THE MECHANICAL UNIVERSE

High School Adaptation

QUAD I

PHYSICS ON EARTH AND IN THE HEAVENS

THE MECHANICAL UNIVERSE

High School Adaptation

A co-production of the
California Institute of Technology
University of Dallas
and
Southern California Consortium

QUAD I

PHYSICS ON EARTH AND IN THE HEAVENS

Newton's Laws
The Apple and the Moon
Harmonic Motion
Navigating in Space

An Annenberg/CPB Project

National Science Foundation

Materials Development Council

Donald J. Barron, Wheaton High School, Wheaton, Maryland
Debra G. Cannon, Grand Prairie High School, Grand Prairie, Texas
Judith B. Healey, Plano Senior High School, Plano, Texas
Dr. Charles R. Lang, Omaha Westside High School, Omaha, Nebraska
William J. Layton, Palisades High School, Pacific Palisades, California
William H. Leader, Loara High School, Anaheim, California
Franceline C. Leary, Troy High School, Troy, New York
Donald A. Martin, Cistercian Preparatory School, Irving, Texas
Katherine E. Mays, Visiting Fellow, AAPT
Wilfred H. Oswald, Napa High School, Napa, California
Donald M. Sparks, North Hollywood High School, North Hollywood, California
Dr. George O. Taylor, Jr., The Baylor School, Chattanooga, Tennessee
Courtney W. Willis, University High School, Greeley, Colorado

Developer and Project Director of The High School Adaptation

Dr. Richard P. Olenick
Associate Professor of Physics, University of Dallas

Curriculum Consultant and Editor

Dr. Kathleen A. Martin, Associate Professor of Education
Texas Christian University

Evaluators

Geraldine R. Grant
J. Richard Harsh

Project Manager

Don Delson

Project Secretaries

Nancy Renwick
Judy Post

The Mechanical Universe, from which these materials have been adapted, is funded by a grant from the Annenberg/CPB Project and is a production of the California Institute of Technology and the Southern California Consortium.

Developer and Project Director of The Mechanical Universe
Dr. David L. Goodstein, Professor of Physics and Applied Physics
California Institute of Technology

Executive Producer

Sally V. Beaty, President, Southern California Consortium

This material is based upon work supported by the National Science Foundation under Grant No.SPE-8318420, MDR-8550178, and MDR-8652023. It was excerpted from the college television course, **The Mechanical Universe**, and re-edited specifically for use in the high school curriculum. **The Mechanical Universe** is funded by The Annenberg/CPB Project.

FOREWORD

Today, scientific and educational leaders are seriously concerned about the quality of science and mathematics education in the United States. It is as though the problems have been rediscovered, 25 years after Sputnik! In addition to those problems which have repeated themselves, today many qualified science and mathematics teachers at the pre-college, college, and university levels are being lured from the classroom by higher-paying jobs in business and industry. Many classrooms, therefore, have become the responsibility of instructors with limited preparation in the subject matter they are called upon to teach. And yet, more than ever the nation's current economic, social, and political needs call for a technologically literate population.

The Mechanical Universe, which served as the basis for the high school materials, addresses one critical need in science education by providing video and print materials that can serve as the basis of a solid, introductory college-level physics course. The video offers an exciting array of audiovisual resources for classroom instruction: close-ups of complicated experiments; extensive computer animation sequences that make abstract concepts and mathematical processes understandable; historical reenact-ments that provide a philosophical fabric for the development of ideas of physics.

The Mechanical Universe, part of the Annenberg/CPB collection, has as its primary purpose the provision of a quality learning experience for those whose lives cannot fit into the traditional campus schedule. This 52-program introduction to physics also offers a partial answer to some of the current problems of science education, for it can be used to upgrade skills of secondary science teachers and to provide supplementary support in the college and university classes.

Through the sponsorship of the National Science Foundation, selected programs of **The Mechanical Universe** have been adapted for use in high school. These materials represent the same quality and innovation as the college series, but they are presented in shorter and less mathematically oriented tapes that can be used in a wide variety of high school curricula. Teachers who find themselves teaching high school physics in spite of limited preparation will discover that, by enrolling in **The Mechanical Universe** course and using the adaptations in their classes, they will enjoy the confident feeling that they are presenting their students with quality instruction.

INTRODUCING

THE MECHANICAL UNIVERSE
High School Adaptation

The adaptations of **The Mechanical Universe** were created by twelve outstanding high school physics teachers (the Materials Development Council) through the generous support of the National Science Foundation. The clear purpose of the Council and the entire staff was to produce quality materials that would be used to improve instruction in physics. No one was satisfied with the goal of producing materials that would simply motivate or fascinate students, or would provide a change of pace. From the start, the challenge was to create materials which could make wise use of the power of television in developing a sound and solid understanding of physics.

Herewith the fruit of these labors: sixteen modules each consisting of a video adaptation from **The Mechanical Universe** with written support materials. Each module stresses conceptual understanding of underlying physical principles. The written materials support the video dimension of the modules. These support materials provide the teacher with additional background information and mathematical derivations, pre-video and post-video questions, applications, demonstrations, and evaluation questions.

The Mechanical Universe was originally developed for lower-division college courses in physics. The materials from **The Mechanical Universe** that have been adapted for use in high schools were field tested in 1984–86 by over 100 high school physics teachers located in schools widely scattered across the county in both urban and rural communities that serve various socio-economic populations. As a result of the assessment of the field testing, the videos were re-edited and the written materials were focused more directly on the videos to provide the best support possible for teachers.

PREFACE

These materials are intended for all teachers of high school physics. Teachers new to the arena of physics will discover rigorous, conceptual video presentations of traditional and not-so-traditional topics in classical physics. We hope that each word of the written materials will be savored. They are your resources and we hope that you tap them to capture the excitement of **The Mechanical Universe**. Experienced teachers will find a different slant to classical physics in the space age: a humanizing, compelling, integrated approach to the greatest revolution in the history of Western civilization. These teachers, too, we hope, will find the written materials continually refreshing resources.

Although **The Mechanical Universe** is a calculus-based course, the excerpts for high school use were selected to focus on concepts. That is not to say that the videos for high school use are not rigorous; they present sound logic at every stage in the development. Mathematics is occasionally used in the high school materials as a language to relate ideas concisely. In many cases the original mathematical derivations have been modified to be appropriate to the high school level. Nonetheless, mathematical derivations go by quickly in the video and we hope that teachers will replay these sections for their students. The mathematical background sections of the modules, we expect, will be read by all teachers even though they may not necessarily present to their classes the same level of mathematics provided in the print materials. We hope that teachers as well as students will gain a better appreciation of the vital role of mathematics in physics.

No laboratory component is currently suggested. The reason is not because we judge a physics laboratory component to be unimportant or uninteresting. On the contrary, we believe that demonstrations and laboratories lie at the heart of a sound education in high school physics. Instead we concentrated on what we could offer best: instruction through television. There are dozens of laboratory manuals which can be appended easily to these materials and we expect that each teacher will decide how best to handle the laboratories. On the other hand, since many demonstrations and applications to everyday life are presented in the video, we identified simple, short, and effective demonstrations that tie into concepts in the video. We hope that all physics teachers will enjoy performing them.

Not all the topics covered in the modules are conventional to high school physics curricula. *Angular Momentum* and *Harmonic Motion*, effectively covered in the videos, are two topics which are not necessarily a part of every curriculum. *Navigating in Space*, on the other hand, represents an exciting application of Kepler's ellipses and Newton's gravity that is not covered in typical curriculum. Other topics, such as *The Fundamental Forces* and *Curved Space and Black Holes*, provide tantalizing looks at twentieth century physics from the perspective of classical physics.

The Mechanical Universe is the story of the Copernican revolution, why it was necessary, and how it unfolded in the work of Galileo, Kepler, and Newton. It is the story of the eventual wedding of the heavens with the earth through the synthesis of mechanics and astronomy. History is presented in the series, not for the sake of historical detail, but for a fuller sense of how scientific thought proceeded through the intellectual searches and triumphs of men who reshaped the society of their times. We hope the infectious spirit of **The Mechanical Universe** will inspire teachers and students and will contribute to a lifelong scientific interest in the workings of the universe.

ACKNOWLEDGEMENTS

The adaptations of these instructional materials for high school use would not have been possible without the assistance of a long list of people who aided through the dedicated use of their diverse and specialized skills.

Heading the list is Professor David L. Goodstein, of Caltech, whose inspiration and guiding force in the creation of **The Mechanical Universe** led to the development of these materials.

Program Direction

Dr. Richard P. Olenick
Nancy Renwick, Secretary

Curriculum Development

Dr. Kathleen A. Martin

Production of Videotapes

Sally V. Beaty, Executive Producer
Dr. James F. Blinn, Computer Animator
Peter F. Buffa, Producer
Jack G. Arnold, Story Editor
Glenn Kammen and Peter Robinson, Videotape Editors

Program Management

Don Delson, Project Manager
Judy Post, Project Secretary

Program Evaluation and Field Testing

Geraldine R. Grant
J. Richard Harsh
Diana D. Price, Secretary

Videotape Distribution

Judy Sullivan, Product Distribution
Jean Coltrin, Director of Marketing

Special Mention

Dr. Howard Hubbard, (Supervisor of Mathematics and Science, Long Beach Unified School District, CA., retired), for review of materials
Lynn Strech, Artist
Judy Sullivan, Southern California Consortium, Design and typesetting

Finally, we offer special thanks to Mary Kohlerman of the National Science Foundation.

Materials Development Council
Irving, Texas
July 1987

STRUCTURE OF THE MATERIALS

The written materials are designed to support and extend the VIDEO presentation of each module. The format and content of the materials are designed to help the user (1) to integrate the concept(s) presented in the VIDEO with traditional high school materials, (2) to supplement and promote conceptual understanding of the phenomena presented in the VIDEO, and (3) to infuse the students with a new spirit of inquiry concerning the mechanics of physics.

Each module is composed of components of written materials. Each component is intended as a resource to promote active engagement of the learner in developing conceptual understanding of the physical phenomena. The five components of the print materials are:

TEACHER'S GUIDE

Pre-Video Activities*

Content and Use of the Video – describes what the VIDEO does and does not cover.

Terms Essential for Understanding the Video – includes the definitions of terms listed in the STUDENT'S GUIDE, discussion of critical elements or relationships.

What to Emphasize and How to do It – includes the objectives of the module, references to demonstrations, possible applications, and suggestions for correcting common misconceptions.

Points to Look for in the Video – includes common misconceptions when relevant; characteristics and questions concerning critical elements presented in the VIDEO. Answers to questions in the STUDENT'S GUIDE are included.

Post Video Activities*

Everyday Connections and Other Things to Discuss – suggests additional questions to promote student participation and discussion. An essential purpose of the questions is to engage students in review and clarification of the concepts.

