What Is Problem Solving?

BY MICHAEL E. MARTINEZ

Errors are part of the process of problem solving, which implies that both teachers and learners need to be more tolerant of them, Mr. Martinez points out. If no mistakes are made, then almost certainly no problem solving is taking place.

To think is constantly to choose in view of the end to be pursued.1

EVERY educator is familiar with the term “problem solving,” and most would agree that the ability to solve problems is a worthy goal of education. But what is problem solving? Its meaning is actually quite straightforward: problem solving is the process of moving toward a goal when the path to that goal is uncertain. We solve problems every time we achieve something without having known beforehand how to do so. We encounter simple problems every day: finding lost keys, deciding what to do when our car won’t start, even improvising a meal from leftovers. But there are also larger and more significant “ill-defined” problems, such as getting an education, becoming a successful person, and finding happiness. Indeed, the most important kinds of human activities involve accomplishing goals without a script.

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Problem solving is a ubiquitous feature of human functioning. Human beings are problem solvers who think and act within a grand complex of fuzzy and shifting goals and changing means to attain them. This has always been true, but it is doubly so today because we live in a time of unprecedented societal transformation. When circumstances change, old procedures no longer work. To adapt is to pursue valued goals even when circumstances—and perhaps the goals themselves—are in flux. Because the pace of societal change shows no signs of slackening, citizens of the 21st century must become adept problem solvers, able to wrestle with ill-defined problems and win. Problem-solving ability is the cognitive passport to the future.

There is no formula for true problem solving. If we know exactly how to get from point A to point B, then reaching point B does not involve problem solving. Think of problem solving as working your way through a maze. In negotiating a maze, you make your way toward your goal step by step, making some false moves but gradually moving closer to the intended end point. What guides your choices? Perhaps a rule like this: choose the path that seems to result in *some* progress toward the goal. Such a rule is one example of a *heuristic*. A heuristic is a rule of thumb. It is a strategy that is powerful and general, but not absolutely guaranteed to work. Heuristics are crucial because they are the tools by which problems are solved.

By contrast, algorithms are straightforward procedures that are guaranteed to work every time. For example, you have in your long-term memory algorithms that enable you to tie your shoelaces, to start up your car, and perhaps even to cook an omelet. Barring broken shoelaces, a dead battery, and rotten eggs, these algorithms serve you very well. An algorithm may even be so automatic that it requires very little conscious processing as you carry out the procedure.

Now here is an important consideration: what constitutes problem solving varies from person to person. For a small child, tying shoelaces will indeed require problem solving, just as cooking an omelet entails problem solving for many adults. Thus problem solving involves an interaction of a person’s experience and the demands of the task. Once we have mastered a skill, we are no longer engaged in problem solving when we apply it. For a task to require necessity of problem solving to all that is important about being a person cannot be overstated.

In addition, problem solving is not an advanced process that is reserved solely for mature learners. Indeed, people of all ages can and must be solvers of problems. Perhaps young children are the most natural problem solvers. Because they continually face circumstances that are novel, they must adapt. It’s their “job.” And they are amazing good at it. Moreover, young children don’t fret about failure the way that school-age children and adults tend to do. They take detours and setbacks in stride because they know intuitively that such obstacles are a part of the problem-solving process. Still, we need to encourage problem solving in children. Whenever possible, this involves letting children find their own ways of reaching their goals. Good parents and other caregivers know when to stand back and let a child figure things out and when to step in and offer the right amount of help.

Armed only with our heuristics, then, we engage in a process of *heuristic search*. Like finding one’s way through a maze, we move closer, haltingly, to where we want to be. We can’t be sure of what lies around the next corner or that the direction that once seemed so promising will pay off. Progress toward important goals is incremental, and each move is informed by our repertoire of heuristics. Because of the possibility of false moves, we need to monitor our progress continually and switch strategies if necessary.

The Power of Heuristics

If heuristics are the problem solver’s best guide, it makes sense to elucidate them as much as possible. First, each learner must know what heuristics are and must be aware of their power. Second, each learner must have both general and specific heuristics at his or her disposal. General heuristics are cognitive “rules of thumb” that are useful in solving a great variety of problems. They are usually content-free and apply across many different situations. Specific heuristics are used in specialized areas, often specific subject domains or professions.

