



Teaching General Relativity with Tensors

Thomas A. Moore, Pomona College
AAPT Theme Conference on Teaching GR, July 21, 2006

Credit: NASA

Acknowledgements

- Edwin Taylor
- Richard Price
- Hans Ohanian
- Bernard Schutz
- James Hartle

Credit: NASA

Math



Physics

Problems: Daunting
Delayed Gratification

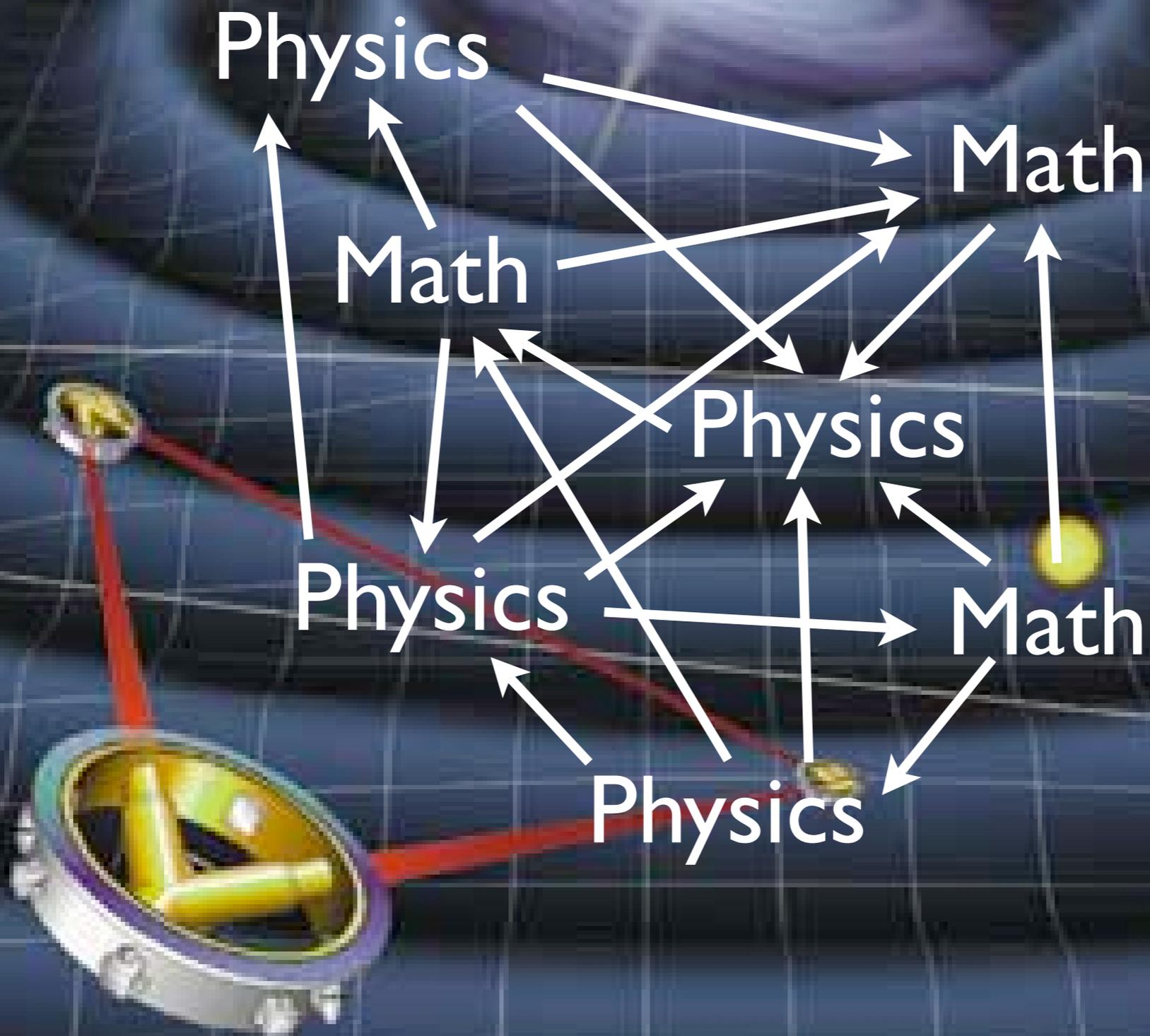
Physics!!