Summary – reviews the key concepts that have been presented.

STUDENT'S GUIDE

(designed for duplication and distribution)

Introduction – a brief statement about the content or purpose of the VIDEO.

Terms Essential to Understanding the Video – includes terms or critical elements of the VIDEO, with definitions and explanations provided in the TEACHER'S GUIDE.

Points to Look for in the Video – includes common misconceptions when relevant; characteristics and questions concerning critical elements presented in the VIDEO along with figures representative of key points in the VIDEO.

TEACHER RESOURCES

Supportive Background Information – summarizes additional historical, physical, and mathematical information that relate to the topics and content presented in the VIDEO.

Additional Resources – includes demonstrations and applications the teacher may use to extend and enrich the treatment of the topic.

Evaluation Questions – provides ten multiple-choice questions dealing with the objectives of the module and two essay questions that require student's explanations of certain concepts related to the topics.

*The repeated showing of the video (in full and part) is essential to student understanding. The division of activities into prevideo and postvideo activities, therefore, is somewhat artificial. It is likely that most, if not all, prevideo activities will precede the initial showing of the video. Sections of the video will undoubtedly be sprinkled throughout the postvideo activities, with a full showing being used for closure where time permits.

QUAD I

NEWTON'S LAWS

WHAT ARE THE CAUSES OF MOTION? Galileo's laws of falling bodies and inertia describe how objects move. However, they do not explain why. In this video, Newton's great work of completing Galileo's kinematics with dynamics – a theory of the causes of motion – is described. Not only did Newto,n's three laws introduce a new order to the chaos of scientific thought, they opened up questions about force and mass which remain central questions of physics even today.
Running Time: 15:45.

THE APPLE AND THE MOON

HOW IS THE MOTION OF THE MOON AROUND THE EARTH LIKE THAT OF A FALLING APPLE? Newton's answer to the question came to be known as his law of universal gravitation. Galileo had described the law of falling bodies – gravity on earth – and Kepler had described planetary orbits – gravity in the heavens. In this video, how the universal law of gravitation emerged from Newton's efforts to reconcile Galileo's new kinematics with Kepler's new astronomy is explored. Together with Newton's three laws of motion, the law of universal gravitation provides the basis for Newton's coherent view of how the universe works.
Running Time: 12:43.

HARMONIC MOTION

WHY DO SOME MOTIONS REPEAT THEMSELVES REGULARLY? A dramatic application of the success of Newton's second law in explaining physics on the earth is simple harmonic motion. In this video, simple harmonic motion is presented as a model which illustrates the scientific process of extracting simple, underlying physical principles from complex behavior.
Running Time: 12:21.

NAVIGATING IN SPACE

HOW DO YOU GET FROM EARTH TO VENUS? Interplanetary travel is introduced as an application of the celestial mechanics of Kepler and Newton. In this video, important principles of classical mechanics are reviewed through an exciting application. The ideas behind transfer orbits, launch opportunities, launch windows, and gravity assists are explained through a conceptual development.
Running Time: 18:04.

QUAD I

NEWTON'S LAWS

WHAT ARE THE CAUSES OF MOTION? Galileo's laws of falling bodies and inertia describe how objects move. However, they do not explain why. In this video, Newton's great work of completing Galileo's kinematics with dynamics – a theory of the causes of motion – is described. Not only did Newto,n's three laws introduce a new order to the chaos of scientific thought, they opened up questions about force and mass which remain central questions of physics even today.
Running Time: 15:45.

THE APPLE AND THE MOON

HOW IS THE MOTION OF THE MOON AROUND THE EARTH LIKE THAT OF A FALLING APPLE? Newton's answer to the question came to be known as his law of universal gravitation. Galileo had described the law of falling bodies – gravity on earth – and Kepler had described planetary orbits – gravity in the heavens. In this video, how the universal law of gravitation emerged from Newton's efforts to reconcile Galileo's new kinematics with Kepler's new astronomy is explored. Together with Newton's three laws of motion, the law of universal gravitation provides the basis for Newton's coherent view of how the universe works.
Running Time: 12:43.

HARMONIC MOTION

WHY DO SOME MOTIONS REPEAT THEMSELVES REGULARLY? A dramatic application of the success of Newton's second law in explaining physics on the earth is simple harmonic motion. In this video, simple harmonic motion is presented as a model which illustrates the scientific process of extracting simple, underlying physical principles from complex behavior.
Running Time: 12:21.

NAVIGATING IN SPACE

HOW DO YOU GET FROM EARTH TO VENUS? Interplanetary travel is introduced as an application of the celestial mechanics of Kepler and Newton. In this video, important principles of classical mechanics are reviewed through an exciting application. The ideas behind transfer orbits, launch opportunities, launch windows, and gravity assists are explained through a conceptual development.
Running Time: 18:04.

TEACHER'S GUIDE TO NEWTON'S LAWS

CONTENT AND USE OF THE VIDEO - Newton's laws of motion are among the most fundamental and useful topics traditionally studied in an introductory physics course. This video treats Newton's laws in a more conceptual manner than the typical high school text, making it suitable for a wide range of audiences. Most teachers will want to supplement this instruction using traditional mathematical applications.

The first law is covered more completely in the module *Inertia*; use of that module should precede this one. The major emphasis here is on Newton's second law: F = ma. The third law is also included in the video, and supporting written materials are found throughout the module. Although the term momentum is briefly mentioned, this topic is extensively covered in a separate module, *Conservation of Momentum*.

Projectile motion is presented in the video as an application of Newton's laws. This segment may serve to reinforce understanding for those students who have already studied the topic. If projectile motion has not yet been studied, you may want to precede the video with a discussion of vectors and components of vectors. Emphasize the independence of the horizontal and vertical components of displacement, velocity, and acceleration in projectile motion.

TERMS ESSENTIAL FOR UNDERSTANDING THE VIDEO - Since the following terms are introduced in the video, it might be helpful to discuss them briefly prior to viewing. All terms are defined as they are used within the context of the video.

mass--a measure of the tendency of an object to resist changes in its state of motion. This mass, also known as inertial mass, appears in Newton's second law as the constant of proportionality between the net force applied on an object and its acceleration, i.e., the *m* in F = ma. Gravitational mass is the property of an object which gives it the ability to attract other masses and appears in Newton's universal law of gravitation (see the modules *The Apple and the Moon* and *Curved Space and Black Holes* for a further treatment). As far as experiments can determine, for any object the inertial and gravitational mass are equal.

vector--a quantity that has *both* direction and magnitude. It can be broken down into components along mutually perpendicular directions. The velocity vector of an object moving through the air, for example, can be analyzed in terms of its vertical and horizontal components. A change in a vector occurs if the magnitude *or* the direction changes.

displacement--a vector which represents a measurement of an object's position from an agreed point of origin.

velocity--the time rate of change of displacement, or how fast an object is moving in a specified direction. Velocity is a vector.

acceleration--the time rate of change of velocity. Acceleration is also a vector.

force--a push or pull that is exerted on one body by another body and contributes to the net push or pull on the body to cause it to accelerate; a net, or unbalanced, force is the vector sum of all the forces acting on a given body and produces an acceleration.

weight--the force on a body due to gravitational attraction of the earth on it; weight is equal to the product of an object's (gravitational) mass and the acceleration of gravity, W = *m*g.

impetus--the 14th century idea of a quantity that caused the motion of objects; it was later replaced by the concept of inertia.

projectile--an object launched at any angle into the air.

trajectory--the path of a projectile.

parabolic--any path that matches a section of a parabola.

WHAT TO EMPHASIZE AND HOW TO DO IT - In 1543, Copernicus had begun a revolution with the publication of his *De Revolutionibus* - a revolution in thinking that forever altered humanity's perception of itself and the universe. In an attempt to solidify Copernican theory, and prove that the earth could move, Galileo formulated a principle of inertia, a description of *how* things move (kinematics). While some evidence suggests that he and Johannes Kepler may have speculated on a *cause* for planetary motion, namely a gravitating force, a dynamic description of motion was yet to come. Newton entered a world desperately in need of order and explanation. He gave us three laws which organize the world and help us to understand how it works. Newton's second law introduced not only a new order, but also the new focus which has characterized physics ever since. It raised penetrating questions, such as the natures of mass and force, and what fundamental forces act in the universe. These questions raised by Newton in the 1600's remain central to physics even today.

The video develops the concept that mechanics, the science of motion, can be summarized by Newton's laws of motion. Projectile motion is a special and important application of these laws. Because of the comprehensive nature of this video, a second viewing is strongly recommended.

Objective 1: State Newton's three laws of motion.

After reviewing the definitions and seeing the video, students should be able to summarize Newton's three laws:

1. An object at rest remains at rest, and an object in motion with constant velocity will continue in that motion unless acted upon by an external, unbalanced force.

2. F = *m*a. An external unbalanced force F acting on an object of mass *m* produces an acceleration a in the direction of the force.

3. For every action there is an equal, and opposite reaction.

These three laws are stated in Newton's original words in the section on SUPPORTIVE BACKGROUND INFORMATION. DEMONSTRATION #1 might be used at this time. Ask students to describe the aspects of the ball's motion which pertain to each of the three laws:

inertia--*the fact that the forward motion of the ball and demonstrator are the same.*

F = *m*a--*the ball accelerates in the vertical direction.*

action-reaction--*the ball exerts a force on the floor, and in turn the floor exerts an equal but opposite force on the ball.*

Reinforce these ideas by asking students to cite examples of Newton's three laws, emphasizing those depicted in the video:

First Law - The asteroid and rocket animation before firing the rockets; Galileo's inclined plane.

Second Law - A diver falling; shot put, discus, pole vault, and high jump; the asteroid and rocket animation during firing of the rockets.

Third Law - Football linemen colliding.

DEMONSTRATION #5 offers a quick application of Newton's third law. Discuss the forces involved. If time permits, this may be reinforced by DEMONSTRATION #6.

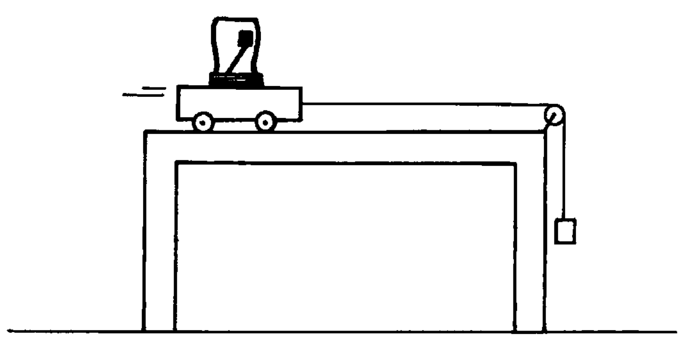
You might suggest to students that they bring a toy to class which in some way demonstrates one of Newton's laws. Discussion of how these toys work provides powerful images of Newton's laws in action.

