2017 PhysicsBowl Solutions

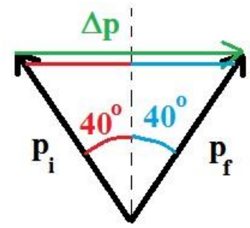
#	Ans	#	Ans	#	Ans	#	Ans	#	Ans
1	A	11	C	21	E	31	A	41	C
2	D	12	D	22	E	32	C	42	C
3	E	13	B	23	E	33	A	43	E
4	A	14	D	24	D	34	C	44	B
5	E	15	C	25	B	35	B	45	A
6	C	16	D	26	A	36	E	46	D
7	B	17	C	27	C	37	B	47	D
8	D	18	E	28	A	38	A	48	C
9	B	19	B	29	E	39	A	49	B
10	B	20	D	30	A	40	D	50	B

1. **A…** $\frac{12\ mm}{3\ \mu s} \times \frac{1\times10^{-3}\ m}{1\ mm} \times \frac{1\ \mu s}{1\times10^{-6} s} = 4 \times 10^3 \frac{m}{s}$. So, this is $4\frac{km}{s}$.
2. **D…** Using constant acceleration kinematics, we have from $y = y_0 + v_0 t + \frac{1}{2}at^2$ that $0 = H + 0 - \frac{1}{2}(10)(4.0)^2$ and so $H = 5(4.0)^2 = 80\ m$.
3. **E…** Acceleration is the only quantity that has a direction associated with it from the list given.
4. **A…** Water undergoes a phase change to a solid at 0 °C which is equivalent to 32 °F and 273 K.
5. **E…** An object with constant linear momentum will have a constant velocity. This relates most directly to Newton's First Law (a body in motion continues in uniform motion).
6. **C…** The period of an ideal mass-spring system does not depend on the amplitude of the oscillation (it depends on spring constant and mass), meaning that the period is unchanged.
7. **B…** Using that wave speed is computed as $v = f\lambda$, we have $\lambda = \frac{v}{f} = \frac{6.0\frac{m}{s}}{3.0\ Hz} = 2.0\ m$. This means that in a one-meter section of string, we would need to find ½ of a full wavelength. Picture C is one full wavelength whereas picture A is ¼ of a wavelength.
8. **D…** For the acceleration to be $0\frac{m}{s^2}$ in a position vs time graph, the position would need to be linear in time. The graph is linear from $0\ s < t < 3\ s$ and $7\ s < t < 10\ s$ whereas the position is changing non-linearly at all other times, meaning that the acceleration is non-zero for $3s < t < 7s$.
9. **B…** Using linear momentum conservation, we have $p_{1i} + p_{2i} = p_{1f} + p_{2f}$ and so
$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \rightarrow (4)(5) + (5)(-3) = (4)(-1) + 5(v_{2f}) \rightarrow$$
$5 = -4 + 5v_{2f} \rightarrow\ v_{2f} = \frac{9}{5} = 1.8\frac{m}{s}$. The positive sign indicates that the mass moves to the right after the collision.
10. **B…** Making the free body diagram of the car at the top of the hill gives just a normal force upward and a gravitational force downward. Since the car is traveling along the arc of a circle, it is accelerating toward the center of that circle… in other words, the acceleration is downward. So, choosing upward as the positive direction, we write $F_{net} = ma \rightarrow\ n + (-mg) = m\left(-\frac{v^2}{r}\right)$. And so, $n = mg - m\frac{v^2}{r} = m\left(g - \frac{v^2}{r}\right) = (2000)\left(10 - \frac{12^2}{25}\right) = 8480\ N$.
11. **C…** We need to find the speeds of the object at the start and end of the motion. From $KE = \frac{1}{2}mv^2 \rightarrow v = \sqrt{2\frac{KE}{m}}$, we find $v_0 = \sqrt{2\left(\frac{50}{5}\right)} = \sqrt{20} = 4.47\frac{m}{s}$ and $v_f = \sqrt{2\left(\frac{20}{5}\right)} = \sqrt{8} = 2.83\frac{m}{s}$. With constant acceleration, the average velocity is $\frac{1}{2}(v_0 + v_f) = \langle v\rangle = \frac{1}{2}(4.47 + 2.83) = 3.65\frac{m}{s}$. Since the object doesn't change direction, the average speed also is $3.65\frac{m}{s}$.
12. **D…** Using kinematics, we have $v = v_0 + at \rightarrow a = \frac{v - v_0}{t} = \frac{2.83 - 4.47}{5} = -0.328\frac{m}{s^2}$

