AAPT UNITED STATES PHYSICS TEAM
AIP 2017

USA Physics Olympiad Exam

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Important Instructions for the Exam Supervisor

- This examination consists of two parts: Part A has four questions and is allowed 90 minutes; Part B has two questions and is allowed 90 minutes.
- The first page that follows is a cover sheet. Examinees may keep the cover sheet for both parts of the exam.
- The parts are then identified by the center header on each page. Examinees are only allowed to do one part at a time, and may not work on other parts, even if they have time remaining.
- Allow 90 minutes to complete Part A. Do not let students look at Part B. Collect the answers to Part A before allowing the examinee to begin Part B. Examinees are allowed a 10 to 15 minutes break between parts A and B.
- Allow 90 minutes to complete Part B. Do not let students go back to Part A.
- Ideally the test supervisor will divide the question paper into 3 parts: the cover sheet (page 2), Part A (pages 3-6), Part B (pages 8-10). Examinees should be provided parts A and B individually, although they may keep the cover sheet. The answer sheets should be printed single sided!
- The supervisor *must* collect all examination questions, including the cover sheet, at the end of the exam, as well as any scratch paper used by the examinees. Examinees may *not* take the exam questions. The examination questions may be returned to the students after April 15, 2017.
- Examinees are allowed calculators, but they may not use symbolic math, programming, or graphic features of these calculators. Calculators may not be shared and their memory must be cleared of data and programs. Cell phones, PDA's or cameras may not be used during the exam or while the exam papers are present. Examinees may not use any tables, books, or collections of formulas.

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INSTRUCTIONS

DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

- Work Part A first. You have 90 minutes to complete all four problems. Each question is worth 25 points. Do not look at Part B during this time.
- After you have completed Part A you may take a break.
- Then work Part B. You have 90 minutes to complete both problems. Each question is worth 50 points. Do not look at Part A during this time.
- Show all your work. Partial credit will be given. Do not write on the back of any page. Do not write anything that you wish graded on the question sheets.
- Start each question on a new sheet of paper. Put your proctor's AAPT ID, your AAPT ID, your name, the question number and the page number/total pages for this problem, in the upper right hand corner of each page. For example,

  Doe, Jamie
  student AAPT ID #
  proctor AAPT ID #
  A1 - 1/3

- A hand-held calculator may be used. Its memory must be cleared of data and programs. You may use only the basic functions found on a simple scientific calculator. Calculators may not be shared. Cell phones, PDA's or cameras may not be used during the exam or while the exam papers are present. You may not use any tables, books, or collections of formulas.
- Questions with the same point value are not necessarily of the same difficulty.
- **In order to maintain exam security, do not communicate any information about the questions (or their answers/solutions) on this contest until after April 8, 2017.**

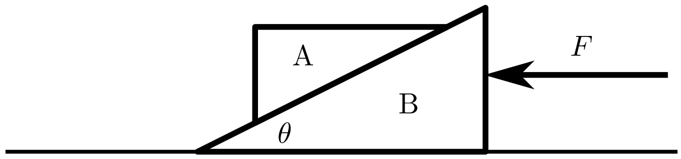
Possibly Useful Information. You may use this sheet for both parts of the exam.