Objective 2: Recognize that a net, or unbalanced, force acting on a body produces an acceleration in the same direction as that force.

Newton's second law of motion is usually expressed as $F = ma$. It is interesting to express it in terms of the acceleration: $a = F/m$, since this readily shows that the acceleration is both directly proportional to the force and inversely proportional to the mass.

It should be emphasized that the force in Newton's second law is the net unbalanced force; this is found by the vector summation of all the forces acting on a body. If the forces are balanced, i.e., the net force is zero, there will be no acceleration.

Perform DEMONSTRATION #3 on the accelerometer at this time. You could hold the jar and exert a force on it, or it could be placed on a cart with a string-pulley-weight system providing the force, as shown below in the following drawing:



The change in motion of a body also involves mass. This may be shown by using DEMONSTRATION #4, where the same force is applied to different masses to produce different accelerations.

Objective 3: Identify action-reaction pairs and recognize that they operate on pairs of bodies.

You will find a discussion of this topic in the SUPPORTIVE BACKGROUND INFORMATION. Students often find it difficult to understand passive (reaction) forces, questioning how an inanimate object such as a chair or floor can exert a force. It is helpful to ask the class what would happen if the floor did not exert an upward force upon them, i.e., if the floor suddenly disappeared. Additional examples may be found in the section on EVERYDAY CONNECTIONS AND OTHER THINGS TO DISCUSS.

Objective 4: Recognize that the trajectory of a projectile is a consequence of Newton's Laws, and that the horizontal and vertical components of the motion are independent of each other.

If your class has already studied projectile motion, this video offers an excellent opportunity to enhance students' understanding of the topic and to reinforce the relationship between projectile motion and Newton's laws. Additional information is available in SUPPORTIVE BACKGROUND INFORMATION and EVERYDAY CONNECTIONS AND OTHER THINGS TO DISCUSS. You might also wish to emphasize the parabolic trajectory of the motion when performing DEMONSTRATION #1.

Perform DEMONSTRATION #2 to develop further an understanding of projectile motion. Discuss any changes that may occur in the horizontal and vertical velocities during the flight. The velocity in the horizontal direction is constant since no force acts in the horizontal direction to change the motion. The velocity in the vertical direction decreases to zero on the way up and increases on the way down because of the force of gravity. Also discuss the velocity and acceleration of the object at the highest point in its trajectory. At the top of the path the velocity is momentarily zero. However, the acceleration is constant at all points along the trajectory and is equal to 9.8 m/s^2 downward.

For those students who have not yet studied projectile motion, additional information is needed. You will probably want to supplement these materials with your text. It is particularly important that students understand how to break a vector in a plane into two components and that in projectile motion the horizontal and vertical components are independent.

In projectile motion, the force that produces the motion is the gravitational force of the earth on the projectile, i.e., the weight of the projectile, **W** = m**g**, that acts vertically downward. This force is substituted for **F** in Newton's second law. Since the force has no component in the horizontal direction (the x-direction), there is no acceleration in that direction and, according to the law of inertia, the horizontal component of the velocity is constant. The mathematics is shown in the video as

$$\mathbf{F} = m\mathbf{a}:\ F_x = ma_x,\ F_x = 0,\ \text{so}\ a_x = 0,\ \text{therefore}\ v_x = \text{constant.}$$

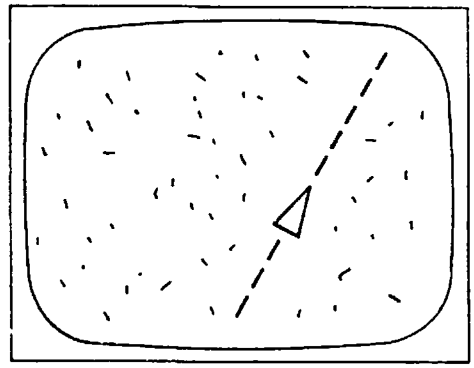
In the vertical direction, the gravitational force of the earth on the projectile produces a constant downward acceleration g:

$$\mathbf{F} = m\mathbf{a}:\ F_z = ma_z,\ F_z = mg,\ \text{so}\ a_z = g = \text{constant.}$$

Note: This video uses the subscripts *x* for horizontal and *z* for vertical components. Boldface print denotes a vector quantity.

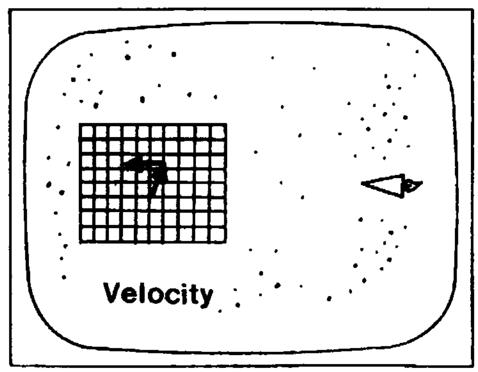
POINTS TO LOOK FOR IN THE VIDEO - Several questions are posed in the STUDENT'S GUIDE. Here are those questions along with suggested responses and frames from the video.

Isaac Newton began with three fundamental principles, Newton's Laws. With his first law, Newton embraced the idea of inertia, an idea he inherited from Galileo. Once a body is in motion, it naturally continues in a straight line unless influenced by some force. When you see this frame, ask your teacher to stop the video so you can answer the following question. How is the rocket's motion an illustration of Newton's First Law?



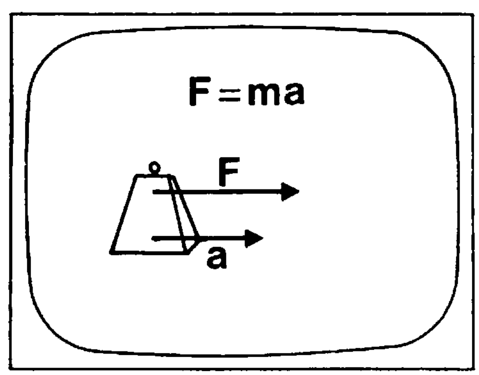
Since no external forces act on the moving rocket, it continues in its straight line motion at a constant speed.

Newton's second law indicates exactly how force changes the motion of an object. Force equals mass times acceleration. In the video the yellow arrow represents the velocity, the red arrow represents the acceleration. The acceleration vector redirects the velocity. Why does the acceleration vector suddenly appear?



The rocket has fired its engines to change direction. The acceleration vector is in the direction of the force, and it redirects the original velocity vector.

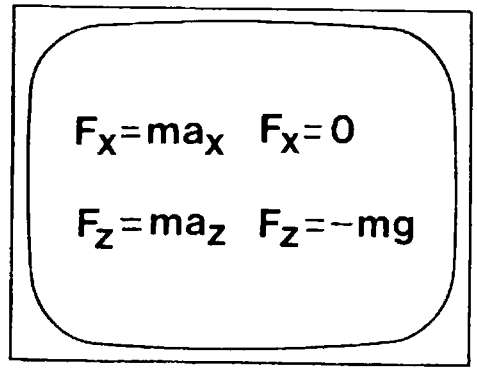
F = ma is a vector equation. Both force and acceleration are vectors. In other words, they have definite directions. What must be true of the directions of F and a?



The acceleration is always in the same direction as the net force.

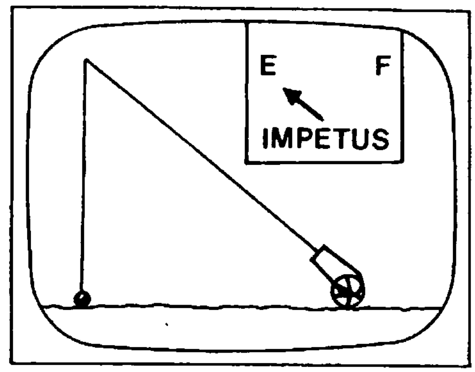
The trajectory of a projectile demonstrates the consequences of Newton's three laws. The vertical component of the vector force is mg downward, or minus mg. What force acts in the horizontal direction?

Since the weight force acts vertically downward, there is no component of the force in the horizontal direction.



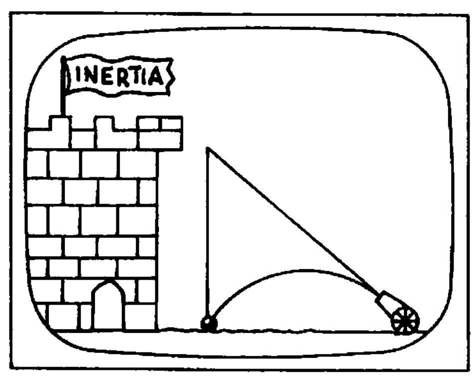
Before Galileo, it was thought that launching a projectile imbued it with a finite amount of impetus, which gave the object its motion. When its impetus was consumed, the object suddenly dropped to earth. What is wrong with this argument?

The argument fails to give the trajectory of a real projectile.



What is the shape of a projectile's path near the surface of the earth, ignoring air resistance?

The shape of the projectile path is parabolic. This characteristic path is a result of the gravitational force acting only in the vertical direction. Since there is no component of the force in the horizontal direction, the velocity component in the horizontal direction is constant. In the vertical direction the projectile accelerates downward with an acceleration given by Newton's second law: $-mg = ma_z$, so $a = -g$. Constant speed in the horizontal direction and constant acceleration in the vertical direction produce the parabolic path.



EVERYDAY CONNECTIONS AND OTHER THINGS TO DISCUSS - To reinforce further the concepts presented in the video, you might pose the following questions to your students.

1. **What kind of motion does a constant force produce?**

   A constant force acting on an object produces a constant acceleration.

2. **If a net force is exerted on an object, in what direction is the object accelerated?**

   An object accelerates in the same direction as the net force acting upon it.

3. **What are the similarities and the differences between action and reaction? Look for some examples in the video.**

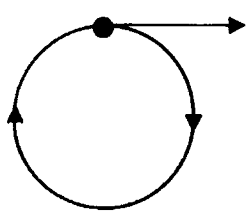
   Action and reaction are both forces. If two objects interact, then the force that one exerts on the second is equal in magnitude but opposite in direction to the force that the second exerts on the first. These forces do not act on the same object. The reaction force is often a passive force.

4. **How does mass differ from weight?**

   Mass is a measure of the inertia of an object and is measured in kilograms. Weight is a force – the force of gravity acting on an object – and is measured in newtons. Near the surface of the earth weight is given by $\mathbf{w} = mg$, *the product of mass times the acceleration due to gravity.*

5. **What would be the path of the planets if suddenly there were no force acting on them?**

   From Newton's first law, in the absence of any force, a body will continue to move in a straight line. Thus each planet would fly off tangent to its orbit, in the direction in which it was traveling.



