Apparatus Title: An Affordable Physical Pendulum: The Pipe Pendulum

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Abstract: Introductory mechanics includes the study of mechanical oscillators, and oscillators with more than one control variable pose a useful challenge to students. We therefore built affordable physical pendulums from sections of PVC pipe suspended by string. We measured the period of small amplitude oscillations using a stopwatch and found that pipes shorter than 45 cm have measured periods that agree with theoretical predictions to within 2%. A 3D-printed clamp assembly facilitates variation of string length.
**Construction of Apparatus:**

A photograph of the entire assembly is shown in Fig. 1a. We assume that a ring stand with a 90 degree clamp is available so that a 0.5” diameter rod can be held horizontally. The 3D-printed clamp assembly [1] can be fastened to the rod with ¼”-20 machine screws, and a 2-m long piece of nylon upholstery thread is held in place by the two string clamping nuts shown in Fig. 1b.

Although the 3D-printed clamp assembly allows the length of the suspending string to be easily changed, it is not necessary. We have also used a setup consisting of C-clamps, wood blocks, and 3” mending plates to support the pipe pendulum and have obtained similar results with both support setups. If using the setup with the mending plates, the experimenter should ensure that the lower edges of the plates are horizontal, to reduce twisting of the pipe section.

![Fig. 1. (a) The pipe pendulum. (b) The 3D printed clamp assembly.](image)
The pipe sections are cut to length using a PVC pipe cutting tool and two sewing needles are attached to the outside surface of the pipe section so that they are located on diametrically opposed sides of the pipe. The needles can be attached to the pipe with one layer of transparent, 2-in. wide packing tape although for longer (i.e., heavier) pipe sections, the needles should be additionally fastened with superglue. The eye of each needle is positioned to be just above the upper end of the pipe section.

**Use of Apparatus:** After checking that the top edge of the pipe section is horizontal, and that the tension in the support string is as uniform as possible, the pendulum is given a small angular displacement and released from rest. The experimenter waits until any twisting or rocking motion of the pipe section ceases and the angular amplitude of the swing is no more than roughly 5 degrees. The experimenter then measures the amount of time for 10 oscillations in order to reduce the uncertainty, due to either reaction time (for hand-timing with a stopwatch) or choosing video frames (for timing via video analysis). The distance $z$ from the rotation axis to the top edge of the pipe section is easily varied by loosening one of the string clamping nuts of the 3D-printed clamp assembly, changing the length of the suspending string, and re-tightening the clamping nut. The distance $z$ is measured with a meter stick.

Using the above procedure, this physical pendulum apparatus can be used by students to experimentally determine the period of small oscillations versus $z$ and pipe section length. Data obtained from 0.5-in. PVC pipe sections of length 8 and 32 cm are shown in Fig. 2. We find that the predicted values agree with the measured values to within 2%. 

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Fig. 2. Plot of experimentally determined period $T$ versus distance from the support to the top of the pipe $z$, for 8 cm and 32 cm pipes.

The theoretically predicted relationship between the period of small oscillations and the physical parameters of the system is not trivial [2], and so students are unlikely to have strong expectations regarding this relationship. Consequently, students realize that they must rely on their own measurements to determine the relationship. Depending on the level of the course, students can be guided to derive the theoretically predicted relationship using the parallel axis theorem, and to compare the values predicted with this relationship to those that they measured.

A classroom of secondary students could be given the same length pipe section and be asked to measure the period of small oscillations for different intervals of string lengths; within an hour, a class of 30 students (10 teams) could look at three different intervals of $z$ values (e.g., 5 to 20 cm, 25 cm to 45 cm, 50 to 70 cm) and combine their data to establish that the aggregated data are not linear. The different teams will need to combine their data to draw correct conclusions about the dependence of small oscillation period versus string length. Figure 3 shows a plot of data...
obtained from three different observers using different string types and different apparatus for suspending a 25 cm pipe pendulum.

Fig. 3. Plot of experimentally determined period $T$ versus distance from the support to the top of the pipe $z$, for a 25 cm pipe. Measurements were performed by three different observers (Obs.), with different suspension systems, and different string types (UT = nylon upholstery thread, GT = general thread, CT = fine coated thread).

An undergraduate laboratory could possibly involve giving different teams of student pipe sections of different lengths. Given the ease and affordability of producing different pipe sections, it is possible to avoid duplication of results within the same lab. It is also possible for students to experiment with pipe sections of equal length but different cross sections to explore the effect of cross section. At the intermediate undergraduate level, video analysis could be used to study dynamics and check the predictions obtained from computational models of the motion.

We find that pipe sections longer than 45 cm can give results that deviate from our predicted small oscillation period by an amount that increases with section length.

The educational value of the apparatus lies in its use to develop a variety of skills. The AAPT Recommendations for the Undergraduate Physics
Laboratory Curriculum [3] identify several skills for development, including technical and practical laboratory skills, analyzing and visualizing data, and communicating physics. Here, students develop these skills by making measurements of time intervals and length, estimating uncertainties, performing error analysis, plotting data (including multiple data sets), and communicating the relationship they have determined between the period and $\pi$ for one or more pipe sections. Additionally, modeling and computational physics skills can be developed, including theoretical modeling, and implementing, testing, and debugging a computational model. As mentioned above, it is possible to compare the dynamics obtained through video analysis to a model of harmonic (small-amplitude) motion. The development of these modeling and computational physics skills is consistent with the performance expectations described in the Next Generation Science Standards and the learning outcomes described in the AAPT Recommendations for Computational Physics in the Undergraduate Physics Curriculum [4,5].

**Parts List:**  

$68.27 \quad $ Total cost for 10 set ups

Here is the detailed list of items for 10 set ups with clamp assemblies:

<table>
<thead>
<tr>
<th>Cost</th>
<th>Description (Vendor)</th>
<th>Vendor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$01.85</td>
<td>½” PVC Pipe, 10 ft section</td>
<td>Home Depot</td>
</tr>
<tr>
<td>$13.98</td>
<td>Ratcheting PVC cutter</td>
<td>Home Depot</td>
</tr>
<tr>
<td>$04.65</td>
<td>Dritz Sharps Hand Sewing Needles, Size 7</td>
<td>Amazon</td>
</tr>
<tr>
<td>$03.97</td>
<td>Super glue, 2-pack</td>
<td>Home Depot</td>
</tr>
<tr>
<td>$05.45</td>
<td>Coats and Clark Extra Strong Upholstery Thread</td>
<td>Amazon</td>
</tr>
<tr>
<td>$04.53</td>
<td>2” Transparent Packing Tape</td>
<td>Amazon.com</td>
</tr>
<tr>
<td>$12.00</td>
<td>20 ¾”-20 Hex-Head Serrated Flange bolts</td>
<td>Home Depot</td>
</tr>
<tr>
<td>$07.36</td>
<td>2 15-pack ¾”-20 nuts</td>
<td>Home Depot</td>
</tr>
<tr>
<td>$09.48</td>
<td>½”-20 tap to thread the clamping holes</td>
<td>Home Depot</td>
</tr>
<tr>
<td>$05.00</td>
<td>eSun 3mm PLA plastic, 1 kg reel</td>
<td>Lulzbot</td>
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</tbody>
</table>
References:

[1] The authors will provide STL files for 3D printing upon request.
[2] Details can be obtained from the first author.