13. **B…** According to current theory, the majority of the Universe consists of dark matter and dark energy with about 5% "normal matter", ~25% dark matter and ~70% dark energy.
14. **D…** Newton's Third Law deals with an interaction between two bodies. Here, the string exerts a force on the block with the block exerting an equal sized force on the string.
15. **C…** Taking the system as the two masses, we write Newton's Second Law to determine the acceleration of the blocks. This gives $F_{net} = ma \rightarrow \; 35 = (21)a \rightarrow \; a = 1.67\frac{m}{s^2}$. So, in analyzing the 15 kg block, we find that there is only the force of the 6 kg onto it acting. This means that $F_{net} = ma \rightarrow \; F_{X\,on\,Y} = (15)(1.67) = 25\,N$. By Newton's Third Law, this is then the force that the 15 kg block exerts onto the 6 kg block ($|F_{Y\,on\,X}|$) .
16. **D…** From $v = f\lambda$, we determine the wavelength of the light to be $\lambda = \frac{v}{f} = \frac{3\times10^{8}}{1\times10^{18}} = 3\times10^{-10}m$ Converting this into nanometers gives $\lambda = 0.3\,nm$. This wavelength is about 1000 times smaller than what one might expect for ultraviolet light (violet light's wavelength is approximately 400 nm). So, we have light much more energetic (higher frequency) than ultraviolet light. Of the choices given, this puts the light in the X-ray range.
17. **C…** Adding the values together gives 1005.839. Because 920. only goes as far as the nearest whole number and the other lengths have values after the decimal, we record the answer as a whole number. Hence, we would write the sum as 1006 with proper significant digits.
18. **E…** The average angular velocity for this motion could be computed as $\langle\omega\rangle = \frac{1}{2}(\omega + \omega_0) = \frac{\Delta\theta}{\Delta t} \rightarrow (0 + \omega_0) = 2\left(\frac{3.5\,rev}{2.50\,s} \times \frac{2\pi\,rad}{1\,rev}\right) \rightarrow \; \omega_0 = 17.6\frac{rad}{s}$.
19. **B…** Using the Ideal Gas Equation, we write $PV = nRT \rightarrow \; P = \frac{nRT}{V} = \frac{(3\,mol)\left(0.0821\frac{L\cdot atm}{mol\cdot K}\right)(373\,K)}{5\,L} = 18.37\,atm$. Converting to Pascals, $18.37\,atm \times \frac{1.013\times10^{5}Pa}{1\,atm} = 1.86\times10^{6}Pa$
20. **D…** cyan light is a combination of blue and green. This means that the object in question reflects green light and absorbs blue light. Since yellow light is made of green and red, we know that green light will reflect and the red light either will reflect (resulting in the object appearing yellow) or be absorbed (resulting in the object appearing green).
21. **E…** By the impulse-momentum theorem, the change in linear momentum is the force multiplied by the time. Hence, since the force is mass multiplied by acceleration, the change in linear momentum is in the same direction as the acceleration.
22. **E…** The density of the fluid in the pool $\left(500\frac{kg}{m^3}\right)$ is less than the density of water $\left(1000\frac{kg}{m^3}\right)$, which is very close to the density of a human being. This means that when the person jumps into the pool, they will sink all the way to the bottom.
23. **E…** The overall kinetic energy associated with the system of the two objects colliding will decrease as a result of the collision. Which object(s) specifically will lose kinetic energy is unknown without more information. For example, if a moving object collides and stick to an initially stationary object, the stationary object will gain kinetic energy from the collision!
24. **D…** Much like linear momentum change is related to force multiplied by time, the change in angular momentum is related to torque multiplied by time.
25. **B…** Taking the system to be the block and incline and Earth, we can write $\Delta KE + \Delta PE + \Delta E_{Thermal} = 0$ which can be expressed as $\frac{1}{2}m\left(v_f^2 - v_0^2\right) + mg\Delta y + f_k d = 0$. From the free body diagram of the block, we can write that $n = mg\cos\theta$. Also, we have that the length $d = \frac{h}{\sin\theta}$. Putting this information together leads to $\frac{1}{2}m(0 - 9^2) + m(10)(2.6) + \mu_k\left(m(10)\cos 30^\circ\right)\left(\frac{2.6}{\sin 30^\circ}\right) = 0$ Solving for the coefficient of friction (after cancelling the mass) leads to $-41.5 + 26 + 45\mu_k = 0 \rightarrow \; \mu_k = \frac{15.5}{45} = 0.322$.