6. **A Lincoln Continental and a Renault Le Car are traveling at the same speed along a level road. How do the forces required to stop them in the same time compare?**

   The Le Car has the smaller mass of the two. The deceleration of both cars must be the same, as they are brought to rest from the same speed in the same time. Therefore the less massive car requires a lesser force:

$$\frac{F_{\text{Le Car}}}{F_{\text{Continental}}} = \frac{M_{\text{Le Car}}A}{M_{\text{Continental}}A} = \frac{M_{\text{Le Car}}}{M_{\text{Continental}}}$$

7. **A rock has a weight of 60 N on the surface of the moon, where the acceleration due to gravity is 1/6 that on earth's surface.**

   (a) Compare the weight of this rock on the earth with its weight on the moon.
   (b) Compare the mass of this rock on the earth with its mass on the moon.

   (a) Weight on earth = 6 times weight on moon (360 N).
   (b) Mass on earth = mass on moon.

8. **Identify the action-reaction pair of forces for each of the following cases:**

   (a) A person standing on the floor.
   (b) A bird flapping its wings.
   (c) Raindrops hitting a roof.
   (d) The earth and the moon.

   (a) Action: Person pushes down on the floor.
   Reaction: Floor pushes up on the person.
   (b) Action: Wings push down on air.
   Reaction: Air pushes up on wings.
   (c) Action: Raindrops push down on roof.
   Reaction: Roof pushes up on raindrops.
   (d) Action: The earth attracts the moon.
   Reaction: The moon attracts the earth.

9. **A marble is dropped into a container of oil and is seen to move at constant velocity to the bottom. Describe how Newton's laws explain this motion.**

   Since the marble moved at constant velocity, according to the first law no unbalanced forces were acting. Therefore, the force of friction between the marble and oil and the pull of gravity (weight) balance. (Note that this is an illustration of terminal velocity.)

10. **A stubborn donkey refuses to pull a wagon. The donkey cites Newton's third law in explaining to Dr. Dolittle: "If I pull on the wagon, the wagon pulls back equally on me, so I could never start it moving." What reaction would Dr. Dolittle have to the donkey?**

    Dr. Dolittle would say, "No, my little equine friend, there is an unbalanced force on the wagon that is due to you. Therefore, according to Newton's second law, you can accelerate the wagon."

11. **(a) Which of Newton's laws of motion describe the *horizontal motion* of a projectile? Explain.**
    (b) Which of Newton's laws of motion describe *vertical motion* of a projectile? Explain.

    (a) Neglecting air friction, the horizontal motion of a projectile is described by Newton's first law, since no unbalanced forces act in this direction.
    (b) The vertical motion is described by Newton's second law since the unbalanced force of gravity acts downward. Thus the projectile accelerated downward. $F = ma = mg$.

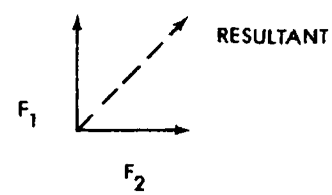
12. **Two identical balls roll off the edge of a table at the same instant. Ball A is moving three times faster than ball B when they leave the table.**

    (a) Which ball, if either, hits the ground first? Why?
    (b) Which ball, if either, travels farther out from the table? Explain.

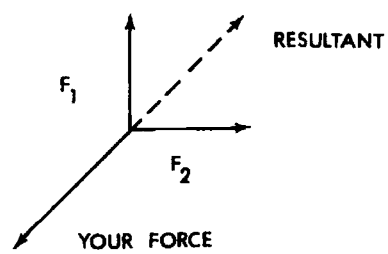
    (a) Both hit the ground in the same time. The vertical motion is the same for both balls. Both are accelerated identically due to the force of gravity.
    (b) Ball B travels three times farther out. In the horizontal direction no unbalanced forces act, so in the same time (time to hit the ground), an object traveling three times faster goes three times farther.

13. **A mass is acted on by two perpendicular forces, equal in strength. One force pulls north, the other pulls east as shown. They result in a net force of 10 N, northeast.**

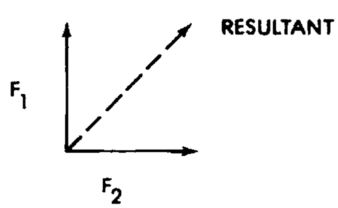
    (a) In the original case, what additional force would be needed to keep the mass stationary?
    (b) What effect would doubling the northward force have on the mass's motion?



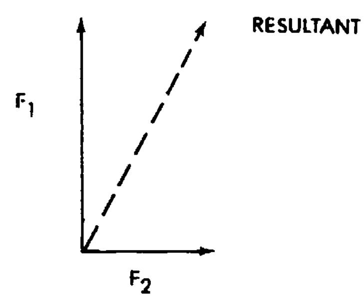
(a) For the mass to remain stationary, there must be no net force. Therefore you would have to exert an equal and opposite force:



(b) Doubling the northward force would increase the magnitude of the resultant and would also change its direction to a more northerly direction:



BEFORE



AFTER

14. A jar of lightning bugs is tightly capped. Does it weigh more, less, or the same when the bugs are flying around compared to when they are at rest?

The system is closed. The total mass has not changed. Inside, the wings of the bugs pushed down on the air and the air pushed on the container. (Note that the air also reacts to the wings.) The interactions inside the jar are all internal forces. The only outside unbalanced force is the weight, regardless of the internal interactions.

SUMMARY - Newton's three laws of motion help us to explain why things move the ways that they do. Galilean physics only described how objects move. Newton explained that forces cause changes in an object's motion. With his first law, Newton restated Galileo's principle of inertia. The second law indicates how a force changes the motion of an object. Bodies don't just act on other bodies, they interact. This is the essence of the third law. Projectile motion is illustrative of the consequences of Newton's three laws of motion.

NOTE OF EXPLANATION REGARDING THE STUDENT'S GUIDE - The following two pages of the STUDENT'S GUIDE should be duplicated and distributed to the students for use in preparation for viewing the video.

In general, the STUDENT'S GUIDE lists topics, terms, and questions, and the TEACHER'S GUIDE provides definitions, discussion, and answers to the questions. It is very important to have the students receive an appropriate "preparatory set" for viewing the VIDEO and also, following the showing of the VIDEO, to have a systematic discussion, analysis, and summarization of the objectives of the module.

The students should be informed that the INTRODUCTION, TERMS ESSENTIAL FOR UNDERSTANDING THE VIDEO, and POINTS TO LOOK FOR IN THE VIDEO should be read and discussed *prior* to viewing the VIDEO. These should also be rediscussed following the viewing.

Answers to the questions listed in the STUDENT'S GUIDE have been included under POINTS TO LOOK FOR IN THE VIDEO in the Teacher's Guide. The questions which follow this section of the Teacher's Guide and deal with EVERYDAY CONNECTIONS AND OTHER THINGS TO DISCUSS as well as the SUMMARY should be discussed as a part of the activities that *follow* the viewing(s) of the VIDEO and give closure to the lesson.

STUDENT'S GUIDE TO NEWTON'S LAWS

INTRODUCTION - This video states Newton's three laws of motion and demonstrates their application to the problem of the motion of a projectile.

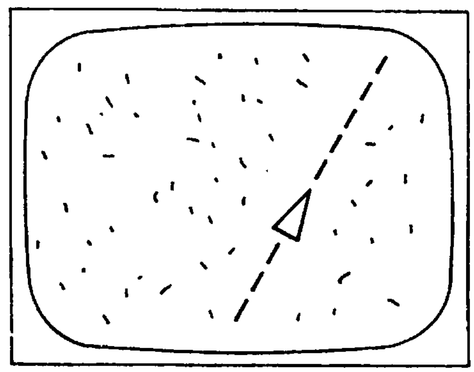
Terms Essential for Understanding the Video

mass
vector
displacement
velocity
acceleration
force
weight
impetus
projectile
trajectory
parabolic

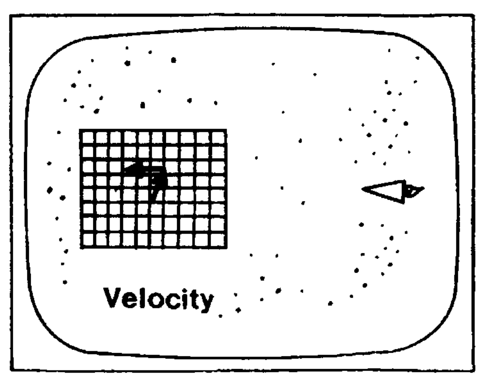
*** *NOTE: Parts of the video, especially mathematical equations, may go by quickly on the screen. If you have questions, you should ask your teacher to replay these sections.* ***

Points to Look for in the Video

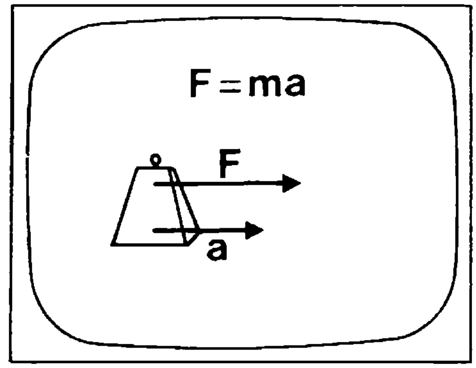
Isaac Newton began with three fundamental principles, Newton's Laws. With his first law, Newton embraced the idea of inertia, an idea he inherited from Galileo. Once a body is in motion, it naturally continues in a straight line unless influenced by some force. When you see this frame, ask your teacher to stop the video so you can answer the following question. How is the rocket's motion an illustration of Newton's First Law?



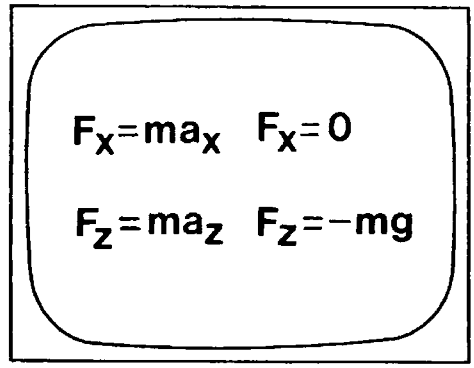
Newton's second law indicates exactly how force changes the motion of an object. Force equals mass times acceleration. In the video the yellow vector represents velocity, the red vector represents the acceleration. The acceleration vector redirects the velocity. Why does the acceleration vector suddenly appear?



