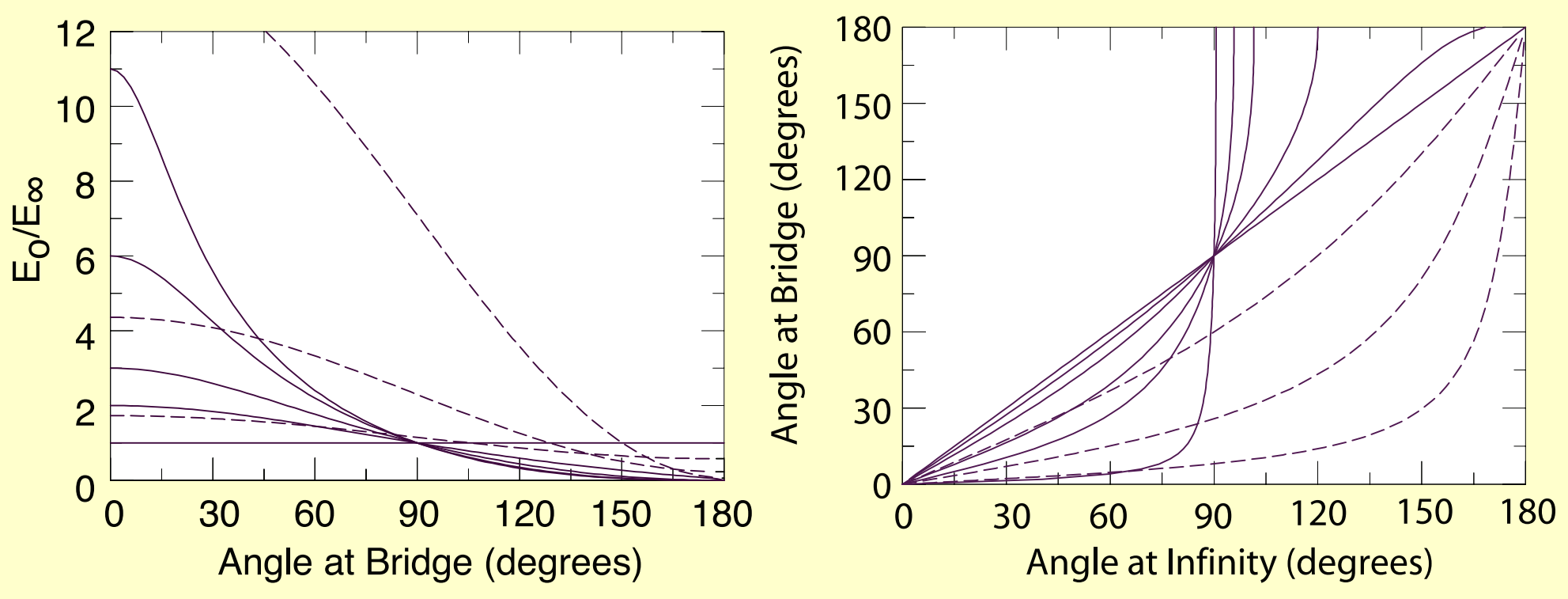
Photon trajectories in the warp drive

⟨Chad Clark, William A. Hiscock & Shane L. Larson⟩
The *aberration of starlight* is a well-known effect in special relativity. Alcubierre proposed a way to travel between distant points in the Universe by warping spacetime. If your starship were riding at the center of such a warp bubble, what would the distant starfield look like to you?

Project: Integrate the null geodesic equation for the warp drive spacetime to determine the angle of deflection and redshift of photons which propagate to the bridge of your starship.

Milestones:

- A code which utilizes a fourth-order Runge-Kutta scheme with adaptive stepsize to integrate a system of ODEs.
- Write down the null geodesic equation for the warp drive spacetime (the real exposure to GR is here, in computing the metric connections and understanding the role they play in the equations of motion).
- Utilizing the previous two milestones, look at a series of photon shots to determine deflection and redshift (illustrated below). To make sure photons hit the bridge, integrate from the starship *outward*.



The required concepts from General Relativity:

- ▷ A rudimentary understanding of the null geodesic equation. At the most base level, all that is really needed to understand the problem is that the null geodesic equation is an equation of motion for photons.
- ▷ Relation between the energy shift of the photons (time component of the null geodesic equation) and the redshift.

Published: Chad Clark, William A. Hiscock, and Shane L. Larson, *Null geodesics in the Alcubierre warp drive spacetime: the view from the bridge*, Classical & Quantum Gravity **16**, 3965 (1999).