26. A… From the free body diagram, we note that there are two forces acting, a string force and the gravitational force. Breaking the forces into components and writing Newton's Second Law, we have: $F_{net_y} = ma_y \rightarrow T\cos\theta + (-mg) = 0 \rightarrow T = \frac{mg}{\cos\theta}$. Now, by looking at the horizontal direction, we write $T\sin\theta = m\left(\frac{v^2}{r}\right)$. Substituting for the tension leads to $mg\tan\theta = \frac{mv^2}{r}$ or $v = \sqrt{gr\tan\theta}$. The only thing left to do is to determine the radius of the circle traversed… and that is found as $r = L\sin 50^\circ$ leading to $v = \sqrt{gL\tan\theta\sin\theta} = \sqrt{(10)(3)\tan 50^\circ \sin 50^\circ} = 5.23\frac{m}{s}$.

27. C… If the picture is rotated slightly so that there is an equal angle on each side between initial and final momentum, we can break down the change in momentum by components. From the figure, we see that the momentum change would be $\Delta p = 2p_i \sin 40^\circ = 2(mv\sin 40^\circ) = 25.71\ Ns$. By the impulse-momentum theorem, we find then that the average force exerted on the object was $F_{net} = \frac{\Delta p}{\Delta t} = \frac{25.71Ns}{0.25s} = 102.8\ N$



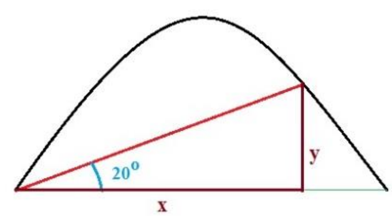
28. A… The unit of a time constant for an RL circuit is $\tau = \frac{L}{R}$ which means $L = R\tau$. Hence, an ohm-second is the same thing as a henry (unit of inductance).

29. E… Bulbs in series have the same current, so the resistor with higher resistance will be brighter (more power) from $P = I^2R$. This means that $R_X > R_Y$. Bulbs connected in parallel have the same potential difference. From $P = \frac{\Delta V^2}{R}$, the smaller resistance bulb has more power and will be brighter. This means that $P_Y > P_X$ and bulb Y is now brighter. Further, in the parallel configuration, both bulbs get the potential difference associated with the battery, whereas in series, each bulb only gets a fraction of the potential difference. This means that both bulbs now are brighter than before.

30. A… The area under an acceleration vs time curve gives the change in velocity. For the curves given, the area under each is equal. This means that at time T, the speeds are the same. Since the acceleration of X is much greater at the start, it gains speed more quickly than Y. This means that X travels further than Y. As Y only matches the speed of X at the end of the interval, X moves faster than Y at all other times and therefore is always moving further ahead of Y.

31. A… Since the object is rolling without slipping, any friction would have to be static on the incline (there is no relative motion between cylinder and surface). As for the direction, one notes that if the friction acts down the incline (as one might suspect being "opposite to the motion"), this would cause the speed of the object to decrease, but it also would cause an increase in the rotational speed of the object. The torque from the center of the cylinder would make the object spin faster and faster which cannot happen. Consequently, in order to slow the rotation, the friction must be pointing up the incline!