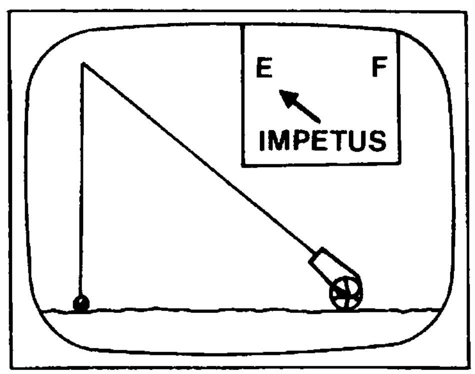
F = *ma* is a vector equation. Both force and acceleration are vectors. In other words, they have definite directions. What must be true of the directions of F and a?



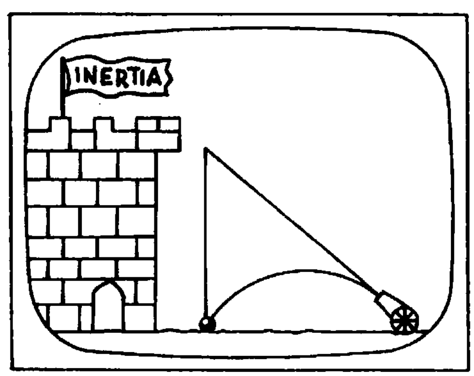
The trajectory of a projectile demonstrates the consequences of Newton's three laws. The vertical component of the vector force is mg downward, or minus mg. What force acts in the horizontal direction?



Before Galileo, it was thought that launching a projectile imbued it with a finite amount of impetus, which gave the object its motion. When its impetus was consumed, the object suddenly dropped to earth. What is wrong with this argument?



What is the shape of a projectile's path near the surface of the earth, ignoring air resistance?



TEACHER RESOURCES

SUPPORTIVE BACKGROUND INFORMATION - Newton drew upon the ideas of Galileo and Descartes in stating his first law of motion.

First law: Every body continues in its state of rest, or of uniform motion in a straight line, unless it is compelled to change that state by forces impressed upon it.

Newton realized that an object's inertia was somehow connected to its mass. The greater the mass of an object, the more difficult it is to change its state of motion. This idea led to his second law of motion. It is stated in the *Principia* as:

Second Law: The rate of change of motion is proportional to the force impressed, and is in the direction in which the force is impressed.

What Newton called motion involved both mass and velocity. This quantity, m times v, is now called *momentum*. It forms the basis of a very important principle – conservation of momentum – which is a direct result of Newton's second and third laws.

According to the second law not the motion but the *rate of change* of motion is proportional to force. Rate of change means how fast something changes. This can be determined by dividing the amount of change of a quantity by the time it takes for the change to take place. The rate of change of motion is proportional (or, in suitable units, is equal) to the force that causes it:

$$\frac{\Delta (mv)}{\Delta t} = F \ .$$

If the mass is constant,

$$\frac{m \Delta v}{\Delta t} = F \ .$$

The quantity $\Delta v / \Delta t$ is the average acceleration. If Δv is measured in a very small time interval that shrinks to zero, i.e., Δt — 0, $\Delta v / \Delta t$ becomes the instantaneous acceleration a. Hence we have the usual form of Newton's second law: $F = ma$. This form was first presented by the Swiss mathematician Leonard Euler 65 years after the publication of the *Principia*. Newton recognized that forces are vector quantities. According to his second law, $\mathbf{F} = m\mathbf{a}$, the acceleration also is a vector and is in the same direction as the force producing it. Acceleration also is the rate of change of velocity. Since velocity is a vector, its magnitude, the speed, may remain constant even though there is an acceleration because only the direction of the velocity may change. $\mathbf{F} = m\mathbf{a}$ says that force creates acceleration. $\mathbf{F}$ is the *net* force acting on a body and is found by taking the vector sum of all the forces acting on the body.

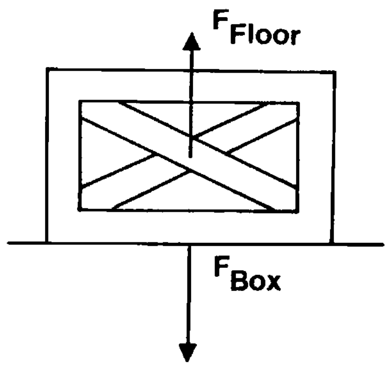
Newton needed one additional law to express what happens when bodies act upon one another.

Third law: To every action there is always opposed an equal reaction: or, the mutual actions of two bodies upon each other are always equal, and directed to contrary parts.

This law may be stated very simply: action-reaction. When you push on something, it pushes back on you. If body 1 exerts a force on body 2, then body 2 exerts an equal but opposite force on body 1. In other words, you can't push without being pushed back:

$$\mathbf{F}_{12} = -\mathbf{F}_{21} \ .$$

This action-reaction pair of forces always act on different bodies. For example, consider a box resting on a floor. If the action force is the downward force of the box, due to its weight, pressing on the floor, $\mathbf{F}_{box}$, then the reaction force is the upward force of the floor acting on the box, $\mathbf{F}_{floor}$. The following diagram shows these action-reaction forces:



Other examples would include the flight of a jet plane and the motion of a rotary lawn sprinkler. In the jet engine, large quantities of gas molecules are heated and driven towards the rear at high speed. The force on the molecules is the action force, but as these molecules are driven out they exert a force on the engine and hence on the plane, in the opposite direction. This reactive force propels the plane forward. In the lawn sprinkler, the force on the water shooting out is the action force, and the force on the arm in the opposite direction is the reactive force.

Sometimes during collisions, the action-reaction pair produce accelerations on the two bodies. Even though the forces are equal and opposite, the accelerations are not. How can this be? Consider a head-on collision of a compact car and a Mack truck. Suppose that the car exerts a force $\mathbf{F}_{car}$ on the truck and the truck exerts a force $\mathbf{F}_{truck}$ on the car. By Newton's third law, these forces are equal and opposite:

$$\mathbf{F}_c = -\mathbf{F}_T .$$

To find the acceleration, we need use Newton's second law of motion:

$$\mathbf{F} = m\mathbf{a} .$$

Substituting this into the previous equation, we find

$$m_c\mathbf{a}_c = -m_T\mathbf{a}_T .$$

In order that there be an equality, the object with the greater mass (truck) must have the smaller acceleration.

Fundamental units such as mass and time are arbitrarily defined. The unit of force is a derived unit defined in terms of acceleration and mass. In SI units a one-newton force will produce an acceleration of one meter per second per second on a one kilogram mass: $1\text{ N} = (1\text{ kg}) \times (1\text{ m/s}^2)$. In the archaic English system, one pound of force will produce an acceleration of one foot per second per second in a mass of one slug.

One force we encounter every day is that of gravity. When an object is in free fall, gravity accelerates it downward with a constant acceleration $g = 9.8$ m/s ^{2}. According to Newton's second law, the force must be

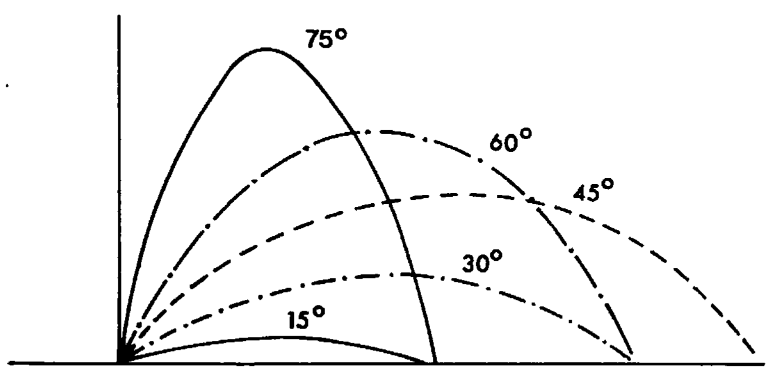
$$\mathbf{F} = m\mathbf{g} ,$$

directed vertically downward, toward the center of the earth. This is what is meant by the weight of an object. It is the force of gravity acting upon an object, whether it is accelerating or not:

$$\mathbf{W} = m\mathbf{g} .$$

Unlike mass, which is an intrinsic property of a body and a measure of its inertia, the weight of a body depends upon its location. The difference between a person's weight in Los Angeles (near sea level) and Denver (about 1600 m above sea level) is very small due to small variations in the value of g on the surface of the earth. The same person's weight on the moon would be only one-sixth of that on the earth because the value of g is about one-sixth that on the earth's surface. This would also mean that falling objects on the moon would have one-sixth the acceleration of gravity at the surface of the earth.

One application of Newton's second law that is presented in the video is projectile motion. Galileo was the first to describe projectile motion correctly. Using his laws of inertia and falling bodies, he was able to show that projectiles follow parabolic trajectories. He also showed that a projectile fired at a given angle will have the same range as an identical projectile fired at the angle's complement, as shown below.



PROJECTILES

Any object that moves through space under the influence of the earth's gravitational force is considered to be a projectile. For a projectile moving in a single plane, the motion may be analyzed in terms of the vertical and horizontal components. These two motions are entirely independent of each other because they are at right angles.

Projectile motion in the absence of air friction is analyzed in terms of Newton's second law in the video. The following details the mathematics that is presented:

Note that z is used to refer to the vertical direction.

In the vertical direction:	Newton's second law	$F_z = ma_z$,
	F_z is the force of gravity	$-mg = ma_z$,
	Therefore	$a_z = -g$.
	Substituting in the kinematic equation	$(s = vt + 1/2\,at^2)$,
		$y = v_z t - 1/2\,gt^2$.
In the horizontal direction:	Newton's second law	$F_x = ma_x$,
	The speed is constant	$\frac{\Delta v_x}{\Delta t} =$ const, or $a = 0$,
	Substituting in the kinematic equation	$(s = vt + 1/2\,at^2)$,
		$x = v_x t$.

Newton's second law, stated as $\boldsymbol{F} = m\boldsymbol{a}$, is probably the single most important equation in classical mechanics. As will be seen in the module *Conservation of Momentum*, Newton's laws take on even more universality when stated in terms of momentum.

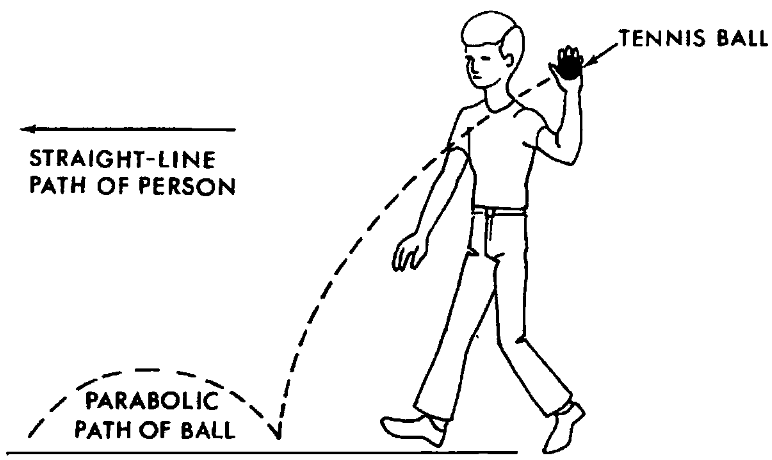
ADDITIONAL RESOURCES

Demonstration #1: Inertia

Purpose: To demonstrate Newton's first law - inertia.