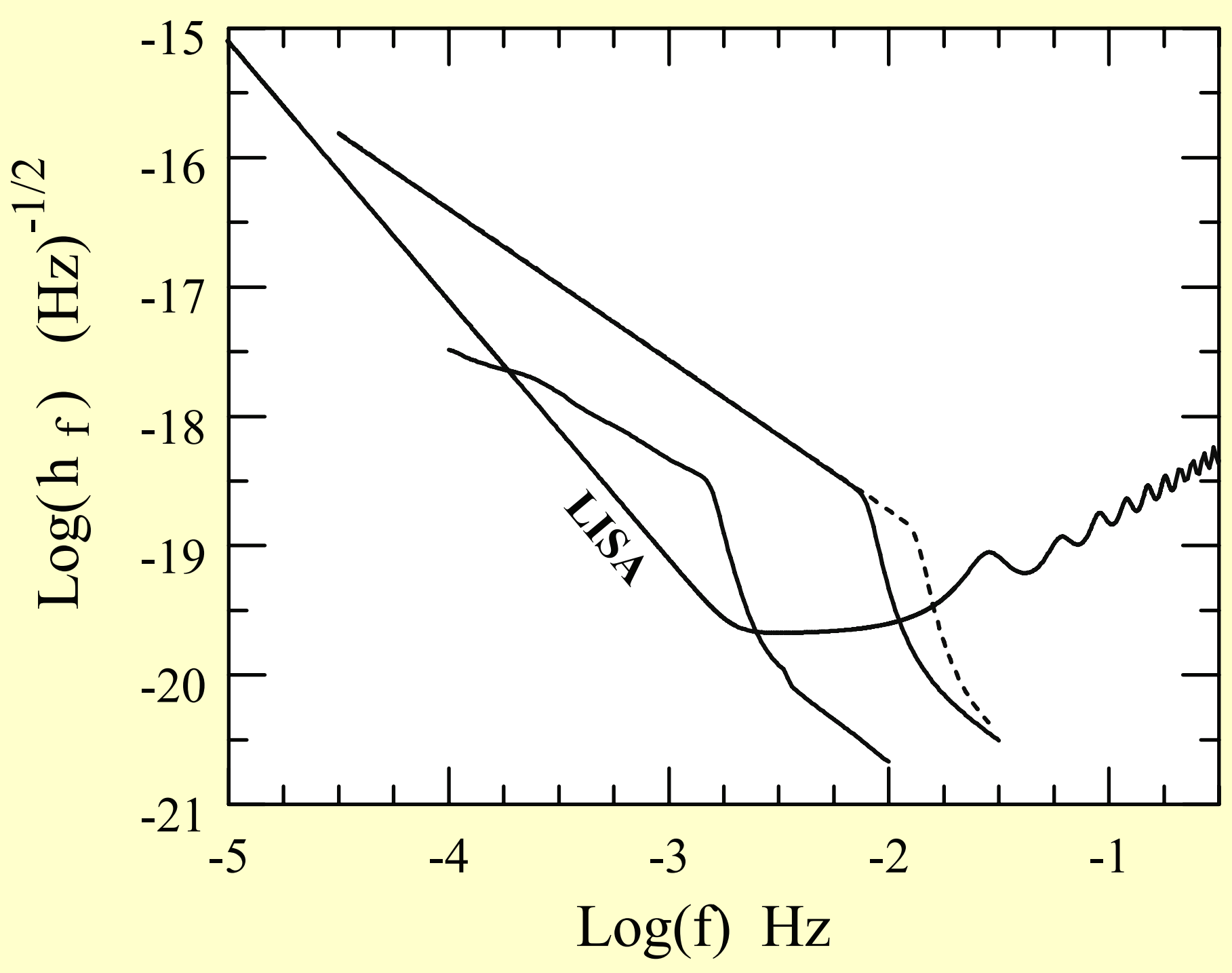
Gravitational waves from a galactic halo

⟨Ben Kulick, Josh Routzahn, W. A. Hiscock, S. L. Larson⟩
White dwarfs in the disk of the galaxy will form a *confusion-limited foreground* in the low-frequency gravitational wave band. Relatively little is known about the binary content of the galactic halo. What would the gravitational wave signal look like if it were filled with binary white dwarfs?

Project: In the absence of concrete knowledge, scale the white dwarf binary distribution from the galactic disk to the galactic halo, and estimate the resulting signal in the LISA band.

Milestones:

- Create an analytic fit to the standard estimate of the gravitational wave strength of the galactic foreground from the disk of the galaxy.
- Construct a scaling law from the disk population of binaries (cylindrically distributed) to a halo population (spherically distributed).
- Utilizing the previous two milestones, construct an estimate of the foreground from a galactic halo filled with white dwarf binaries. Since the halo size is uncertain, scale the results to different halo radii.



Published: William A. Hiscock, Shane L. Larson, Joshua Routzahn, and Ben Kulick, *Low frequency gravitational waves from binary white dwarf MACHOs*, Astrophysical Journal Letters **540**, L5 (2000).