Probably the most powerful general heuristic, alluded to in the maze example, is “means-ends analysis.” Essentially, the heuristic is this: form a subgoal to reduce the discrepancy between your present state and your ultimate goal state. Phrased more colloquially: do something to get a little closer to your goal.

Problems defy one-shot solutions; they must be broken down. Means-ends analysis accepts incremental advancement toward a goal. The method is not fail-safe, of course, because positive results are not guaranteed with any heuristic. However, if all goes well, this heuristic will help move you incrementally toward your ultimate goal. You apply it again and again, trying to reduce the discrepancy further. By means of this less-than-direct path, you find your way to the ends you seek. Such a search is not simply a process of trial and error, because the steps taken are not blind or random. Rather, the application of a series of tactical steps leads you ever closer toward the goal. Mistakes made along the way must be accepted as inextricable from the problem-solving process.

The benefits conferred by means-ends analysis may be as much emotional as intellectual. If a large and complex problem seems daunting as a whole, perhaps one can summon the will to accomplish a small piece of it. And that success can motivate one to persist. Thus starting a task can make
the effort self-sustaining. Sometimes when we tackle a difficult project, it's as if we are trying to start a car on a cold winter morning. We encounter resistance. Once begun, however, the task becomes marginally easier and doesn't require a constant exertion of will to sustain it. At some point, we "cross the Rubicon"—we reach the point where it seems more difficult to stop than to carry on to completion. That is when a problem-solving activity becomes self-sustaining and bears us along by its momentum. "Just do it!" is not solely a great marketing slogan; it can also be seen as a directive to disregard the ominous hulking problem that looms ahead and simply take the first step.

Heuristics are usually picked up incidentally—often identified and taught explicitly in school. This situation is not ideal. A curriculum that encourages problem solving needs to provide more than just practice in solving problems; it needs to offer explicit instruction in the nature and use of heuristics. Herbert Simon has written:

In teaching problem solving, major emphasis needs to be directed toward extracting, making explicit, and practicing problem-solving heuristics—both general heuristics, like means-ends analysis, and more specific heuristics, like applying the energy conservation principle in physics.'

What are some other heuristics? One that is probably familiar to most readers goes by the name of "working backward." First, consider your ultimate goal. From there, decide what would constitute a reasonable step just prior to reaching that goal. Then ask yourself, What would be the step just prior to that? Beginning with the end, you build a strategic bridge backward and eventually reach the initial conditions of the problem.

An illustration of the use of this approach can be taken from the Tower of Hanoi problem. A number of disks are placed on a peg in an arrangement like this:

![Diagram of the Tower of Hanoi problem]

The rules are simple. Only one disk can be moved at a time, and a larger disk may never be placed on top of a smaller disk. The goal is to move the entire stack of disks from the first peg to the third. Working backward helps us understand that at some point we must find a way to place the largest disk at the bottom of the third peg. Working backward from there, we would infer that all the smaller disks would eventually need to be placed on the middle peg, according to the rules, so that the largest disk is free to move. That step also has logical precursors, and so on. Working backward makes the problem more manageable and its solutions much more efficient than following a less reasoned approach.

Or take another example. My daughter came home from school with a story about a provocative exchange between a teacher and a student:

Teacher: What do you want to be when you are an adult?  
Student: I want to be rich.  
Teacher: No, but what do you want to be?  
Student: I don't care. I just want to be rich.

This student certainly had a clear goal in mind, though some might question its value independent of the means for achieving it. In any case, the student has some serious "working backward" to do. If his goal is to be rich, what kind of career might allow him to achieve it? Becoming a movie star? A Wall Street investor? An entrepreneur? A criminal? Some combination of these? If an entrepreneur, that might imply that majoring in business in college would be in order. In turn, that goal might suggest that tonight the student should study his mathematics a little harder than is his custom. Working backward makes "next steps" plainer than simply wishing and hoping that dreams will materialize.