32. C… From the figure, we see that $\tan 20^\circ = \frac{y}{x}$. Using constant acceleration kinematics, we write $\tan 20^\circ = \frac{y_0 + v_{0y}t + \frac{1}{2}a_y t^2}{x_0 + v_{0x}t + \frac{1}{2}a_x t^2} = \frac{v_0 \sin 40^\circ T - \frac{1}{2}gT^2}{v_0\cos 40^\circ T} \rightarrow \tan 20^\circ = \tan 40^\circ - \frac{1}{2}\frac{g}{v_0\cos 40^\circ}T$ And so by solving for time, we have $T = \frac{2\,v_0 \cos 40^\circ}{g}(\tan 40^\circ - \tan 20^\circ) = 2.18\ s$



33. A… From the right-hand rule, the thumb points along the direction of current and the right fingers wrap in the sense of the magnetic field lines. Consequently, for the 2-A current, the field points downward anywhere to the left of the wire and upward anywhere to the right of the wire on the x-axis. Likewise, the 3-A current produces fields that point downward anywhere to the right of the wire and upward everywhere to the left of the wire. To have no total field, the individual magnetic fields must cancel (point in opposite directions) and that only occurs in Regions I and III. However, since points in Region III always are closer to the larger current, there is no way for the smaller current, acting from further away, to cancel the field from the larger current. The only way the fields can cancel is if the larger current is acting from a greater distance which corresponds to a location in Region I.

34. C… As there is no friction on the pond, there are no horizontal forces acting on the plank-person-box system. This means that the center-of-mass of the system remains stationary throughout. Locating the box at $x = 0\ m$, the person at $x = 9.20\ m$, and the center of the plank at $x = 4.60\ m$, we have $x_{cm} = \frac{m_{box}x_{box}+m_{pl}x_{pl}+m_{per}x_{per}}{M_{total}} = \frac{(0)+m_{pl}(4.60)+(71)(9.2)}{52+71+m_{pl}} = \frac{653.2+4.6\ m_{pl}}{123+m_{pl}}$. After the person moves, this same expression can be rewritten as $x_{cm} = \frac{m_{box}x_{box}+m_{pl}x_{pl}+m_{per}x_{per}}{M_{total}} = \frac{(52)(3.84)+m_{pl}(4.60+3.84)+(71)(3.84)}{52+71+m_{pl}} = \frac{472.32+8.44\ m_{pl}}{123+m_{pl}}$. Equating the c.o.m. expressions gives $\frac{653.2+4.6\ m_{pl}}{123+m_{pl}} = \frac{472.32+8.44\ m_{pl}}{123+m_{pl}} \rightarrow 180.88 = 3.84\ m_{pl} \rightarrow\ \ m_{pl} = \frac{180.88}{3.84} = 47.1\ kg.$

35. B… Since the area is changing size at the same rate on both sides (related to the speed of the bar, $\xi = Blv$), the currents are equal since the induced emf is the same. Because the change in flux is into the page in the right loop, a counterclockwise current will exist resulting in current up the plane through Y. Likewise, as there is a lessening flux in the left loop, there will be a clockwise current resulting in current directed upward through X.

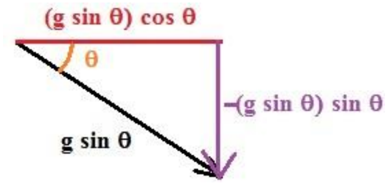
36. E… From the expression for a tube closed at one end, $f_n = n\left(\frac{v}{4L}\right)$. In the experiment, only first harmonics are being sought, so $n = 1$. Rearranging this expression, we write it as $L = \left(\frac{v}{4}\right)\frac{1}{f}$. Plotting L vs. $\frac{1}{f}$ will give a straight line of slope $m = \left(\frac{v}{4}\right)$. So, the speed of the waves is $v = 4m$.