$g = 9.8$ N/kg	$G = 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$
$k = 1/4\pi\epsilon_0 = 8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2}$	$k_\mathrm{m} = \mu_0/4\pi = 10^{-7}\ \mathrm{T \cdot m/A}$
$c = 3.00 \times 10^8$ m/s	$k_\mathrm{B} = 1.38 \times 10^{-23}$ J/K
$N_\mathrm{A} = 6.02 \times 10^{23}\ (\mathrm{mol})^{-1}$	$R = N_\mathrm{A} k_\mathrm{B} = 8.31\ \mathrm{J/(mol \cdot K)}$
$\sigma = 5.67 \times 10^{-8}\ \mathrm{J/(s \cdot m^2 \cdot K^4)}$	$e = 1.602 \times 10^{-19}$ C
$1\ \mathrm{eV} = 1.602 \times 10^{-19}$ J	$h = 6.63 \times 10^{-34}\ \mathrm{J \cdot s} = 4.14 \times 10^{-15}\ \mathrm{eV \cdot s}$
$m_e = 9.109 \times 10^{-31}\ \mathrm{kg} = 0.511\ \mathrm{MeV/c^2}$	$(1+x)^n \approx 1 + nx$ for $\lvert x \rvert \ll 1$
$m_p = 1.673 \times 10^{-27}\ \mathrm{kg} = 938\ \mathrm{MeV/c^2}$	$\ln(1+x) \approx x$ for $\lvert x \rvert \ll 1$
$\sin\theta \approx \theta - \frac{1}{6}\theta^3$ for $\lvert\theta\rvert \ll 1$	$\cos\theta \approx 1 - \frac{1}{2}\theta^2$ for $\lvert\theta\rvert \ll 1$

Part A

Question A1

A pair of wedges are located on a horizontal surface. The coefficient of friction (both sliding and static) between the wedges is μ, the coefficient of friction between the bottom wedge B and the horizontal surface is μ, and the angle of the wedge is θ. The mass of the top wedge A is m, and the mass of the bottom wedge B is $M = 2m$. A horizontal force F directed to the left is applied to the bottom wedge as shown in the figure.



Determine the range of values for F so that the the top wedge does not slip on the bottom wedge. Express your answer(s) in terms of any or all of m, g, θ, and μ.

Question A2

Consider two objects with equal heat capacities C and initial temperatures T_1 and T_2. A Carnot engine is run using these objects as its hot and cold reservoirs until they are at equal temperatures. Assume that the temperature changes of both the hot and cold reservoirs is very small compared to the temperature during any one cycle of the Carnot engine.

a. Find the final temperature T_f of the two objects, and the total work W done by the engine.

Now consider three objects with equal and constant heat capacity at initial temperatures $T_1 = 100$ K, $T_2 = 300$ K, and $T_3 = 300$ K. Suppose we wish to raise the temperature of the third object.

To do this, we could run a Carnot engine between the first and second objects, extracting work W. This work can then be dissipated as heat to raise the temperature of the third object. Even better, it can be stored and used to run a Carnot engine between the first and third object in reverse, which pumps heat into the third object.

Assume that all work produced by running engines can be stored and used without dissipation.

b. Find the minimum temperature T_L to which the first object can be lowered.

c. Find the maximum temperature T_H to which the third object can be raised.

Question A3

A ship can be thought of as a symmetric arrangement of soft iron. In the presence of an external magnetic field, the soft iron will become magnetized, creating a second, weaker magnetic field. We want to examine the effect of the ship's field on the ship's compass, which will be located in the middle of the ship.

Let the strength of the Earth's magnetic field near the ship be B_e, and the orientation of the field be horizontal, pointing directly toward true north.

The Earth's magnetic field B_e will magnetize the ship, which will then create a second magnetic field B_s in the vicinity of the ship's compass given by

$$\vec{\mathbf{B}}_s = B_e \left(-K_b \cos\theta \, \hat{\mathbf{b}} + K_s \sin\theta \, \hat{\mathbf{s}} \right)$$

where K_b and K_s are positive constants, θ is the angle between the heading of the ship and magnetic north, measured clockwise, $\hat{\mathbf{b}}$ and $\hat{\mathbf{s}}$ are unit vectors pointing in the forward direction of the ship (bow) and directly right of the forward direction (starboard), respectively.

Because of the ship's magnetic field, the ship's compass will no longer necessarily point North.

a. Derive an expression for the deviation of the compass, $\delta\theta$, from north as a function of K_b, K_s, and θ.

b. Assuming that K_b and K_s are both much smaller than one, at what heading(s) θ will the deviation $\delta\theta$ be largest?

A pair of iron balls placed in the same horizontal plane as the compass but a distance d away can be used to help correct for the error caused by the induced magnetism of the ship.

