

2013 $F = ma$ Contest

25 QUESTIONS - 75 MINUTES

INSTRUCTIONS

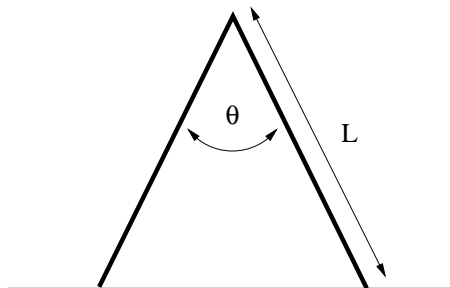
DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

- Use $g = 10 \text{ N/kg}$ throughout this contest.
- You may write in this booklet of questions. However, you will not receive any credit for anything written in this booklet.
- Your answer to each question must be marked on the optical mark answer sheet.
- Select the single answer that provides the best response to each question. Please be sure to use a No. 2 pencil and completely fill the box corresponding to your choice. If you change an answer, the previous mark must be completely erased.
- Correct answers will be awarded one point; incorrect answers will result in a deduction of $\frac{1}{4}$ point. There is no penalty for leaving an answer blank.
- A hand-held calculator may be used. Its memory must be cleared of data and programs. You may use only the basic functions found on a simple scientific calculator. Calculators may not be shared. Cell phones may not be used during the exam or while the exam papers are present. You may not use any tables, books, or collections of formulas.
- This test contains 25 multiple choice questions. Your answer to each question must be marked on the optical mark answer sheet that accompanies the test. Only the boxes preceded by numbers 1 through 25 are to be used on the answer sheet.
- All questions are equally weighted, but are not necessarily the same level of difficulty.
- **In order to maintain exam security, do not communicate any information about the questions (or their answers or solutions) on this contest until after February 20, 2013.**
- The question booklet and answer sheet will be collected at the end of this exam. You may not use scratch paper.

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Contributors to this year's exam include David Fallest, Jiajia Dong, Paul Stanley, Warren Turner, Qiuzi Li, and former US Team members Marianna Mao, Andrew Lin, Steve Byrnes.

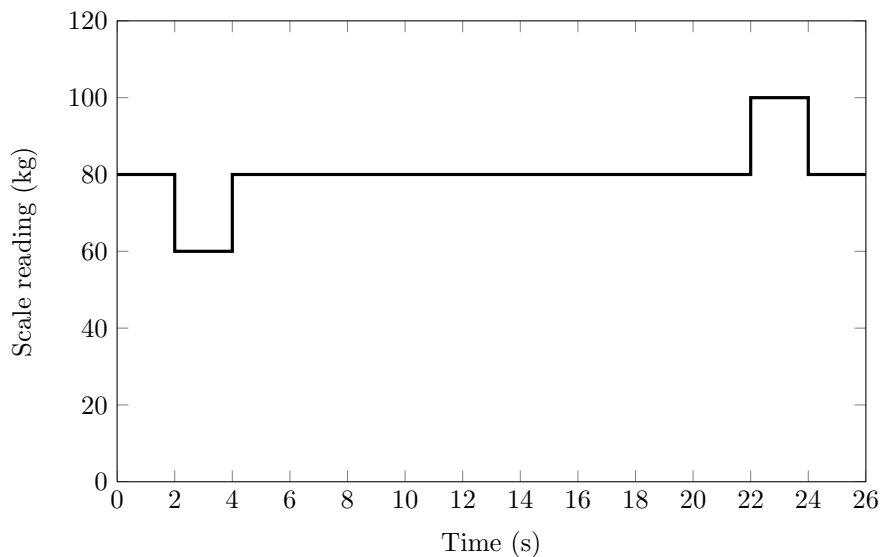
- An observer stands on the side of the front of a stationary train. When the train starts moving with constant acceleration, it takes 5 seconds for the first car to pass the observer. How long will it take for the 10th car to pass?
 - 1.07s
 - 0.98s
 - 0.91s
 - 0.86s
 - 0.81s
- Jordi stands 20 m from a wall and Diego stands 10 m from the same wall. Jordi throws a ball at an angle of 30° above the horizontal, and it collides elastically with the wall. How fast does Jordi need to throw the ball so that Diego will catch it? Consider Jordi and Diego to be the same height, and both are on the same perpendicular line from the wall.
 - 11 m/s
 - 15 m/s
 - 19 m/s
 - 30 m/s
 - 35 m/s
- Tom throws a football to Wes, who is a distance l away. Tom can control the time of flight t of the ball by choosing any speed up to v_{max} and any launch angle between 0° and 90° . Ignore air resistance and assume Tom and Wes are at the same height. Which of the following statements is **incorrect**?
 - If $v_{max} < \sqrt{gl}$, the ball cannot reach Wes at all.
 - Assuming the ball can reach Wes, as v_{max} increases with l held fixed, the minimum value of t decreases.
 - Assuming the ball can reach Wes, as v_{max} increases with l held fixed, the maximum value of t increases.
 - Assuming the ball can reach Wes, as l increases with v_{max} held fixed, the minimum value of t increases.
 - Assuming the ball can reach Wes, as l increases with v_{max} held fixed, the maximum value of t increases.
- The sign shown below consists of two uniform legs attached by a frictionless hinge. The coefficient of friction between the ground and the legs is μ . Which of the following gives the maximum value of θ such that the sign will not collapse?