A third heuristic seeks to solve problems through "successive approximation." Initial tries at solving a problem may result in a product that is less than satisfying. Writing is a good example. Few accomplished writers attempt to write perfect prose the first time they set pen to paper (or fingertips to keyboard). Rather, the initial goal is a rough draft or an outline or a list of ideas. Over time, a manuscript is gradually molded into form. New ideas are added. Old ones are removed. The organization of the piece is reshaped to make it flow better. Eventually, a polished form emerges that finally approximates the effect that the author intended.

Given time and effort, what started out as rough and approximate can become art. In fact, successive approximation seems to be an important heuristic in producing outstanding creative works of all kinds. This model is relevant to many pursuits other than writing. Inventions, theories, stories, recipes, and even personal and group identities start out rough but are restructured and refined over time. Think of the bicycle, whose various designs over the decades have metamorphosed toward greater efficiency and lighter weight. Successive approximation accepts the design process as problem solving, a series of ziggs and zags toward something better. Not only is such a process compatible with human information processing, but awareness of the principle can sustain a half-baked idea that initially seems raw, wild, and foolish but is just possibly the germ of an eventual marvel.

George Polya's advice was "Draw a figure." In that spirit, I offer a fourth and final example of a heuristic: portray the problem at hand in an explicit "external representation." List, describe, diagram, or otherwise render the main features of a problem. This heuristic has several important features. First, it allows us to represent more complexity than we can hold in mind at once. Depicting a problem on paper, whiteboard, or computer screen relieves short-term memory of the burden of representing the problem and allows the processing capacity of our brains to be directed toward solving it. An incidental benefit is that often the very attempt to
Metacognition

All heuristics help break down a problem into pieces. The problem as a whole is thus transformed. It is no longer a chaotic mass, like a ton of cooked spaghetti. Rather, through the creation of various subgoals, each of the pieces becomes manageable. The problem does become more complex in one sense because the pieces themselves must somehow be borne in mind. If a large goal is broken down into subgoals, then one cognitive challenge becomes goal management — keeping track of what to do and when. Goal management is probably a major aspect of intelligent thought. Patricia Carpenter, Marcel Just, and Peter Shell regard goal management as a central feature of problem solving.

A key component of analytic intelligence is goal management, the process of spawning subgoals from goals, and then tracking the ensuing successful and unsuccessful pursuits of the subgoals on the path to satisfying higher-level goals. . . . The decomposition of complexity . . . consists of the recursive creation of solvable subproblems. . . . But the cost of creating embedded subproblems, each with [its] own subgoals, is that they require management of a hierarchy of goals.

The importance of monitoring subgoals is an example of a more general phenomenon: one common feature of problem solving is the capacity to examine and control one’s own thoughts. This self-monitoring is known as metacognition. Metacognition is essential for any extended activity, especially problem solving, because the problem solver needs to be aware of the current activity and of the overall goal, the strategies used to attain that goal, and the effectiveness of those strategies. The mind exercising metacognition asks itself, What am I doing? and How am I doing? These self-directed questions are assumed in the application of all heuristics. However, in practice, teachers cannot simply assume that students will engage in metacognition; it must be taught explicitly as an integral component of problem solving.

Problem solving requires both the vigilant monitoring and the flexibility permitted by metacognition. When solving problems, means shift continually depending on one’s position relative to desired goals. Even goals change as old goals are superseded by new and better ones. Maintaining flexibility is essential. Too often we feel wedded to a chosen strategy and continue to apply that strategy even if it leads us wildly astray. When this happens, it is usually wrong to conclude that we must start over. The important question is always “What do I do now, given my goal, my current position, and the resources available to me?” Getting off course along the way is fully expected. Cool-headed reappraisal is the best response — not mindless consistency, panic, or surrender.

A New Mindset

In pursuit of the goal of improving problem-solving ability, I have advocated the use of heuristics and have suggested a few. There are countless others. Some are general and apply to many problem situations, but most are specific and apply in specialized fields. Heuristics are vital, but they are not necessarily the most important aspect of problem solving.