37. B… To compute the angular momentum we are going to approximate the Earth as point-like since its radius is small compared to that of its orbit radius. So, $L = I\omega = (MR^2)\omega$. The distance from the Earth to the Sun can be approximated as the distance that light travels in about 10 minutes. That is, $R = vt = \left(3 \times 10^8 \frac{m}{s}\right)\left(10\ min\ \times \frac{60\ s}{1\ min}\right) = 1.8 \times 10^{11} m$. The mass is on the constants sheet and the angular speed of the Earth around the Sun is approximately $\omega = \frac{2\pi\ rad}{1\ yr} \times \frac{1\ yr}{365\ dy} \times \frac{1\ dy}{24\ hr} \times \frac{1\ hr}{60\ min} \times \frac{1\ min}{60\ s} = 1.99 \times 10^{-7} \frac{rad}{s}$. Putting this together gives the angular momentum as $L = (6.0 \times 10^{24})(1.8 \times 10^{11})^2(1.99 \times 10^{-7}) = 3.87 \times 10^{40} \frac{kgm^2}{s}$. Hence, the answer is 10^{40}. The angular momentum associated with the Earth's rotation about its axis is several orders of magnitude smaller than this number.

38. A… For an object to "just make it" over the top of the arc means that only the gravitational force is needed to maintain the circular motion. That is, $F_{net} = m\frac{v_{top}^2}{r} \rightarrow\ \ mg = m\frac{v_{top}^2}{L} \rightarrow v_{top}^2 = gL$. As the mass continues its motion, it is picking up speed. Using mechanical energy conservation, we find $\Delta KE + \Delta PE = 0 \rightarrow \frac{1}{2}m(v^2 - v_0^2) + mg\Delta y = 0$. From the construction, we have that $\Delta y = -(L - L\cos\theta)$. So, $\frac{1}{2}(v^2 - gL) - gL(1 - \cos\theta) = 0 \rightarrow v^2 = 2gL(1 - \cos\theta) + gL$ or $v^2 = gL(3 - 2\cos\theta)$. Now, from the free body diagram of the mass at this location, we have $F_{net} = ma \rightarrow n + mg\cos\theta = m\frac{v^2}{r} \rightarrow n = \frac{m\left(gL(3-2\cos\theta)\right)}{L} - mg\cos\theta = 3mg(1 - \cos\theta)$. With the information given, we find $n = 3(2)(10)\left(1 - \cos 30^\circ\right) = 8.0\ N$

39. A… Since the ratio of the wavelengths is 3:2, this makes the ratio of the corresponding frequencies to be 2:3 as $v = f\lambda$. Using the Doppler Shift expression, we have $f_o = f_s\left(\frac{v}{v \pm v_{src}}\right)$ where the sign in the denominator relates to whether the source is approaching or receding from the observer. For our ratio, we have $\frac{f_b}{f_f} = \frac{2}{3} = f_s\left(\frac{v}{v+v_s}\right) / f_s\left(\frac{v}{v-v_s}\right) = \frac{v-v_s}{v+v_s}$. Solving for the source speed gives $2v + 2v_s = 3v - 3v_s \rightarrow 5v_s = v \rightarrow v_s = \frac{1}{5}v$.

40. D… Taking the free body diagram of the incline+mass system yields only 2 vertical forces… normal and gravitational. From Newton's Second Law, we then write $F_{net} = ma \rightarrow \ \ n - (2Mg) = (2M)a$. Since the box on the incline is accelerating downward while the incline remains stationary, this means that the center-of-mass of the system is accelerating, which is the "a" we need to find in Newton's Second Law.

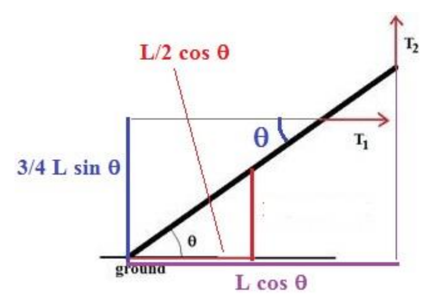


Looking at only the sliding mass, we write $F_{net_x} = ma_x \rightarrow \ \ Mg\sin\theta = Ma$ leading to the acceleration down the incline of $a = g\sin\theta$. Breaking this acceleration into components parallel and perpendicular to the ground gives: $a_x = (g\sin\theta)\cos\theta$ and $a_y = -(g\sin\theta)\sin\theta$. This makes the center of mass acceleration of the system as $(2M)\,a_{cm_y} = M_1 a_{1y} + M_2 a_{2y} \rightarrow a_{cm_y} = \frac{1}{2M}\left(M(-g\sin^2\theta) + M(0)\right) = -\frac{1}{2}g\sin^2\theta$.

