AAPT UNITED STATES PHYSICS TEAM
AIP 2020

2020 $F = ma$ Exam

25 QUESTIONS - 75 MINUTES

INSTRUCTIONS

DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

- Use $g = 10$ N/kg throughout this contest.
- You may write in this booklet of questions. However, you will not receive any credit for anything written in this booklet. The only scratch paper you may use is scratch paper provided by the proctor. You may not use your own.
- This test contains 25 multiple choice questions. Select the answer that provides the best response to each question. Please be sure to use a No. 2 pencil and completely fill the box corresponding to your choice. If you change an answer, the previous mark must be completely erased. Only the boxes preceded by numbers 1 through 25 are to be used on the answer sheet.
- All questions are equally weighted, but are not necessarily of the same level of difficulty.
- Correct answers will be awarded one point; incorrect answers or leaving an answer blank will be awarded zero points. There is no additional penalty for incorrect answers.
- A hand-held calculator may be used. Its memory must be cleared of data and programs. You may use only the basic functions found on a simple scientific calculator. Calculators may not be shared. Cell phones cannot be used during the exam or while the exam papers are present. You may not use any tables, books, or collections of formulas.
- The question booklet, the answer sheet, and all scratch paper will be collected at the end of this exam.
- **In order to maintain exam security, do not communicate any information about the questions (or their answers or solutions) on this contest until after February 1, 2020.**

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We acknowledge the following people for their contributions to this year's exams (in alphabetical order):

Ariel Amir, JiaJia Dong, Mark Eichenlaub, Abijith Krishnan, Daniel Longenecker, Kye Shi, Brian Skinner, Paul Stanley, Mike Winer, and Kevin Zhou.

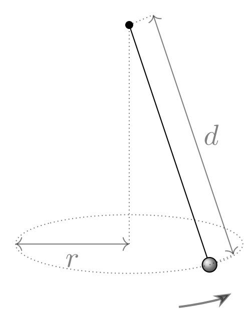
1. A ball is bouncing vertically between a floor and ceiling, which are both horizontal and separated by 4 m. All collisions are perfectly elastic, and when the ball hits the floor, it has a speed of 12 m/s. How long does a complete up-down cycle take?

   (A) 0.3 s
   (B) 0.4 s
   (C) 0.6 s
   (D) 0.8 s
   (E) 2.4 s

2. A uniform rod of mass m and length ℓ has moment of inertia $m\ell^2/12$ about an axis that is perpendicular to the rod and passes through its center. What is the moment of inertia of a uniform square plate with mass M and side length L about the axis along its diagonal?

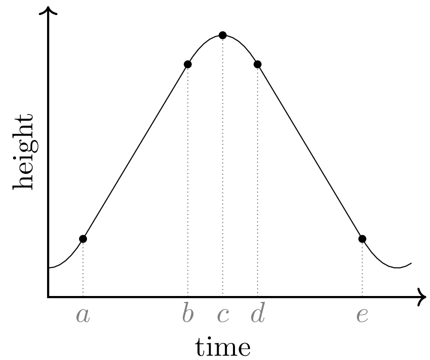
   (A) $ML^2/12$
   (B) $\sqrt{2}\,ML^2/12$
   (C) $ML^2/6$
   (D) $ML^2/4$
   (E) $ML^2/3$

3. A conical pendulum of length d swings in a horizontal circle of radius r, as shown. If ω is the angular frequency of this motion, what is ω^2?



   (A) g/d
   (B) g/r
   (C) $g/\sqrt{d^2 + r^2}$
   (D) $g/\sqrt{d^2 - r^2}$
   (E) $g/\sqrt{dr}$

4. In "zero-g" airplane rides, passengers can float around the cabin, as if they were weightless. A flight trajectory for such a ride is shown below, with a few points during the journey labelled. Which of the following statements is correct? Take **a** to be the acceleration of the plane during the "zero-g" part of the flight.



(A) The "zero-g" flight begins at a and ends at c, during which $|\mathbf{a}| = g$ and $\mathbf{a}$ points up.

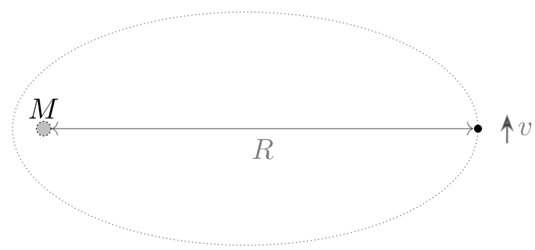
(B) The "zero-g" flight begins at a and ends at e, during which $|\mathbf{a}| = 0$.