- $\sin \theta = 2\mu$
- $\sin \theta/2 = \mu/2$
- $\tan \theta/2 = \mu$
- $\tan \theta = 2\mu$
- $\tan \theta/2 = 2\mu$

The following information applies to questions 5 and 6

A student steps onto a stationary elevator and stands on a bathroom scale. The elevator then travels from the top of the building to the bottom. The student records the reading on the scale as a function of time.

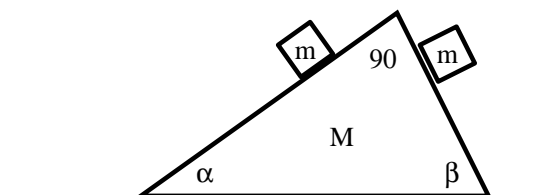


5. At what time(s) does the student have maximum downward velocity?
- (A) At all times between 2 s and 4 s
 - (B) At 4 s only
 - (C) At all times between 4 s and 22 s
 - (D) At 22 s only
 - (E) At all times between 22 s and 24 s
6. How tall is the building?
- (A) 50 m
 - (B) 80 m
 - (C) 100 m
 - (D) 150 m
 - (E) 400 m
7. A light car and a heavy truck have the same momentum. The truck weighs ten times as much as the car. How do their kinetic energies compare?
- (A) The truck's kinetic energy is larger by a factor of 100
 - (B) The truck's kinetic energy is larger by a factor of 10
 - (C) They have the same kinetic energy
 - (D) The car's kinetic energy is larger by a factor of 10
 - (E) The car's kinetic energy is larger by a factor of 100

The following information applies to questions 8 and 9

A truck is initially moving at velocity v . The driver presses the brake in order to slow the truck to a stop. The brake applies a constant force F to the truck. The truck rolls a distance x before coming to a stop, and the time it takes to stop is t .

8. Which of the following expressions is equal the initial kinetic energy of the truck (i.e. the kinetic energy before the driver starts braking)?
- (A) Fx
 - (B) Fvt
 - (C) Fxt
 - (D) Ft
 - (E) Both (a) and (b) are correct
9. Which of the following expressions is equal the initial momentum of the truck (i.e. the momentum before the driver starts braking)?
- (A) Fx
 - (B) $Ft/2$
 - (C) Fxt
 - (D) $2Ft$
 - (E) $2Fx/v$
10. Which of the following can be used to distinguish a solid ball from a hollow sphere of the same radius and mass?
- (A) Measurements of the orbit of a test mass around the object.
 - (B) Measurements of the time it takes the object to roll down an inclined plane.
 - (C) Measurements of the tidal forces applied by the object to a liquid body.
 - (D) Measurements of the behavior of the object as it floats in water.
 - (E) Measurements of the force applied to the object by a uniform gravitational field.
11. A right-triangular wooden block of mass M is at rest on a table, as shown in figure. Two smaller wooden cubes, both with mass m , initially rest on the two sides of the larger block. As all contact surfaces are frictionless, the smaller cubes start sliding down the larger block while the block remains at rest. What is the normal force from the system to the table?