Math (?)

Problems: No foundation
Partial Gratifica

There is a Third Way--



Sample Syllabus

Conceptual Overview	Review of Relativity	Four-Vectors in SR
Index Notation	Arbitrary Coordinates	Tensor Equations
Maxwell's Equations	Geodesics	Schwarzschild Metric
Particle Orbits	Perihelion Precession	Photon Orbits
Bending of Light	Event Horizon	Alternative Coordinates
BH Thermodynamics	Kerr Metric	Kerr Particle Orbits
Ergoregion and Horizon	Negative Energy Orbits	Penrose Process
The Absolute Gradient	Geodesic Deviation	The Riemann Tensor
Stress-Energy Tensor	Einstein Equation	Interpreting the Equation
Schwarzschild Solution	The Observed Universe	A Cosmic Metric
Evolution of Universe	Cosmic Implications	The Early Universe
Linearized Gravity	Gauge Freedom	Gravitational Waves
"Energy" in GWs	Generation of GWs	Applications

Credit: NASA

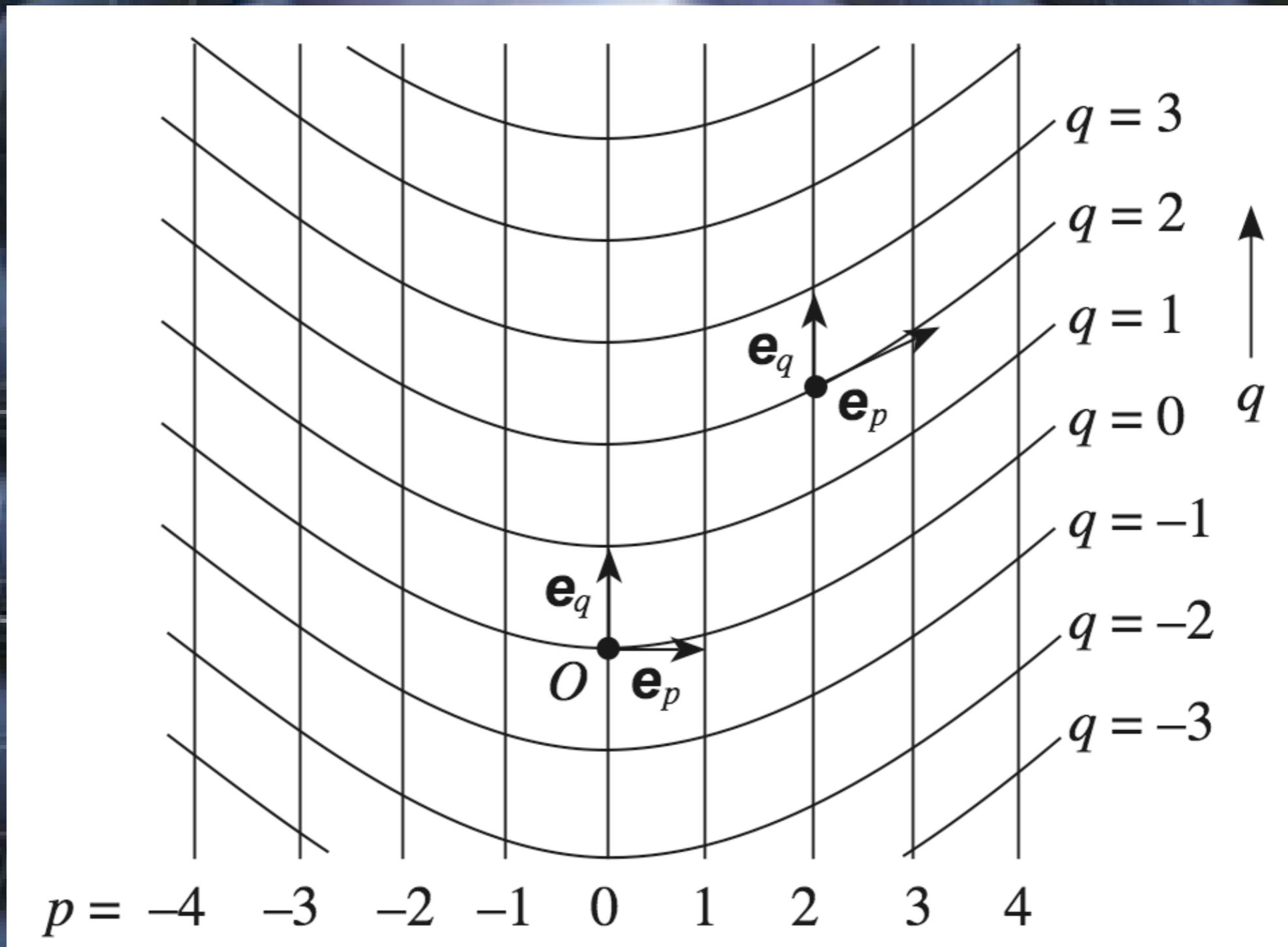
How to make it work: the 7-Fold Way

- Blend the math and physics
- Use (lots of) 2D examples
- KISS (Keep it Suitably Simple)
- Drill 'em (AKA Index Boot Camp)
- Develop “Ownership” through Activity
- Let Homework = practice
- Use Tools to Avoid Tedium

2D Examples

- Flat Space Polar Coordinates
- θ - Φ Coordinates on Spheres
- r - Φ Coordinates on Spheres
- Metrics for weird 2D surfaces
- “Warp-drive” spacetime