Materials: Tennis ball.

Procedure and Notes: Hold the tennis ball in your hand. While continuing to walk at a constant speed, drop the ball. Even though you walk away from the point where you dropped it, the ball will continue to move alongside you in a series of parabolas.



Explanation: Both you and the ball are initially moving at the same constant speed in the same direction. Therefore, during the time it is falling, the ball continues to move forward with the same constant speed. It follows a parabolic trajectory and lands beside you at each bounce. This shows that the horizontal and vertical motions are independent of one another.

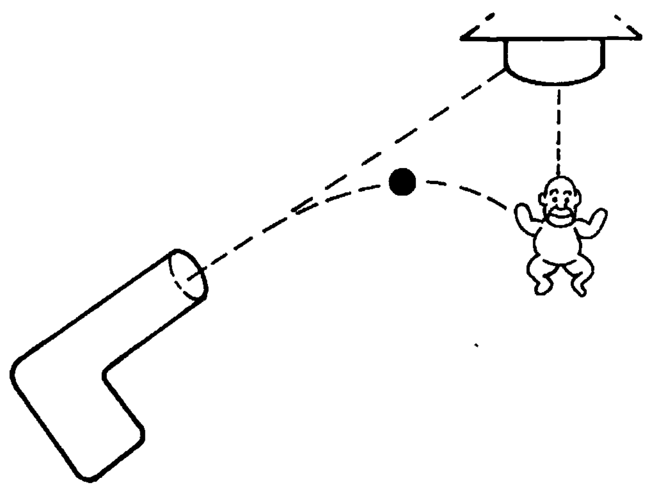
Note: Additional demonstrations on inertia may be found in the module *Inertia.*

Demonstration #2: Monkey and Hunter

Purpose: To demonstrate the independence of the horizontal and vertical components of projectile motion.

Materials: Monkey-and-hunter apparatus.

Procedure and Notes: The apparatus consists of an air gun with a sight system for aiming, and a monkey held up by an electromagnet. After aim has been taken, the monkey is released simultaneously with the firing of the gun. The projectile will strike the monkey. Different initial velocities can be obtained for the projectile by using various charges of compressed air.



Explanation: Without the effects of gravity, the projectile would follow a straight line as it leaves the gun (see diagram). Because of gravity, however, the projectile falls below that straight line by an amount identical to the distance the monkey falls. The projectile, therefore, always hits the monkey when it is aimed directly at him.

Demonstration #3: Accelerometer

Purpose: To demonstrate the vector natures of force and acceleration, and to show that they have the same direction, as required by Newton's second law, $F = ma$.

Materials: Accelerometer, commerically available or homemade. (See the module *Moving in Circles)*

Procedure and Notes: When a force is applied, the cork in the accelerometer leans in the direction of the acceleration. By exerting forces on the accelerometer in different directions and noting the direction in which the cork leans, the students can readily see that the force and acceleration are always in the same direction.

Explanation: For a detailed explanation, refer to the module *Moving in Circles* and/or *The Physics Teacher*, vol. 2, no. 4, p. 176.

Demonstration #4: Force and Acceleration

Purpose: To demonstrate that the same force applied to different masses will produce different accelerations.

Materials: Two identical shoe boxes or other containers, spring scale.

Procedure and Notes: Each box should be filled with material of a different mass, e.g., sand in one and styrofoam packing in the other. Seal the containers tightly so that students cannot see what is inside. Attach a spring scale to each box. Ask student volunteers to pull the boxes horizontally, simultaneously and with equal force. One container will move easily and quickly, but not the other.

Explanation: Since the students must exert equal forces on the boxes, they will not be able to compensate for the larger mass. Exerting the same force on the two will accelerate the smaller mass much more than the larger ($a \propto F$, $a \propto 1/m$).

Demonstration #5: Balloon Launch

Purpose: To demonstrate Newton's third law.

Materials: One balloon.

Procedure and Notes: Blow up a balloon and release it, allowing the air to escape. The balloon will "fly" around the room. Ask students to explain what happens in terms of Newton's laws. As an interesting aside, you might ask students what would happen if the balloon were released in space or in a vacuum.

Explanation: For every action there is an equal and opposite reaction. The escaping air causes the balloon to move in the opposite direction. This is also space or a vacuum, even though some students may protest that there is nothing for the escaping air to "push against."

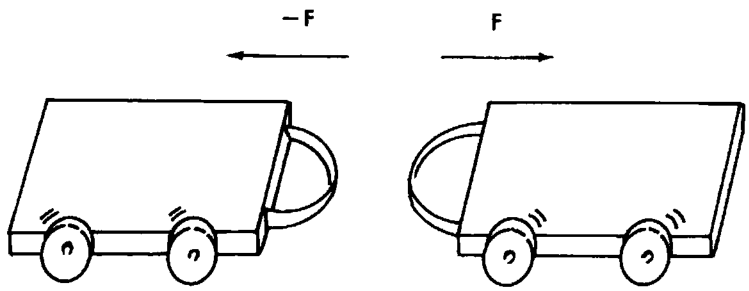
Demonstration #6: Spring Carts

Purpose: To demonstrate Newton's third law that for every action there is an equal and opposite reaction.

Materials: A pair of spring carts.

Procedure and Notes: The spring on each cart is depressed and the carts are brought together. When the springs are released, the carts depart in opposite directions, but with the same magnitude of force. With equal masses, their accelerations will be the same.

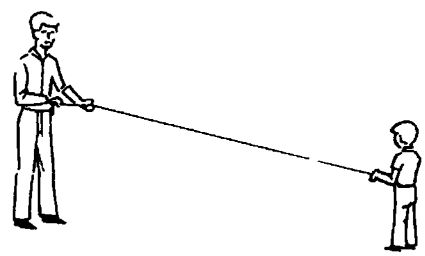
Newton's second law can also be investigated by varying the amount of mass on the carts and examining the resulting accelerations. Again, forces of equal magnitude but opposite in direction should be found to be acting on the carts.



Explanation: Newton's third law says that forces exist in action-reaction pairs; therefore, the forces on the carts should be found to have equal magnitude but opposite directions. Since the carts were initially at rest, there is no net force acting and the action-reaction pair cancel. Newton's second law states that $F = ma$ and can be used to calculate the resulting forces if the masses and accelerations are known. Here, the force on each cart at a given time is not known. But it is known that the two *forces*, whatever they are, are equal and opposite; therefore, so are the resulting *rates of change of momentum*. If the *masses* are equal, it follows that the resulting *accelerations* will be equal and opposite at all times; therefore, so are the resulting *velocities* and *positions*, since each cart's motion is a mirror image of the other cart's motion.

EVALUATION QUESTIONS

A man and his son are on a frictionless frozen pond. They are separated by a certain distance and each holds one end of a stretched rope in his hand. The man has twice the mass of his son. The father pulls with a constant nonzero force and keeps the rope taut. Questions 1 - 3 below refer to this situation.



1. Which of the following best describes the motion of the boy?

A. Constant speed.
B. Constant velocity.
C. Constant acceleration.
D. At rest.

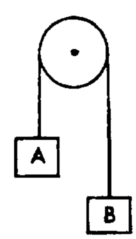
2. What is the force on the boy?

A. Half the force on the father, directed *towards* the father.
B. Equal to the force on the father, directed *towards* the father.
C. Twice the force on the father, directed *towards* the father.
D. Twice the force on the father, directed *away* from the father.

3. What is the acceleration of the boy?

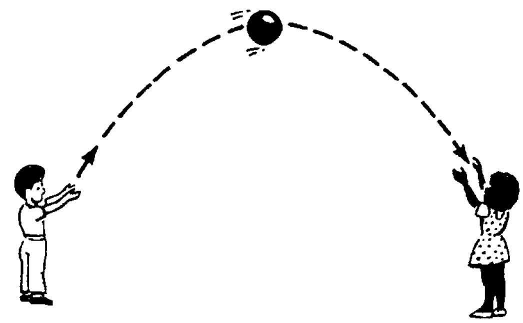
A. Twice the acceleration of the father, directed *towards* the father.
B. Half the acceleration of the father, directed *towards* the father.
C. Equal to the acceleration of the father, directed *away* from the father.
D. Zero, since the boy moves with a constant speed.

4. Two objects, Object A and Object B, are connected by a string over a frictionless pulley as shown. Object A is accelerating downward. Which of the following statements is true?



A. Object A is *more* massive than Object B.
B. Object A is *less* massive than Object B.
C. The magnitude of the acceleration of Object B is *greater* than that of Object A.
D. The magnitude of the acceleration of Object B is *less* than that of Object A.

5. **Two children are playing catch. The ball follows a parabolic trajectory. Which of the following best represents the direction of the force on the ball, the ball's acceleration, and its velocity at the *top* of its trajectory?**



	FORCE	ACCELERATION	VELOCITY
A.	→	O	→
B.	↓	↓	→
C.	→	→	→
D.	O	O	O

6. **A sheet of paper can be withdrawn from under a bottle of cola without toppling it if the paper is jerked quickly. This can be done because of the property of**

   A. weight.
   B. inertia.
   C. acceleration.
   D. Newton's third law of motion.

7. **An automobile that is pulling a trailer is accelerating on a level highway. The force that the automobile exerts on the trailer is**

   A. equal to the force the trailer exerts on the automobile.
   B. equal to the force the road exerts on the trailer.
   C. greater than the force the trailer exerts on the automobile.
   D. equal to the force the trailer exerts on the road.

8. **When the mass of an object is halved and the net force acting on it doubled, then the acceleration is**

   A. 1/4.
   B. 1/2.
   C. 2 times as great.
   D. 4 times as great.

9. A bug splatters on the windshield of a moving car. Compared to the magnitude of the force acting *on* the car, the magnitude of the force acting on the bug is

A. larger.
B. smaller.
C. equal.
D. not able to be determined with the information given.

10. A ball is thrown into the air at some angle. Neglecting air resistance, the horizontal velocity

A. increases.
B. decreases.
C. decreases, is zero at the top of the trajectory, then increases.
D. remains constant.

ESSAY QUESTIONS

11. To simulate walking on the moon, astronauts were trained by being suspended from a cable, with their feet on the ground. Why could such a cable be used to simulate situations in lunar gravity?