(C) The "zero-g" flight begins at b and ends at d, during which $|\mathbf{a}| = g$ and $\mathbf{a}$ points down.

(D) The "zero-g" flight begins at c and ends at e, during which $|\mathbf{a}| = g$ and $\mathbf{a}$ points down.

(E) The "zero-g" flight begins at d and ends at e, during which $|\mathbf{a}| = 0$.

5. A mote of dust is initially located at distance R from the sun, which has mass M. At this point, the mote has a small tangential velocity v. Which of the following is a good approximation for the distance of closest subsequent approach between the mote and the sun?



(A) $\dfrac{R^3v^4}{G^2M^2}$

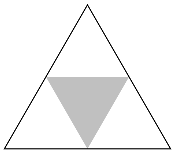
(B) $\dfrac{R^3v^4}{2G^2M^2}$

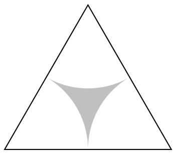
(C) $\dfrac{R^2v^2}{2GM}$

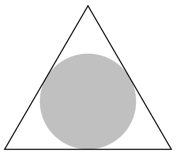
(D) $\dfrac{R^2v^2}{GM}$

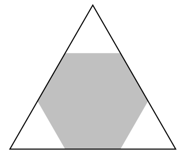
(E) $\dfrac{2R^2v^2}{GM}$

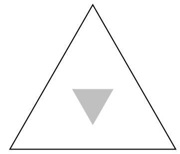
6. A three-legged table is shaped like a uniform equilateral triangle, and has identical legs at each corner. When the mass of an object placed on the center of the table exceeds $m_{\max}$, the table's legs will all simultaneously break. Which of the following shaded regions shows the area within which an object of $2m_{\max}/3$ can be placed without breaking any legs of the table?

(A) 

(B) 

(C) 

(D) 

(E) 

7. Two satellites are initially in identical circular orbits around the Sun, with orbital speed $v = 1 \times 10^4\,\mathrm{m/s}$. The first satellite fires its thrusters toward the Sun, and quickly obtains a radial velocity of $\Delta v_r = 1\,\mathrm{m/s}$. The second satellite instead fires its thrusters behind it, and quickly increases its tangential velocity by Δv_t. If the two satellites subsequently perform orbits with the same period, approximately what was Δv_t?

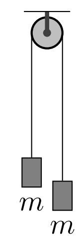
(A) $0.000\,05\,\mathrm{m/s}$
(B) $0.005\,\mathrm{m/s}$
(C) $0.5\,\mathrm{m/s}$
(D) $1\,\mathrm{m/s}$
(E) $50\,\mathrm{m/s}$

8. A conveyor belt is moving with velocity v to the east. A block with velocity v to the south slides from the ground onto the conveyor belt. The coefficient of friction between the block and the belt is μ. The block stops slipping after a time

(A) $\dfrac{v}{\sqrt{2}\,\mu g}$
(B) $\dfrac{v}{\mu g}$
(C) $\dfrac{\sqrt{2}\,v}{\mu g}$
(D) $\dfrac{2v}{\mu g}$
(E) The block never stops slipping.

The following information is relevant to problems 9 and 10.

9. Two equal masses m are connected by an elastic string that acts like an ideal spring with spring constant k and unstretched length l. The two masses are hung over a frictionless pulley. What is the total length of the string at equilibrium? (Diagram not necessarily to scale.)



(A) $2mg/k$
(B) $l + mg/k$
(C) $l + mg/2k$
(D) $l + 2mg/k$
(E) There is not enough information to decide.

10. The two masses are both displaced downward by a small vertical distance x and simultaneously released from rest. What is the period of oscillation?

(A) $2\pi\sqrt{l/g}$
(B) $\pi\sqrt{2m/k}$
(C) $2\pi\sqrt{m/k}$
(D) $2\pi\sqrt{m/k + l/g}$
(E) There is not enough information to decide.

11. An escalator can carry passengers up a vertical distance of 10 m in 30 s. A mischievous person of mass 50 kg walks down the up-escalator so that they stay in place with respect to the building. If the child does this for 30 s, the total work the child performs on the escalator, in the frame of the building, is

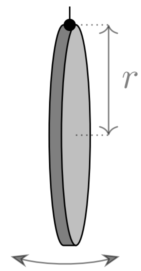
(A) -10^4 J
(B) -5×10^3 J
(C) 0 J
(D) 5×10^3 J
(E) 10^4 J

12. A platform juts out horizontally from the edge of a building. If the platform is modeled as a uniform metal rod, which one of the following statements about the tensile and compressive stress is correct?