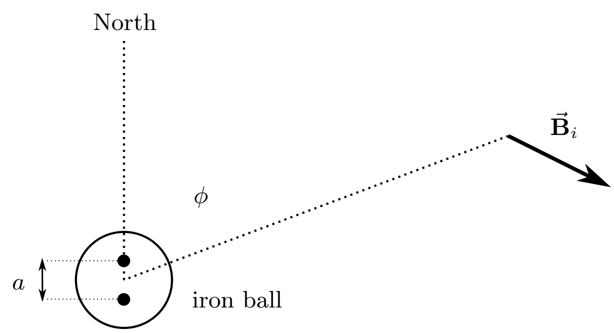
A binnacle, protecting the ship's compass in the center, with two soft iron spheres to help correct for errors in the compass heading. The use of the spheres was suggested by Lord Kelvin.

Just like the ship, the iron balls will become magnetic because of the Earth's field B_e. As spheres, the balls will individually act like dipoles. A dipole can be thought of as the field produced by two magnetic monopoles of strength $\pm m$ at two different points.

The magnetic field of a single pole is

$$\vec{\mathbf{B}} = \pm m \frac{\hat{\mathbf{r}}}{r^2}$$

where the positive sign is for a north pole and the negative for a south pole. The dipole magnetic field is the sum of the two fields: a north pole at $y = +a/2$ and a south pole at $y = -a/2$, where the y axis is horizontal and pointing north. a is a small distance much smaller than the radius of the iron balls; in general $a = K_i B_e$ where K_i is a constant that depends on the size of the iron sphere.



c. Derive an expression for the magnetic field $\vec{\mathbf{B}}_i$ from the iron a distance $d \gg a$ from the center of the ball. Note that there will be a component directed radially away from the ball and a component directed tangent to a circle of radius d around the ball, so using polar coordinates is recommended.

d. If placed directly to the right and left of the ship compass, the iron balls can be located at a distance d to cancel out the error in the magnetic heading for any angle(s) where $\delta\theta$ is largest. Assuming that this is done, find the resulting expression for the combined deviation $\delta\theta$ due to the ship and the balls for the magnetic heading for all angles θ.

Question A4

Relativistic particles obey the mass energy relation

$$E^2 = (pc)^2 + (mc^2)^2$$

where E is the relativistic energy of the particle, p is the relativistic momentum, m is the mass, and c is the speed of light.

A proton with mass m_p and energy E_p collides head on with a photon which is massless and has energy E_b. The two combine and form a new particle with mass m_Δ called Δ, or "delta". It is a one dimensional collision that conserves both relativistic energy and relativistic momentum.

a. Determine E_p in terms of m_p, m_Δ, and E_b. You may assume that E_b is small.

b. In this case, the photon energy E_b is that of the cosmic background radiation, which is an EM wave with wavelength 1.06 mm. Determine the energy of the photons, writing your answer in electron volts.

c. Assuming this value for E_b, what is the energy of the proton, in electron volts, that will allow the above reaction? This sets an upper limit on the energy of cosmic rays. The mass of the proton is given by $m_pc^2 = 938$ MeV and the mass of the Δ is given by $m_\Delta c^2 = 1232$ MeV.

The following relationships may be useful in solving this problem:

velocity parameter	$\beta = \frac{v}{c}$
Lorentz factor	$\gamma = \frac{1}{\sqrt{1-\beta^2}}$
relativistic momentum	$p = \gamma\beta mc$
relativistic energy	$E = \gamma mc^2$
relativistic doppler shift	$\frac{f}{f_0} = \sqrt{\frac{1-\beta}{1+\beta}}$

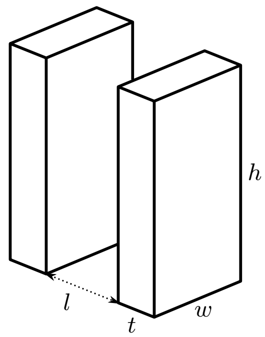
STOP: Do Not Continue to Part B

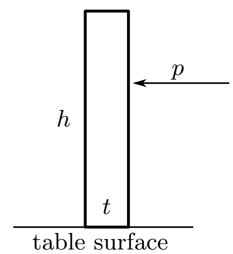
If there is still time remaining for Part A, you should review your work for Part A, but do not continue to Part B until instructed by your exam supervisor.