12. The acceleration of a rocket fired into space increases as it travels. Why do you suppose this happens?

KEY

1. **C**
2. **B**
3. **A**
4. **A**
5. **B**
6. **B**
7. **A**
8. **D**
9. **C**
10. **D**

SUGGESTED ESSAY RESPONSES

11. The tension in the cable exerts an upward force on the astronaut. This upward force vector added to the astronaut's downward force effectively makes the astronaut appear to have less weight.

12. As the rocket travels, it consumes fuel, which is part of its mass, at an enormous rate. Consequently, the rocket is losing mass. According to Newton's second law, the acceleration is inversely proportional to the mass. So if the mass decreases, then the acceleration must increase.

TEACHER'S GUIDE TO THE APPLE AND THE MOON

CONTENT AND USE OF THE VIDEO - This video should be shown only after a study of kinematics that has covered: (1) speed, velocity, and acceleration; (2) uniformly accelerated motion; (3) falling bodies as a special case of uniformly accelerated motion, specifically that the distance fallen depends on the square of the time spent falling; and (4) projectile motion, emphasizing that the horizontal and vertical components of motion are independent and that the horizontal component of velocity determines the distance traveled. An understanding of circular motion is not a necessary prerequisite for this video. However, the video assumes that from their study of dynamics students understand Newton's laws of motion.

This video is particularly effective if shown immediately following a presentation of Newton's law of universal gravitation and should be shown only after this topic has been introduced.

TERMS ESSENTIAL FOR UNDERSTANDING THE VIDEO - The video introduces several terms with which students may not be familiar. Therefore, it might be helpful to list these terms and discuss them briefly.

inertial mass--the property of an object which gives it resistance to acceleration, i.e., the m in F = ma. For further treatment of this concept, refer to Objective #1 in the module entitled *Curved Space and Black Holes*.

gravitational mass--the property of an object which gives it the ability to attract other masses, i.e., the m in $F_g = GMm/r^2$; although they exhibit different properties, inertial mass and gravitational mass are equal.

unit vectors--a unit vector can be defined as any vector divided by its magnitude; hence, a unit vector has a magnitude of 1 and is dimensionless but retaines the direction of the original vector. For example, a unit displacement vector can be written as $\hat{\mathbf{r}} = \mathbf{r}/|\mathbf{r}|$. A negative sign with $\hat{\mathbf{r}}$ indicates direction toward the origin. The gravitational force is an example,

$$\mathbf{F}_g = \frac{-Gm_1m_2}{r^2}\,\hat{\mathbf{r}}\,,$$

that implies an attractive force on m_2 toward m_1.

weightlessness--a state in which an object is at the same acceleration as its reference frame; objects that seem to float in space are really falling at the same rate as the objects around them.

WHAT TO EMPHASIZE AND HOW TO DO IT - The video develops the concept that the gravitational force which exists between all masses is the same force responsible for the acceleration of a falling apple as well as the acceleration of the moon.

Objective 1: Recognize that a gravitational force exists between any two objects and that the force is directly proportional to the product of the masses and inversely proportional to the square of the distance between them.

When Newton proposed this theory, he had no way of verifying it. How could he show that gravity was correctly described by his law? The video presents Newton's calculation of the distance the moon falls toward the earth during each second; his analysis postulates a relationship of force to the inverse square of the distance between their centers of mass. When this relationship was combined with his second law, Newton was able to verify his law of universal gravitation. The SUPPORTIVE BACKGROUND INFORMATION provides a detailed consideration paralleling the calculation shown in the video.

Objective 2: Investigate the variation in gravitational force for attracting bodies of different masses and at different distances.

The next to last question in the section on EVERYDAY CONNECTIONS AND OTHER THINGS TO DISCUSS encourages students to explore the relationship of mass and distance to gravitational force. Most texts provide appropriate problems for a fuller examination of the relationships.

Objective 3: Understand the relationship inherent in the following expressions:

$$F = GMm/r^2, \quad F = ma, \quad a = GM/r^2.$$

Prior to viewing the video, you might want to indicate how the last formula above is derived from the first two formulas. Stress that the gravitational attraction force between two masses depends on *both* masses, whereas the acceleration of one mass toward the other *only* depends on the mass of the body causing that acceleration.

Objective 4: Recognize that, for small enough velocities, the time for a projectile to fall to earth is independent of its horizontal velocity, but for very large velocities, the effect of the earth's curvature must be taken into consideration.

There is a myth in physics that the fall of an apple inspired Isaac Newton to formulate his law of universal gravitation. Since Newton did spend time walking among the orchards of his family farm in Lincolnshire, it is not unlikely that he had something like a falling apple in mind when he pondered the connection between the way things fall on earth with the way the moon "falls." The same questions that provoked Newton are the ones presented and discussed in the video. However, the video makes the connection between the apple and the moon, not in an orchard but through the extension of projectile motion to an object in orbit. A large horizontal velocity imparted to a falling object, combined with the curvature of the earth results in the object never touching the ground. Thus, the moon falls freely in the same way as a dropped apple.

A projectile motion demonstration can offer a good introduction to the video. Use a second-law-of-motion apparatus which allows one object to fall as another is projected horizontally (Sargent- Welch 0877). If one is not available, the phenomenon can be shown, using DEMONSTRATION #1 from the resource section of the module. After the demonstration, review the physics involved. Ask the question: *What if the horizontal object were propelled by an ultra-high velocity cannon?* Encourage the students to discuss the possibilities.

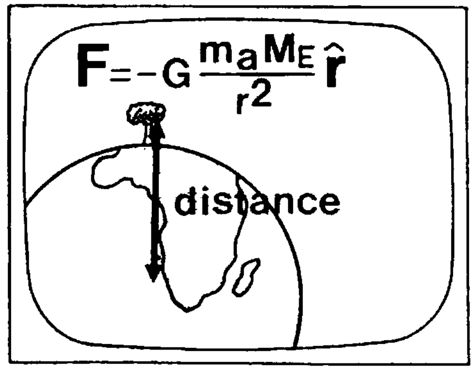
Objective 5: Describe orbital motion in terms of the laws of universal gravitation and inertia.

The notion of "weightlessness" fascinates students and lends itself to several excellent demonstrations. The discussion surrounding the demonstrations reinforces an understanding of the forces acting on falling bodies. Both DEMONSTRATION #2 and DEMONSTRATION #3 are recommended. Some applications of weightlessness are cited in the ADDITIONAL RESOURCES and might be discussed at this time. The resource section also offers further questions for discussion. Most of these questions focus on a mathematical explanation of Newton's law of universal gravitation and perhaps are better discussed after students have had time to work with the law on their own.

Since most students are interested in planets and satellites, a few selected questions about "what holds them in orbit" or "what would happen if the gravitational force suddenly shut off" could also be used here.

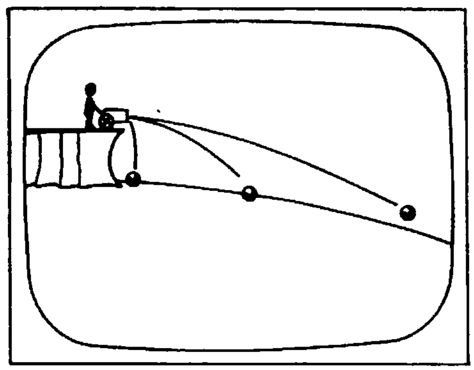
POINTS TO LOOK FOR IN THE VIDEO - Several questions are posed in the STUDENT'S GUIDE. Here are those questions along with suggested responses and frames from the video.

Newton assumed that every two bits of matter in the universe attract each other. The force of attraction is proportional to each of the masses and weakens, inversely, as the square of the distance between the two bodies. This law can be represented as a vector equation with a constant of proportionality called "G", which is the same for any two bodies in the universe. What does the negative sign in this frame of the video indicate?



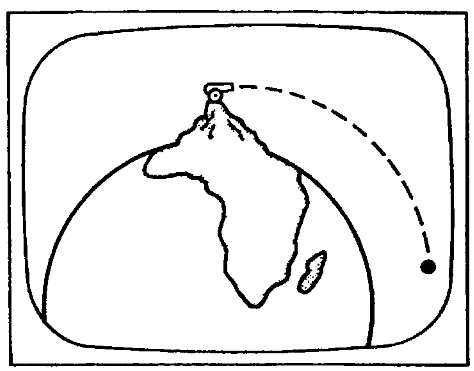
If we say that the earth is exerting a gravitational force on a mass m, the vector $\hat{r}$ goes from the center of the mass of the earth to m, but the gravitational force is attractive, so the force on m is directed toward the earth, i.e., in the negative $\hat{r}$ direction.

Newton could imagine a cannonball moving so fast, it would never strike the ground. It would just keep falling forever, while the earth curved away forever beneath it. In other words, it would be in orbit. Is the orbit a parabola or a circle?



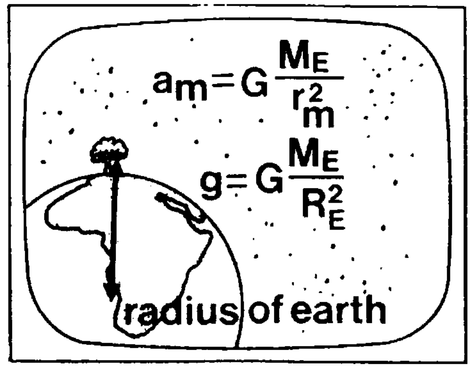
Since the acceleration is constant in magnitude and always pointing toward the center of the earth (thus changing direction with the curvature of the earth) the orbit becomes circular.

Would the orbiting cannonball have the same acceleration toward the center of the earth as the moon would have?



The accelerations differ because their distances to the center of the earth are different. Because the cannonball is orbiting much nearer to the earth than is the moon, its acceleration is greater.

Since the moon is falling, why doesn't it hit the surface of the earth?



The moon effectively remains at the same distance from the earth's surface because as it falls toward the earth, the earth constantly curves away beneath it.

EVERYDAY CONNECTIONS AND OTHER THINGS TO DISCUSS - To reinforce further the concepts presented in the video, you might pose the following questions to your students.