- Weird Flat-space Coordinates

Weird Coordinates



$$p = x$$

$$q = y - cx^2$$

Credit: NASA

KISS (Keep It Suitably Simple)

- Treat tensors as generalized vectors
 - Like vectors, they represent *physical* objects
 - Like vectors, be naturally represented by collections of components relative to some basis
 - Raising or lowering indices “embeds” factors of the metric relative to natural definition
 - Some forms more “natural” than others, but placement of indices is really physically irrelevant

KISS (Keep It Suitably Simple)

- The main point:
 - Build on student intuition about vectors
 - Avoid dual bases, the Levi-Civita tensor, basis vectors as gradient operators and *any other complexity that is not absolutely necessary.*

Some Sample Drills

Consider the following examples of renaming an index:

a) $\Lambda^{\alpha}_{\beta} A^{\beta} = A^{\alpha'}$ renamed to $\Lambda^{\alpha}_{\beta} A^{\beta} = A^{\mu'}$ (4.26a)

b) $\eta_{\mu\nu} A^{\mu} B^{\nu} = 0$ renamed to $\eta_{\mu\mu} A^{\mu} B^{\mu} = 0$ (4.26b)

c) $\frac{dp^{\mu}}{d\tau} = 0$ renamed to $\frac{dp^{\alpha}}{d\tau} = 0$ (4.26c)

d) $\eta_{\alpha\beta} = \eta_{\mu\nu} \Lambda^{\mu}_{\alpha} \Lambda^{\nu}_{\beta}$ renamed to $\eta_{\mu\nu} = \eta_{\alpha\beta} \Lambda^{\alpha}_{\mu} \Lambda^{\beta}_{\nu}$ (4.26d)

