Physics Challenge for Teachers and Students

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Solution to February 2016 *Challenge*

Lord of the ring

A cube with edge a is placed at the bottom of a cylindrical vessel with the cross-sectional area $3a^2$. The vessel also contains oil and water that behave, well, like oil and water. The densities of water, oil, and the cube are, respectively, d, $0.8d$, and $2d$. The depths of the water and oil layers are, respectively, $0.5a$ and a.

Find the minimum amount of work required to pull the cube completely out of the liquid.

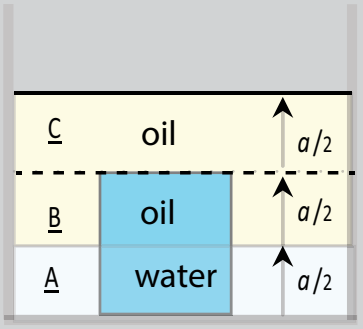
Solution:

The work required to raise the cube out of the liquids is equal to the change in the gravitational potential energy U of the cube and the fluids when the cube is on top of the fluids. From the initial conditions, one can determine the following:

$$V_w = \text{volume of water} = 3a^2\left(\frac{a}{2}\right) - \frac{a^3}{2} = a^3,$$

$$V_o = \text{volume of oil} = 3a^3 - \frac{a^3}{2} = \frac{5a^3}{2}.$$

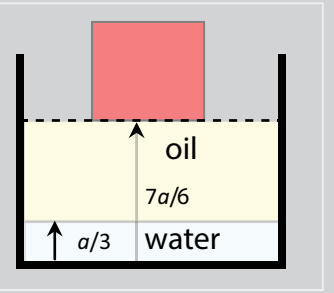
In order to determine U for the initial configuration, you first locate the centers of mass of the cube and each of the regions of fluid (A, B, C, as shown in the figure below).



Since the density of each object/region is uniform, its center of mass (COM) is located at its geometric center, as shown in the table below.

After the cube has emerged from the liquids, the liquid levels can be recalculated.

For the water, $h = a^3/(3a^2) = a/3$. For the entire fluid (water and oil), $H = (3.5a^3) / (3a^2) = 7a/6$. (See the figure below.)



The positions of the centers of mass of cube, water, and oil in this final configuration can then be calculated, as shown in the table. (We will assume that the densities given are *weight* densities.)

(The base of the vessel has been chosen as the zero level for calculating gravitational potential energy.)

OBJECT	Weight density	Volume	COM height	GPE
cube	$2d$	a^3	$.5a$	da^4
A (water)	d	a^3	$.25a$	$.25da^4$
B (oil)	$.8d$	a^3	$.75a$	$0.6da^4$
C (oil)	$.8d$	$1.5a^3$	$1.25a$	$1.5da^4$
Initial configuration:			U_o	$3.35da^4$
cube	$2d$	a^3	$5a/3$	$(10/3)da^4$
water	d	a^3	$a/6$	$(1/6)da^4$
oil	$.8d$	$2.5a^3$	$0.75a$	$1.5da^4$
Final configuration:			U_f	$5da^4$

Therefore, the work done by the lifting force is $W = U_f - U_o = 1.65\ da^4$.

Note: if d is assumed to be the mass density, $W = 1.65\ dga^4$.

(Submitted by Norman Derby, Curry Community College, Brookings, OR)

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Many thanks to all contributors; we hope to hear from many more in the future. We also hope to see more submissions of the original problems – thank you in advance!

**(Adapted from "Olimpiadnye Zadachi po Fizike" by M. Bakunov and S. Biragov, R&C Dynamics, Moscow-Izhevsk, 2006)*

Guidelines for contributors:

- We ask that all solutions, preferably in Word format, be submitted to the dedicated email address *challenges@aapt.org*. Each message will receive an automatic acknowledgment.
- The subject line of each message should be the same as the name of the solution file (see the instructions below).
- The deadline for submitting the solutions is the last day of the corresponding month.
- We can no longer guarantee that we'll publish every successful solver's name; each month, a representative selection of names will be published, both in print and on the web.
- If your name is—for instance—Donald Duck, please name the file "**Duck16May**" (do not include your first initial) when submitting the May 2016 solution.
- If you have a message for the Column Editor, you may contact him at korsunbo@post.harvard.edu; however, please do not send your solutions to this address.

As always, we look forward to your contributions and hope that they will include not only solutions but also your own *Challenges* that you wish to submit for the column.

Many thanks to all contributors and we hope to hear from many more of you in the future!

–Boris Korsunsky