(A) There is a horizontal compression throughout the rod.
(B) There is a horizontal tension throughout the rod.
(C) There is horizontal tension in the top of the rod and a compression in the bottom.
(D) There is a horizontal tension near the middle of the rod and a compression near the end.

(E) There is a horizontal compression near the middle of the rod and a tension near the end.

13. A uniform disk of mass m and radius r is attached at its edge to a flexible pivot on the ceiling. It is given a small displacement *perpendicular* to the plane of the disk, so that it begins to oscillate perpendicular to the plane of the disk. What is the period of oscillation? The moment of inertia of a disk about the axis going through its center and perpendicular to the plane it's in is $I_{\text{disk}} = \frac{1}{2}mr^2$

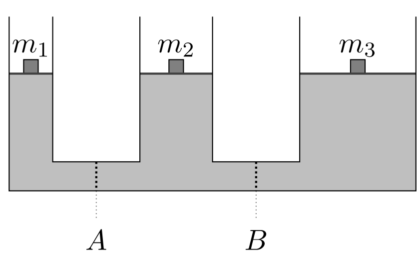


(A) $\pi\sqrt{2r/5g}$
(B) $\pi\sqrt{5r/g}$
(C) $\pi\sqrt{6r/g}$
(D) $2\pi\sqrt{r/g}$
(E) $2\pi\sqrt{2r/g}$

14. A large block of mass 5 kg is moving to the right at a velocity of 10 m/s. Spaced out every meter are smaller, initially stationary blocks of mass 1 kg. All collisions are perfectly inelastic. Neglecting friction, how far will the large block travel before its velocity has decreased to below 3 m/s?

(A) 5 m
(B) 8 m
(C) 12 m
(D) 17 m
(E) 50 m

15. In the device shown, blocks of various masses are placed on pistons so that the device is in equilibrium. (The fluid in the drawing is to scale.) There are valves that are both initially open at locations A and B. One of the valves is closed, and the system is allowed to come to equilibrium. How will this affect the height of the mass m_2?



(A) Closing either valve will have no effect on the height of m_2.
(B) Closing either valve causes m_2 to rise.
(C) Closing either valve causes m_2 to fall.
(D) Closing valve A causes m_2 to rise, and closing valve B causes m_2 to fall.
(E) Closing valve A causes m_2 to fall, and closing valve B causes m_2 to rise.

The following information is relevant to problems 16 and 17.

16. A mass M sits on top of a vertical spring of spring constant k, in equilibrium. A mass m is held a height h above it. The mass M is then pushed downward by a distance Δx, and both masses are released from rest simultaneously. For what value of h will the two masses first collide when M first returns to its equilibrium position?

(A) $\dfrac{\pi M^2 g^2}{4k^2 \Delta x}$

(B) $\dfrac{8Mg}{\pi^2 k}$

(C) $\dfrac{Mg}{k}$

(D) $\dfrac{\pi^2 Mg}{8k}$

(E) $\dfrac{\pi k \Delta x^2}{4Mg}$

17. Assume that h takes the value found in the previous question, and that the collision between the two masses is perfectly elastic. For what value of Δx will m rebound to a maximum height that is exactly equal to its original height?

(A) $\dfrac{\pi g}{2k}\sqrt{\dfrac{m^3}{M}}$

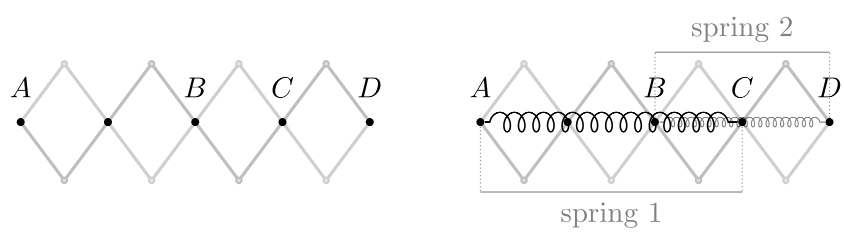
(B) $\dfrac{\pi g}{k}\sqrt{\dfrac{m^3}{2M}}$

(C) $\dfrac{2g}{\pi k}\sqrt{\dfrac{2m^3}{M}}$

(D) $\dfrac{4mg}{\pi k}$

(E) $\dfrac{\pi mg}{2k}$

18. An extendable arm is made from rigid beams free to pivot around the dots shown. Spring 1, with equilibrium length L_1, is attached between points A and C while spring 2, with equilibrium length L_2 is attached between B and D. The system is allowed to come to equilibrium. In equilibrium, what is the ratio of the tension in spring 1 to the tension in spring 2?