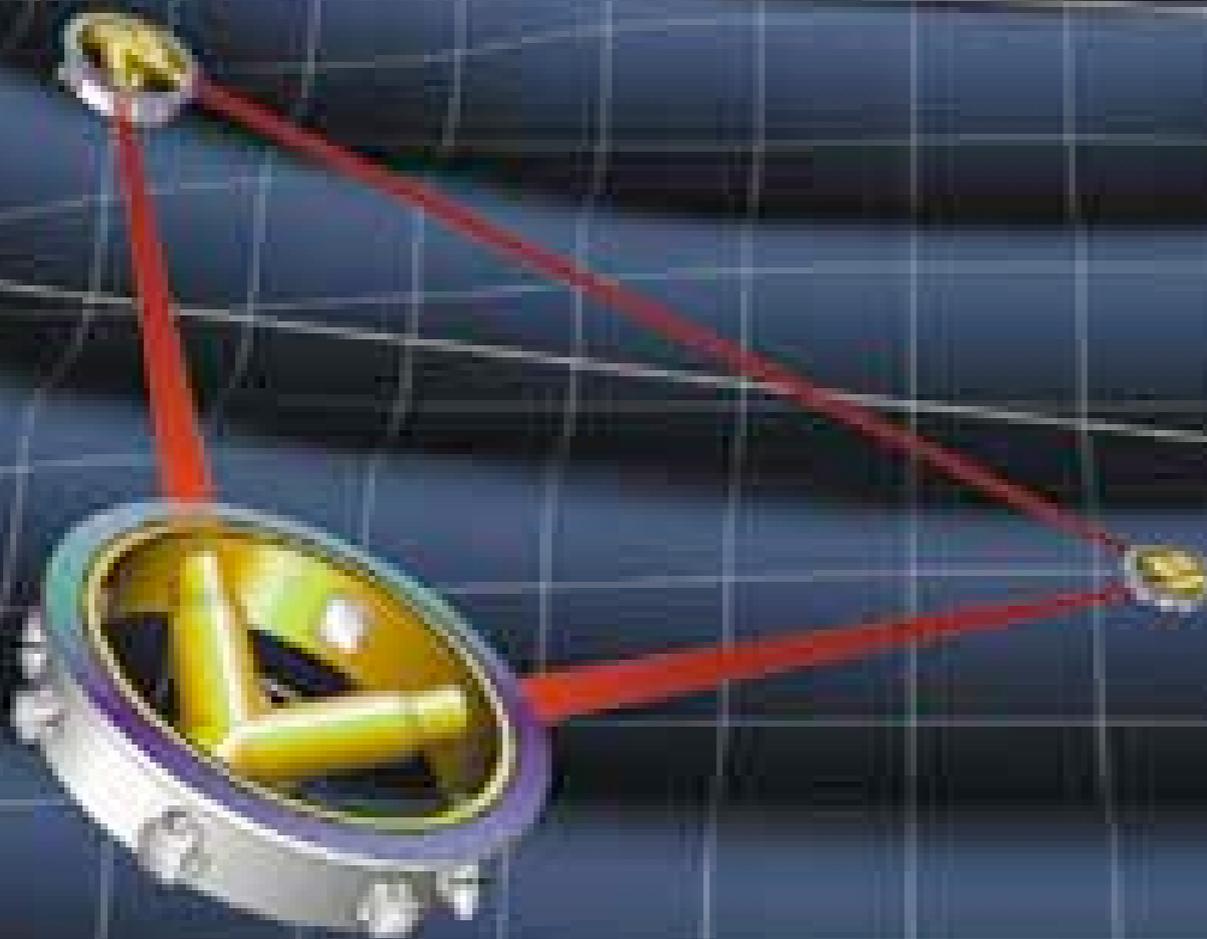
e) $\eta_{\mu\nu} A^{\mu} B^{\nu} + \eta_{\alpha\beta} A^{\alpha} C^{\beta} = 0$ renamed to $\eta_{\mu\nu} A^{\mu} B^{\nu} + \eta_{\mu\nu} A^{\mu} C^{\nu} = 0$ (4.26e)

f) $\delta^{\alpha}_{\mu} \delta^{\beta}_{\nu} F^{\mu\nu} = F^{\alpha\beta}$ renamed to $\delta^{\mu}_{\mu} \delta^{\nu}_{\nu} F^{\mu\nu} = F^{\mu\nu}$ (4.26f)

Exercise 4.5.2. Which of these equations violate Rule 2 about renaming indices? Why do those equations violate the rule?