Part B

Question B1

Suppose a domino stands upright on a table. It has height h, thickness t, width w (as shown below), and mass m. The domino is free to rotate about its edges, but will not slide across the table.





a. Suppose we give the domino a sharp, horizontal impulsive push with total momentum p.

   i. At what height H above the table is the impulse p required to topple the domino smallest?

   ii. What is the minimum value of p to topple the domino?

b. Next, imagine a long row of dominoes with equal spacing l between the nearest sides of any pair of adjacent dominos, as shown above. When a domino topples, it collides with the next domino in the row. Imagine this collision to be completely inelastic. What fraction of the total kinetic energy is lost in the collision of the first domino with the second domino?

c. After the collision, the dominoes rotate in such a way so that they always remain in contact. Assume that there is no friction between the dominoes and the first domino was given the smallest possible push such that it toppled. What is the minimum l such that the second domino will topple?

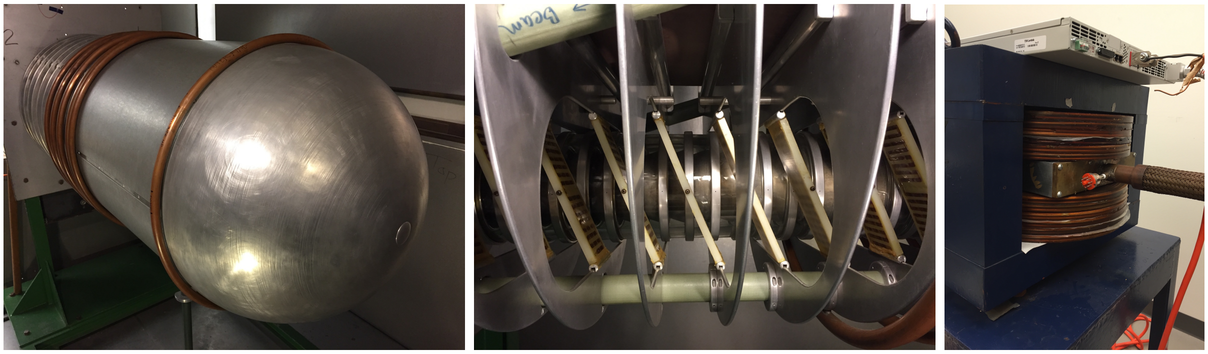
   You may work to lowest nontrivial order in the angles through which the dominoes have rotated. Equivalently, you may approximate $t,\ l \ll h$.

d. A row of toppling dominoes can be considered to have a propagation speed of the length $l + t$ divided by the time between successive collisions. When the first domino is given a minimal push just large enough to topple and start a chain reaction of toppling dominoes, the speed increases with each domino, but approaches an asymptotic speed v.

   Suppose there is a row of dominoes on another planet. These dominoes have the same density as the dominoes previously considered, but are twice as tall, wide, and thick, and placed with a spacing of $2l$ between them. If this row of dominoes topples with the same asymptotic speed v previously found, what is the local gravitational acceleration on this planet?