Hence, $n = 2Mg + 2Ma \rightarrow 2Mg + 2M\left(-\frac{1}{2}g\sin^2\theta\right) = Mg(2 - \sin^2\theta)$

41. C… Red light can have a wavelength of $650 - 750\ nm$ depending on the textbook. Assuming that the angle associated with the bright spots is less than about 15°, we could write $\Delta y = \frac{\Delta m \lambda L}{d} = \frac{(2)(700)(5)}{8333.3} = 0.84\ m$. As a check, bright locations from a diffraction grating are found as $d\sin\theta = m\lambda$. We find the angles of bright spots to be $\theta_1 = \sin^{-1}\left(\frac{650}{8333.3}\right) = 4.47^\circ$ and $\theta_3 = \sin^{-1}\left(\frac{3(650)}{8333.3}\right) = 13.5^\circ$ where $d = \frac{1\ cm}{1200} \times \frac{1\times10^{-2}\ m}{1\ cm} \times \frac{1nm}{1\times10^{-9}m} = 8333.3\ nm$. The location of the bright spots is found from $\tan\theta = \frac{y}{L}$ where L is the distance to the screen and y is the vertical location on the screen. We need to find $y_3 - y_1 = L(\tan\theta_3 - \tan\theta_1)$. Using $\lambda = 650\ nm$, and finding that. This leads to $y_3 - y_1 = (5)\left(\tan(13.5^\circ) - \tan(4.47^\circ)\right) = 0.81\ m$. If one chose red light at $\lambda = 750\ nm$, the resulting spacing is $0.90\ m$.

42. C… Computing the torque from the pivot at the ground gives $\tau_{net} = -mg\left(\frac{L}{2}\right)\cos\theta - T_1\left(\frac{3}{4}L\right)\sin\theta + T_2 L\cos\theta = 0$. As $T_1 = T_2 = mg$, we have $-\frac{1}{2}\cos\theta - \frac{3}{4}\sin\theta + \cos\theta = 0 \rightarrow \frac{1}{2}\cos\theta = \frac{3}{4}\sin\theta \rightarrow$ $\tan\theta = \frac{2}{3} \rightarrow \ \theta = 33.7^\circ$



43. E… While one *can* use calculus to solve this, since the charge is all positive as described, there is a non-zero field at P directed downward. Looking at the non-zero answers, one notes that they all have different units. The only choice with electric field units is (E).

44. B… From the information given and symmetry, the total time of flight would be $4.80\ s$. This means it is $2.40\ s$ to the peak of the motion. We treat the downward motion for simplicity as the object starts from rest. From kinematics, this means that $\frac{H}{2} = \frac{1}{2}g\,t_1^2$ and $H = \frac{1}{2}g\,t_2^2$. With substitution, $\frac{1}{2} = \frac{t_1^2}{t_2^2} \rightarrow t_1 = \frac{t_2}{\sqrt{2}} = \frac{2.40}{\sqrt{2}} = 1.70\ s$. In other words, subtracting 1.70 seconds from when the mass reaches its peak gives the time at which it first reaches it's half-way point. This gives $T = 2.40 - 1.70 = 0.70\ s$!

45. A… A converging lens produces either a real image with $M < 0$ or a virtual image with $M > 1$.

46. D… In equilibrium, there is no electric field in the wire connecting the spheres which means that everything is at the same potential. As the potential outside of a sphere is given as $V = \frac{kQ}{r}$, the potentials are $V_X = \frac{kQ_X}{R_x}$ and $V_Y = \frac{kQ_Y}{R_y}$ and these must be equal. Hence, $\frac{kQ_X}{R_x} = \frac{kQ_Y}{R_y} \to Q_X = \frac{R_x}{R_y} Q_Y$. Since $R_x > R_y$, we have $Q_x > Q_y$ and sphere X will lose electrons to increase its overall charge.