Don't Forget:

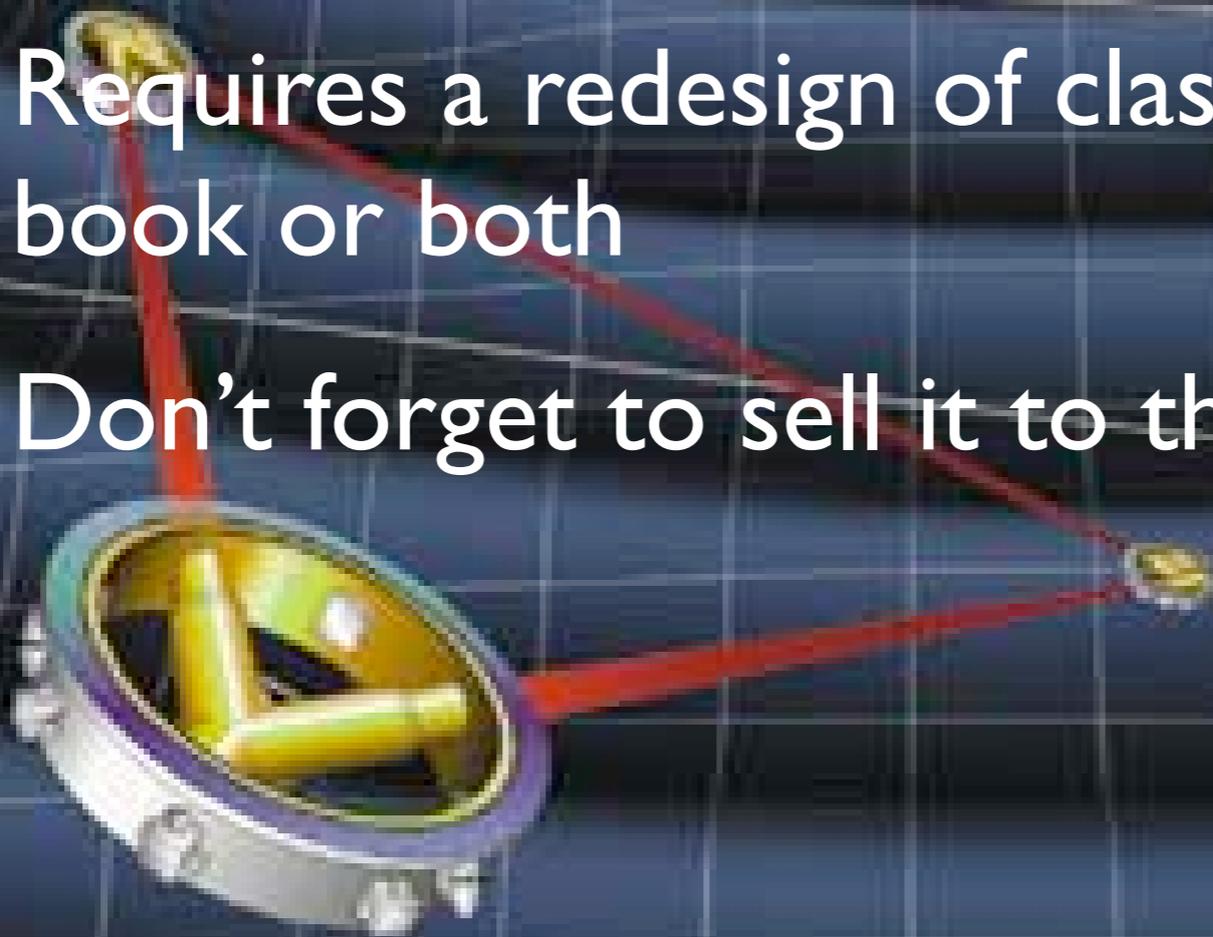
“When in doubt, write it out”



Credit: NASA

Building Ownership through Active Learning

- Active learning is *essential*
- Make them derive (virtually) everything!!
- Requires a redesign of class sessions or the book or both
- Don't forget to sell it to the *students!*



Homework = Practice

- Reconceptualize homework as being like a sports practice
- Part of the point of practice is the freedom to try something hard and fail
- Requires reorienting homework grading to reward good effort

Tricks and Tools for Avoiding Tedium

- Computer programs (but students must own the algorithm!)
- Ricci Chart
- General Cleverness



Why Teach Tensors?

- ~~Because they are there~~
- Because doing it provides a firm foundation for understanding the GR's big picture
- Experience of empowerment

Thanks!

- Alpha-draft book on display here (3 copies)
 - 3 people will be able to take copies home
 - more can get a CD with PDFs (ask me)
- (Please recognize that the draft is very rough)
- For more information: tmoore@pomona.edu