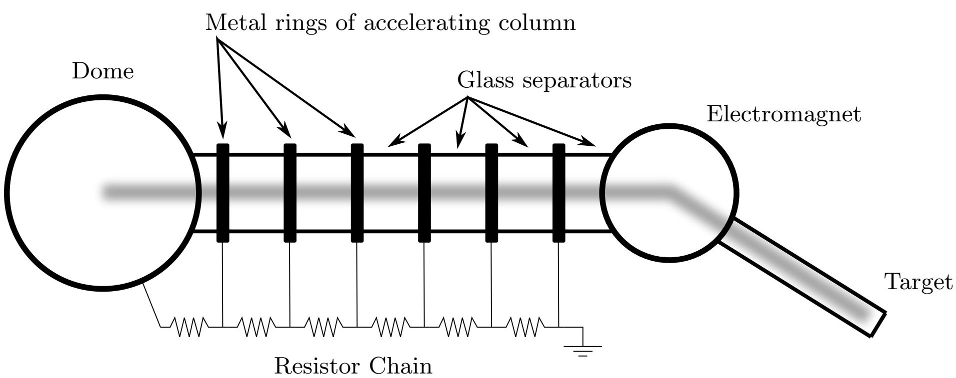
Question B2

Beloit College has a "homemade" 500 kV VanDeGraff proton accelerator, designed and constructed by the students and faculty.



Accelerator dome (assume it is a sphere); accelerating column; bending electromagnet

The accelerator dome, an aluminum sphere of radius $a = 0.50$ meters, is charged by a rubber belt with width $w = 10$ cm that moves with speed $v_b = 20$ m/s. The accelerating column consists of 20 metal rings separated by glass rings; the rings are connected in series with 500 MΩ resistors. The proton beam has a current of 25 μA and is accelerated through 500 kV and then passes through a tuning electromagnet. The electromagnet consists of wound copper pipe as a conductor. The electromagnet effectively creates a uniform field B inside a circular region of radius $b = 10$ cm and zero outside that region.



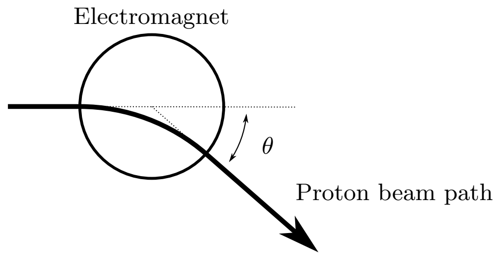
Only six of the 20 metals rings and resistors are shown in the figure. The fuzzy grey path is the path taken by the protons as they are accelerated from the dome, through the electromagnet, into the target.

a. Assuming the dome is charged to 500 kV, determine the strength of the electric field at the surface of the dome.

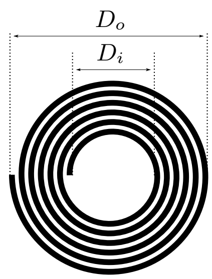
b. Assuming the proton beam is off, determine the time constant for the accelerating dome (the time it takes for the charge on the dome to decrease to $1/e \approx 1/3$ of the initial value.

c. Assuming the 25 μA proton beam is on, determine the surface charge density that must be sprayed onto the charging belt in order to maintain a steady charge of 500 kV on the dome.

d. The proton beam enters the electromagnet and is deflected by an angle $\theta = 10°$. Determine the magnetic field strength.



e. The electromagnet is composed of layers of spiral wound copper pipe; the pipe has inner diameter $d_i = 0.40$ cm and outer diameter $d_o = 0.50$ cm. The copper pipe is wound into this flat spiral that has an inner diameter $D_i = 20$ cm and outer diameter $D_o = 50$ cm. Assuming the pipe almost touches in the spiral winding, determine the length L in one spiral.



f. Hollow pipe is used instead of solid conductors in order to allow for cooling of the magnet. If the resistivity of copper is $\rho = 1.7 \times 10^{-8} \Omega \cdot \mathrm{m}$, determine the electrical resistance of one spiral.

g. There are $N = 24$ coils stacked on top of each other. Tap water with an initial temperature of $T_c = 18°$ C enters the spiral through the copper pipe to keep it from over heating; the water exits at a temperature of $T_h = 31°$ C. The copper pipe carries a direct 45 Amp current in order to generate the necessary magnetic field. At what rate must the cooling water flow be provided to the electromagnet? Express your answer in liters per second with only one significant digit. The specific heat capacity of water is 4200 J/°C · kg; the density of water is 1000 kg/m^3.

h. The protons are fired at a target consisting of Fluorine atoms ($Z = 9$). What is the distance of closest approach to the center of the Fluorine nuclei for the protons? You can assume that the Fluorine does not move.