- (A) $2mg$
- (B) $2mg + Mg$
- (C) $mg + Mg$
- (D) $Mg + mg(\sin \alpha + \sin \beta)$
- (E) $Mg + mg(\cos \alpha + \cos \beta)$

12. A spherical shell of mass M and radius R is completely filled with a frictionless fluid, also of mass M . It is released from rest, and then it rolls without slipping down an incline that makes an angle θ with the horizontal. What will be the acceleration of the shell down the incline just after it is released? Assume the acceleration of free fall is g .

The moment of inertia of a thin shell of radius r and mass m about the center of mass is $I = \frac{2}{3}mr^2$; the moment of inertia of a solid sphere of radius r and mass m about the center of mass is $I = \frac{2}{5}mr^2$.

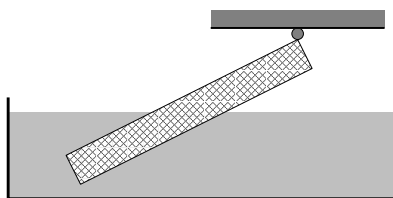
- (A) $a = g \sin \theta$
 (B) $a = \frac{3}{4}g \sin \theta$
 (C) $a = \frac{1}{2}g \sin \theta$
 (D) $a = \frac{3}{8}g \sin \theta$
 (E) $a = \frac{3}{5}g \sin \theta$
13. There is a ring outside of Saturn. In order to distinguish if the ring is actually a part of Saturn or is instead part of the satellites of Saturn, we need to know the relation between the velocity v of each layer in the ring and the distance R of the layer to the center of Saturn. Which of the following statements is correct?

- (A) If $v \propto R$, then the layer is part of Saturn.
 (B) If $v^2 \propto R$, then the layer is part of the satellites of Saturn.
 (C) If $v \propto 1/R$, then the layer is part of Saturn.
 (D) If $v^2 \propto 1/R$, then the layer is part of Saturn.
 (E) If $v \propto R^2$, then the layer is part of the satellites of Saturn.

14. A cart of mass m moving at 12 m/s to the right collides elastically with a cart of mass 4.0 kg that is originally at rest. After the collision, the cart of mass m moves to the left with a velocity of 6.0 m/s. Assuming an elastic collision in one dimension only, what is the velocity of the center of mass (v_{cm}) of the two carts *before* the collision?

- (A) $v_{\text{cm}} = 2.0$ m/s
 (B) $v_{\text{cm}} = 3.0$ m/s
 (C) $v_{\text{cm}} = 6.0$ m/s
 (D) $v_{\text{cm}} = 9.0$ m/s
 (E) $v_{\text{cm}} = 18$ m/s

15. A uniform rod is partially in water with one end suspended, as shown in figure. The density of the rod is $5/9$ that of water. At equilibrium, what portion of the rod is above water?



- (A) 0.25
 (B) 0.33
 (C) 0.5
 (D) 0.67
 (E) 0.75

16. *Inspired by a problem from the 2012 International Physics Olympiad, Estonia.*

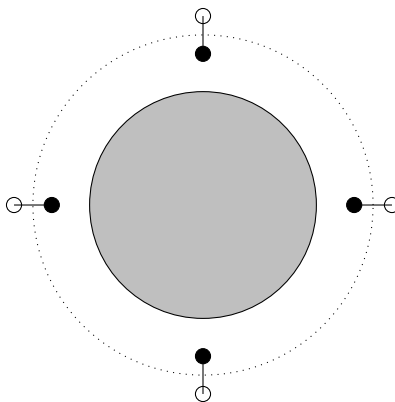
A very large number of small particles forms a spherical cloud. Initially they are at rest, have uniform mass density per unit volume ρ_0 , and occupy a region of radius r_0 . The cloud collapses due to gravitation; the particles do not interact with each other in any other way.

How much time passes until the cloud collapses fully? (The constant 0.5427 is actually $\sqrt{\frac{3\pi}{32}}$.)