Perhaps more powerful than any heuristic is an understanding that, by its very nature, problem solving involves error and uncertainty. Even if success is achieved, it will not be found by following an unerring path. The possibilities of failure and of making less-than-optimal moves are inseparable from problem solving. And the loftier the goals, the more obvious will be the imperfection of the path toward a solution. The necessity of uncertainty is recognized implicitly whenever we commend someone for being a risk taker. It is not the taking of risks itself that is commendable; rather, taking risks is a means to an end. What we actually applaud is the courage to adopt a difficult and commendable goal and then to enter the thorny thicket of problem solving where the only way out is through heuristic search and nerve.

The willingness to suspend judgment — to accept temporary uncertainty — is an important aspect of thinking in general. John Dewey linked tolerance of uncertainty to reflective thinking:

Reflective thought involves an initial state of doubt or perplexity. . . . To many persons both suspense of judgment and intellectual search are disagreeable; they want to get them ended as soon as possible. . . . To be genuinely thoughtful, we must be willing to sustain and protract the state of doubt, which is the stimulus to thorough inquiry.

How then is it possible to improve problem-solving ability? First, we need to recognize when we are engaged in problem solving and accept as natural, normal, and expected the stepwise and discursive path
toward a goal through the application of general and specific heuristics. Second, we must not let anxiety take hold. Anxiety is a spoiler in the problem-solving process. It stalks right behind uncertainty, ready to pounce. Demanding and uncertain environments, the seedbeds of all problem solving, are fertile ground for anxiety. Uncertainty is an integral part of the business of solving problems. Those who cannot bear situations in which it is impossible to see the way clearly to the end are emotionally ill-prepared to solve problems.

Errors are a part of the process of problem solving, which implies that both teachers and learners need to be more tolerant of them. If no mistakes are made, then almost certainly no problem solving is taking place. Unfortunately, one tradition of schooling is that perfect performance is often exalted as an ideal. Errors are seen as failures, as signs that the highest marks are not quite merited. Worse still, errors are sometimes ridiculed or taken as ridiculous. Mistakes and embarrassment often go hand in hand. Perfect performance may be a reasonable criterion for evaluating algorithmic performance (though I doubt it), but it is incompatible with problem solving.

What so often counts most in schools is the important but incomplete cognitive resource of knowledge. Fixed knowledge and algorithms are easier to teach, learn, and test than is the tangled web of processes that make up problem solving. Typically, it is not before graduate school that problem solving really becomes the focus of an educational program. Even in graduate school a student may not get to wrestle with the true problems of a field of study until the dissertation.

What can reverse this sorry state of affairs? A better understanding of the nature of problem solving is a place to start. Ultimately, we will have to change the culture of schooling. In the workplace as well, we need to revise our attitude toward errors — at least toward those that are a reasonable consequence of significant problem solving. (Errors in balancing the books don’t count.) But if a job requires fluid intelligence — the ability to operate within the flux of continually changing demands and challenges — even the corporate culture must accept and deal with the multitude of paths toward solutions and the necessary existence of error.

For educators to accept errors, uncertainty, and indirect paths toward solutions is itself a difficult problem because doing so contradicts our ingrained beliefs and expectations about teaching and learning. But problem solving must be understood and promoted if the next generation is to be prepared for the unprecedented challenges (i.e., problems) that it will face. Yet great things are accomplished when great things are attempted, and in our efforts we do not face total uncertainty. We have, in fact, our experience and its dividend, our knowledge, to support us. Heuristics and knowledge are what Herbert Simon has called the "two blades" of effective professional education, and he reminds us that "two-bladed scissors are still the most effective kind."[1] I would add that what is good for professional education is good for education of all kinds at all levels. By combining what we do know with our understanding of the problem-solving process, we can move toward our goals — perhaps not unerringly, but by the sort of wending progress that is the signature of problem solving.

5. George Polya, How to Solve It, 2nd ed. (Garden City, N.Y.: Doubleday, 1957).
10. It is not impossible to solve a problem without error, but it is misleading to think that this experience is the normal character of problem solving.