47. D… In equilibrium, there is no current through the branch with the capacitor. Taking a Kirchhoff Loop Rule around the outside of the circuit gives $\xi - RI - \frac{\xi}{3} - 2RI = 0 \to IR = \frac{2}{9}\xi$. Now, doing a loop around the right half gives $\xi + \Delta V_C - 2RI = 0 \to \ |\Delta V_C| = \xi - 2\left(\frac{2}{9}\xi\right) = \frac{5}{9}\xi$.

48. C… Writing Newton's Second Law for the mass M gives $F_{net} = Ma \to \frac{GM(4M)}{D^2} = M\frac{v^2}{r}$. The center of mass of the system is $x_{cm} = \frac{M(0)+4M(D)}{5M} = \frac{4}{5}D$. Hence, $\frac{GM(4M)}{D^2} = M\frac{\left(\frac{2\pi r}{T}\right)^2}{r} \to$ $\frac{4GM}{D^2} = \frac{4\pi^2\left(\frac{4}{5}D\right)}{T^2} \to \frac{GM}{D^2} = \frac{4\pi^2 D}{5T^2} \to 5M = \frac{4\pi^2 D^3}{GT^2}$. The total system mass, recall, is $5M$.

49. B… First, we calculate a few needed quantities: $v_1 = 2.4 \times 10^8 \frac{m}{s} = \frac{4}{5}\ c \to \gamma_1 = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-0.8^2}} = \frac{5}{3}$ and $v_2 = 1.8 \times 10^8 \frac{m}{s} = \frac{3}{5}\ c \to \gamma_2 = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-0.6^2}} = \frac{5}{4}$.
Using momentum conservation, we have $\gamma_1 m v_1 + \gamma_2 m v_2 = \gamma M v_f \to \frac{5}{3}\ m\left(\frac{4}{5}c\right) + \frac{5}{4}\ m\ \left(\frac{3}{5}c\right) = \gamma M\ v_f \to \left(\frac{4}{3} + \frac{3}{4}\right) mc = \gamma M\ v_f \to \frac{25}{12} mc = \gamma M\ v_f$ where M is the rest mass of the new particle. From energy conservation: $\gamma_1 mc^2 + \gamma_2 mc^2 = \gamma M c^2 \to \frac{5}{3}\ mc^2 + \frac{5}{4}\ mc^2 = \gamma M\ c^2 \to \left(\frac{5}{3} + \frac{5}{4}\right)\ m = \gamma M \to \frac{35}{12}\ m = \gamma M$. Substituting for γM gives $\frac{25}{12} mc = \frac{35}{12} m\ v_f$ so that

$$v_f = \frac{25}{35}c\ = \frac{5}{7}c = 2.14 \times 10^8 \frac{m}{s}$$

50. B… The efficiency is found as $e = 1 - \left|\frac{Q_c}{Q_h}\right|$ and we have an adiabat which has $Q = 0$. Since the heat is positive from C to A (no work, temperature increase), that is $Q_h = nc_v(T_A - T_C)$ with $Q_c = nc_p(T_C - T_B)$ So, the efficiency is computed as $e = 1 - \left|\frac{nc_p(T_C - T_B)}{nc_v(T_A - T_C)}\right| = 1 - \gamma\frac{T_B - T_C}{T_A - T_C}$ where $\gamma = \frac{c_p}{c_v} = \frac{5}{3}$ for a monatomic gas. We have that $T_A = T_0$ and therefore from the ideal gas equation, $T_C = \frac{1}{3}T_0$. At B, we need to use that for an adiabat, $PV^\gamma = Con$. So, $P_A V_A^\gamma = P_B V_B^\gamma \to V_B = \left(\frac{P_A}{P_B}\right)^{1/\gamma} V_A = (3)^{\frac{3}{5}} V_0 = 1.933\ V_0$. For the process BC at constant pressure, this means that the temperature at B is 1.933 times as large as it is at C. So, we have

$$e = 1 - \gamma\frac{T_B - T_C}{T_A - T_C} = 1 - \frac{\frac{5}{3}\left(\left(\frac{1.933}{3}\right) - \frac{1}{3}\right)T_0}{\left(1 - \frac{1}{3}\right)T_0} = 1 - 0.7775 = 0.223$$