- (A) $\frac{0.5427}{r_0^2 \sqrt{G\rho_0}}$
- (B) $\frac{0.5427}{r_0 \sqrt{G\rho_0}}$
- (C) $\frac{0.5427}{\sqrt{r_0} \sqrt{G\rho_0}}$
- (D) $\frac{0.5427}{\sqrt{G\rho_0}}$
- (E) $\frac{0.5427}{\sqrt{G\rho_0}} r_0$

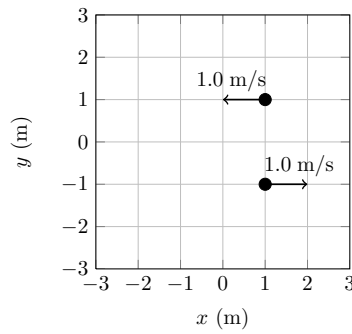
17. Two small, equal masses are attached by a lightweight rod. This object orbits a planet; the length of the rod is smaller than the radius of the orbit, but not negligible. The rod rotates about its axis in such a way that it remains vertical with respect to the planet.

- Is there a force in the rod? If so, is it tension or compression?
- Is the equilibrium stable, unstable, or neutral with respect to a small perturbation in the angle of the rod? (Assume this perturbation maintains the rate of rotation, so that in the co-rotating frame the rod is still stationary but at an angle to the vertical.)

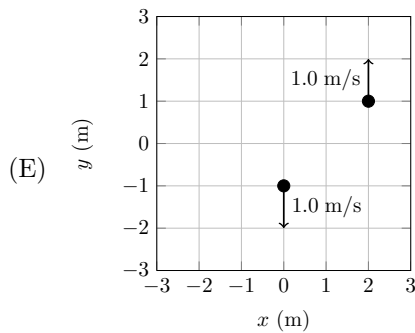
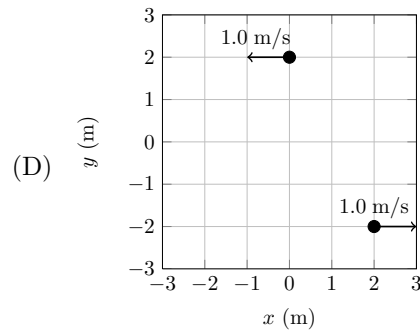
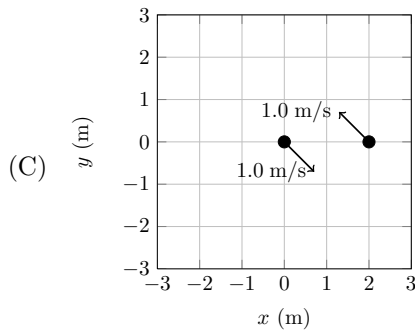
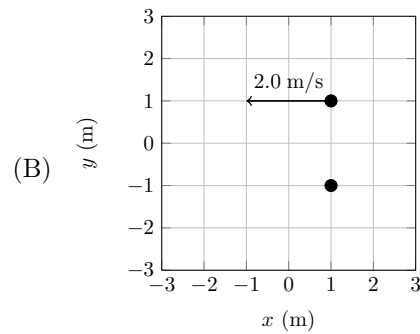
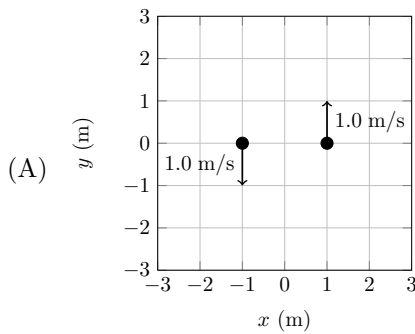


- (A) There is no force in the rod; the equilibrium is neutral.
- (B) The rod is in tension; the equilibrium is stable.
- (C) The rod is in compression; the equilibrium is stable.
- (D) The rod is in tension; the equilibrium is unstable.
- (E) The rod is in compression; the equilibrium is unstable.