(A) $-2/3$

(B) $-3/4$

(C) -1

(D) $-3/2$

(E) There is not enough information to determine the ratio.

19. Consider an axially symmetric object that experiences no external forces and initially rotates about its symmetry axis. The object then changes its shape while remaining axially symmetric. Afterward, it is found that its moment of inertia about the symmetry axis has increased. How have its kinetic energy T and angular velocity ω changed?

(A) T has decreased, and ω has decreased.

(B) T has decreased, and ω stays the same.

(C) T stays the same, and ω has decreased.

(D) T has increased, and ω has increased.

(E) T has increased, and ω stays the same.

20. A well-calibrated scale reads zero when nothing is placed on it. When a deflated balloon is placed on the scale, it reads mg. When a non-airtight box is placed on the scale, it reads Mg, where $M > m$. The balloon is inflated with helium, so that it floats upward in air. The balloon is placed in the box, and the box is placed on the scale. If the scale reads W, which of the following is true?

(A) $W \leq (M - m)g$

(B) $(M - m)g < W < (M + m)g$

(C) $W = (M + m)g$

(D) $W > (M + m)g$

(E) None of the above are necessarily true.

21. A person stands on the seat of a swing and squats down, so that the distance between their center of mass (CM) and the swing's pivot is ℓ. As the swing gets to the lowest point, the speed of their CM is v. At this moment, they quickly stand up, and thus decrease the distance from their CM to the swing's pivot to ℓ'. Immediately after they finish standing up, their CM speed is v'. Which of the following statements is correct? You may neglect friction, the change in moment of inertia of the person about their CM, and the time taken to stand up.

(A) $v/\ell = v'/\ell'$

(B) $v = v'$

(C) $v\ell = v'\ell'$

(D) $\frac{1}{2}v^2 = \frac{1}{2}v'^2 + g(\ell - \ell')$

(E) Multiple statements are correct.

22. A point mass m sits on a long block, also of mass m, which rests on the floor. The coefficient of static and kinetic friction between the mass and the block is μ, and the coefficient of static and kinetic friction between the block and the floor is $\mu/3$. An impulse gives a horizontal momentum p to the point mass. After a long time, how far has the point mass moved relative to the block? Assume the mass does not fall off the block.

(A) $\dfrac{3p^2}{8m^2\mu g}$

(B) $\dfrac{15p^2}{32m^2\mu g}$

(C) $\dfrac{9p^2}{16m^2\mu g}$

(D) $\dfrac{3p^2}{10m^2\mu g}$

(E) $\dfrac{3p^2}{4m^2\mu g}$

23. Assume that the drag force for a fish in water depends only on the typical length scale of the fish R, its velocity v, and the density of water ρ. A pufferfish is about 10 cm in length and swims at about 5 m/s. How fast does a clown fish, about 1 cm in length, need to swim such that it experiences the same drag force as the pufferfish?

(A) 5 m/s

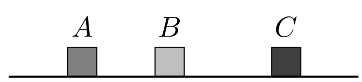
(B) 16 m/s

(C) 50 m/s

(D) 500 m/s

(E) 2500 m/s

24. Three boxes A, B and C lie along a straight line on a horizontal frictionless surface, as shown. Box A is initially moving to the right with speed v while the other two boxes are initially at rest. If all collisions are elastic, and the masses of the boxes can be chosen freely, which of the following is closest to the maximum possible final speed of box C?



(A) v

(B) $2v$

(C) $3v$

(D) $4v$

(E) $5v$

25. If a certain radioactive decay process happens n times per hour on average, then in any given hour, one expects to observe n decays with an uncertainty of $\sqrt{n}$. Each hour can be assumed independent of the previous one, and n can be assumed constant over time. How many hours do you need to conduct the observation so that you can determine n within an uncertainty of 1%?

(A) $n/10^2$

(B) $n/10^4$

(C) $10^2/n$

(D) $10^4/n$

(E) $10^8/n^2$