18. Two point particles, each of mass 1 kg, begin in the state shown below.



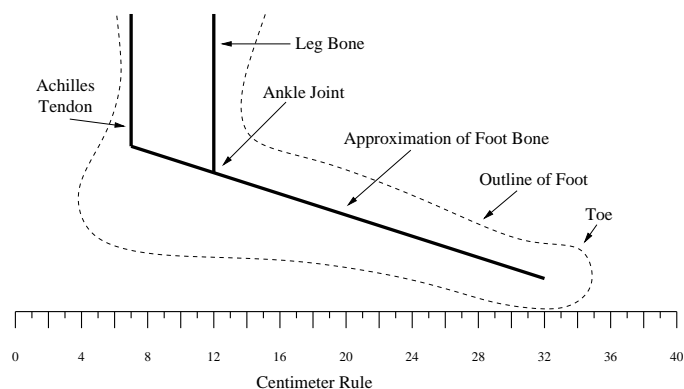
The system evolves through internal forces only. Which of the following could be the state after some time has passed?



The following information applies to questions 19, 20, and 21.

A simple pendulum experiment is constructed from a point mass m attached to a pivot by a massless rod of length L in a constant gravitational field. The rod is released from an angle $\theta_0 < \pi/2$ at rest and the period of motion is found to be T_0 . Ignore air resistance and friction.

19. At what angle θ_g during the swing is the tension in the rod the greatest?
- (A) The tension is the greatest at the point $\theta_g = \theta_0$.
 (B) The tension is the greatest at the point $\theta_g = 0$.
 (C) The tension is the greatest at an angle θ_g with $0 < \theta_g < \theta_0$.
 (D) The tension is constant.
 (E) None of the above is true for all values of θ_0 with $0 < \theta_0 < \pi/2$.
20. What is the maximum value of the tension in the rod?
- (A) mg
 (B) $2mg$
 (C) $mL\theta_0/T_0^2$
 (D) $mg \sin \theta_0$
 (E) $mg(3 - 2 \cos \theta_0)$
21. The experiment is repeated with a new pendulum with a rod of length $4L$, using the same angle θ_0 , and the period of motion is found to be T . Which of the following statements is correct?
- (A) $T = 2T_0$ regardless of the value of θ_0 .
 (B) $T > 2T_0$ with $T \approx 2T_0$ if $\theta_0 \ll 1$.
 (C) $T < 2T_0$ with $T \approx 2T_0$ if $\theta_0 \ll 1$.
 (D) $T > 2T_0$ for some values of θ_0 and $T < 2T_0$ for other values of θ_0 .
 (E) T_0 and T are undefined because the motion is not periodic unless $\theta_0 \ll 1$.
22. A simplified model on the foot is shown. When a student of mass $m = 60$ kg stands on a single toe, the tension T in the Achilles Tendon is closest to



- (A) $T = 600$ N
 (B) $T = 1200$ N
 (C) $T = 1800$ N
 (D) $T = 2400$ N
 (E) $T = 3000$ N

The following information applies to questions 23 and 24

A man with mass m jumps off of a high bridge with a bungee cord attached to his ankles. The man falls through a maximum distance H at which point the bungee cord brings him to a momentary rest before he bounces back up. The bungee cord is perfectly elastic, obeying Hooke's force law with a spring constant k , and stretches from an original length of L_0 to a final length $L = L_0 + h$. The maximum tension in the Bungee cord is four times the weight of the man.

23. Determine the spring constant k .

(A) $k = \frac{mg}{h}$

(B) $k = \frac{2mg}{h}$

(C) $k = \frac{mg}{H}$

(D) $k = \frac{4mg}{H}$

(E) $k = \frac{8mg}{H}$

24. Find the maximum extension of the bungee cord h .

(A) $h = \frac{1}{2}H$

(B) $h = \frac{1}{4}H$

(C) $h = \frac{1}{5}H$

(D) $h = \frac{2}{5}H$

(E) $h = \frac{1}{8}H$

25. A box with weight W will slide down a 30° incline at constant speed under the influence of gravity and friction alone. If instead a horizontal force P is applied to the box, the box can be made to move *up* the ramp at constant speed. What is the magnitude of P ?

(A) $P = W/2$

(B) $P = 2W/\sqrt{3}$

(C) $P = W$

(D) $P = \sqrt{3}W$

(E) $P = 2W$