1. **According to Newton's universal law of gravitation, each bit of matter attracts every other bit in the universe. For example, every atom in an astronaut attracts each bit of matter in the moon. Now this property is also true of gas atoms, such as hydrogen, out in the far reaches of space from which stars are formed. Does gravity play a role in the formation of stars? Based upon what you know about the universal law of gravitation, do you expect the force between two gas atoms to be very large? Will the associated acceleration be great or small?**

   The gravitational force is the force which forms stars and the force which ultimately determines the fate of the universe. Through the mutual attraction of gravity, gas atoms collect together and condense to form stars. Because the masses of atoms are small and the distances in interstellar space are vast, the gravitational force, GMm/r^2, *is very small. This small force produces small acceleration of the atoms toward each other according to* $a = F/m$. *Although this acceleration is small, the atoms, given enough time, eventually "fall together" and form stars. The time scale is not seconds or minutes but rather billions of years!*

2. **To construct faster computers and electronic circuits to perform more complex functions, perfect crystals of germanium, silicon, iron, etc., are needed. How is the environment onboard a Space Shuttle better for growing perfect crystals?**

   In an orbiting Space Shuttle everything is in freefall and appears weightless. Crystals that grow in such an environment don't have gravity causing them to sag downward as they do on the surface of the earth, and so they can be free of imperfections in their structure. See DEMONSTRATION #2 and DEMONSTRATION #3.

3. **The density of substances varies; for example, oil is less dense than gold. How does a geologist use these variations and a knowledge of the universal law of gravity to locate oil or gold?**

   In the video we saw that the acceleration due to gravity depends on the mass of the earth, its radius, and a universal constant according to $g = GM_E / R_E^2$. *Since the density of the earth is not the same everywhere over the globe, g varies over the surface of the earth. Large deposits of oil make the density and hence mass of earth in those regions to be smaller than the average (rock) and results in a smaller value of g. On the other hand, a deposit of gold would result in a larger value of g than normally expected. So by carefully measuring variations in g, geologists can detect the presence of oil and minerals. (Isn't that why they're called "g"-ologists?)*

4. **NASA scientists plan to construct space stations out of aluminum foil girders that are fabricated in space from aluminum foil. Do these structures have to be as strong as buildings on earth?**

   The space stations will be in free fall as they orbit the earth. That means you can build large structures without worrying about their crushing under their own weight. See DEMONSTRATION #2.

5. **Since gravitational force exists between any two bodies, why are you and your neighbor not moving closer together?**

   Because the masses involved are tiny compared to the mass of the earth, the gravitational force is extremely small and other forces, such as friction, prevent you from "falling" toward your neighbor.

6. **If a satellite is in orbit around the earth, are the astronauts really weightless?**

   No. Gravitational force always exists between two masses. However, the spacecraft is falling along with the astronaut. No contact forces exist between the astronaut and the spacecraft, so the astronaut has a sense of weightlessness, but only with respect to the spacecraft. The gravitational force is causing both the spacecraft and the astronaut to orbit.

7. **Which weighs more--an orbiting spacecraft or its astronaut?**

   The spacecraft weighs more because the gravitational force (weight) is proportional to the mass of the object.

8. **How does the gravitational acceleration of the orbiting spacecraft compare with that of its astronaut?**

   Gravitational acceleration is independent of the mass of the falling object, so the accelerations are the same.

9. **When the universal law of gravity was combined with Newton's second law, we tacitly assumed that the mass of the object appearing in both was the same and cancelled it. However, does the mass in $F = ma$ measure something different from mass in $F = GMm/r^2$?**

   In Newton's second law mass is a measure of an object's ***inertia*** *- its resistance to changes in motion. In the universal law of gravity mass is a measure of the* ***pull of gravity****. So mass is a measure of two different properties of matter. Einstein wondered about this equivalence between inertial mass (the m in $F = GMm/r^2$) and used it as a starting point in his theory of gravity--the general theory of relativity. As far as experiments can reveal, these two masses are identical in value.*

10. **An apple is suspended 110 km above the earth's surface. If the earth's radius were somehow expanded by 100 km while its mass remained constant, how would the gravitational force on the apple change? Why?**

    The gravitational force would remain constant, because neither the masses nor the distance between their centers would be changed.

11. Assuming the ratio of the earth's mass to that of the moon is 81 to 1, and the separation between the earth and the moon is 240,000 mi, calculate the distance from the moon where the gravitational pull of the earth and the moon would be equal and opposite on some mass. Would this point make an ideal location for a permanent space colony?

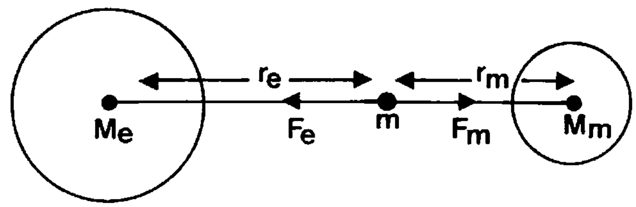
Let

F_e = gravitational force due to the earth

F_m = gravitational force due to the moon

r_e = distance of mass to earth center

r_m = distance of mass to moon center



so that

$$F_e = \frac{GM_e m}{(r_e)^2}, \quad F_m = \frac{GM_m m}{(r_m)^2}.$$

At the point required, $F_e = F_m$, which implies

$$\frac{GM_e m}{(r_e)^2} = \frac{GM_m m}{(r_m)^2}$$. Rearranging them we get

$$\frac{M_e}{M_m} = \frac{(r_e)^2}{(r_m)^2}$$. Substituting and solving for r_m, we find

$$\frac{81}{1} = \frac{(240{,}000\text{ mi} - r_m)^2}{(r_m)^2}, \text{ or}$$

$$\frac{9}{1} = \frac{240{,}000\text{ mi} - r_m}{r_m}, \text{ and solving for } r_m$$

$$9r_m = 240{,}000\text{ mi} - r_m,$$

$$10r_m = 240{,}000\text{ mi},$$

$$r_m = 24{,}000\text{ mi}.$$

At this point the total force is zero, but let's examine what happens if an object there moved even slightly closer to the earth or to the moon. If it moved closer to the earth, the gravitational force from the earth would be greater than that from the moon, and so the object would fall to the earth. On the other hand, if the object moved slightly closer to the moon, the moon's gravity would be greater than the earth's gravity acting on the object and so the object would fall to the moon. The point where there is no gravitational force is not a stable one, and so a space station would not remain suspended there.

12. **Could a golf ball be placed into "orbit" just above the surface of the earth, assuming that there were no mountains to block its path and ignoring effects of air resistance? If so, would it have to move faster or slower than the moon does in its orbit?**

If a golf ball were hit hard enough and imparted a great enough speed, it could orbit the earth. From the video we know that the acceleration of a falling golf ball (just like an apple) will be about 3600 times greater than that of the moon. Since the golf ball is much closer to the earth, it has a greater acceleration toward the center of the earth than the moon does. Therefore, the golf ball would have to move much faster than the moon in order to remain a constant distance from the center of the earth as it falls.

13. **Does the earth fall farther in a second toward the sun than the moon falls in one second toward the earth? Think about the factors that determine this and reason an answer; use a result from the video to back up your conclusion.**

The sun is much more massive than the earth, so you might expect the gravitational force of the sun on the earth to be greater than that of the earth on the moon. However, the earth is much farther away from the sun than the moon is from the earth. A combination of mass and distance determines how far any object falls in a second. The sun is about a million times more massive than the earth ($M_{sun} = 2.0 \times 10^{30}$ kg, $M_E = 6.0 \times 10^{24}$ kg) *and about 400 times farther away* ($r_{ES} = 1.5 \times 10^{11}$ m, $r_{mE} = 3.8 \times 10^{8}$ m). *That means that the acceleration of the earth is about* $10^6/(400)^2$ *or about 6 times greater than the acceleration of the moon toward the earth. Consequently, the earth falls farther toward the sun in one second than the moon falls toward the earth in a second. From the video we know that* $s = d^2/2r$ *into which numbers can be substituted to confirm this prediction.*

The following thought experiment provides an opportunity for students to apply the concepts learned in the video to a new situation.

Thought Experiment: Suppose you have a lead brick and a ping-pong ball.

Which is more difficult to throw?

The lead brick is more difficult to move because it has greater inertia due to its greater mass.

On which is the earth pulling more?

The gravitational force is greater on the lead brick because it has greater mass.

In a vacuum which falls faster?

They both fall with the same constant acceleration.

Why, if the lead brick is harder to get moving, does it fall in a vacuum with the same acceleration as the ping-pong ball?

Although the brick is harder to move, gravitational force is greater on it than on the ping-pong ball. This larger force compensates for the greater inertia and the amazing result is that the brick and ball fall with the same constant acceleration.

Suppose you placed the lead brick on a scale in a vacuum and dropped them. What would the scale read?

Since the brick and scale fall at the same rate, the brick can't push against the scale. Therefore, it is "weightless." Like astronauts orbiting the earth, objects that are in free fall are "weightless."

How much would you weight in an elevator which is freely falling?

You would be "weightless" (temporarily) since your environment is freely falling, as would the scale that you are standing on.

SUMMARY - The video ties together two seemingly different phenomena. The moon falls in its orbit around the earth in the same way that an apple falls to earth. This relationship of apple to moon is dramatically demonstrated through the development of Newton's universal law of gravitation. If we apply Newton's second law to the apple (moon) falling to the ground, then the mass m in $F = ma$ is the mass of the apple (moon). The force F is given by Newton's law of gravitation, $F = -GMm/r^2 \hat{r}$, where M is the mass of the earth and m that of the apple (moon). When this force is substituted into the Newton's second law, the mass of the apple (moon) cancels out and the acceleration of the apple (moon) is dependent only on the mass of the earth and the distance from the earth to the apple (moon).

There are many relationships of a mathematical nature illustrated in this video. All of these use algebraic treatments that are within the comprehension of most students. However, they go by rapidly on the screen. Therefore, it is important for the teacher to use the pause capabilities of the video recorder for a more detailed analysis of the equations and to write the various equations on the board for discussion of the relationships among quantities.

NOTE OF EXPLANATION REGARDING THE STUDENT'S GUIDE - The following two pages of the STUDENT'S GUIDE should be duplicated and distributed to the students for use in preparation for viewing the video.

In general, the STUDENT'S GUIDE lists topics, terms, and questions, and the TEACHER'S GUIDE provides definitions, discussion, and answers to the questions. It is very important to have the students receive an appropriate "preparatory set" for viewing the VIDEO and also, following the showing of the VIDEO, to have a systematic discussion, analysis, and summarization of the objectives of the module.

The students should be informed that the INTRODUCTION, TERMS ESSENTIAL FOR UNDERSTANDING THE VIDEO, and POINTS TO LOOK FOR IN THE VIDEO should be read and discussed *prior* to viewing the VIDEO. These should also be rediscussed following the viewing.

Answers to the questions listed in the STUDENT'S GUIDE have been included under POINTS TO LOOK FOR IN THE VIDEO in the Teacher's Guide. The questions which follow this section of the Teacher's Guide and deal with EVERYDAY CONNECTIONS AND OTHER THINGS TO DISCUSS as well as the SUMMARY should be discussed as a part of the activities that *follow* the viewing(s) of the VIDEO and give closure to the lesson.

STUDENT'S GUIDE TO THE APPLE AND THE MOON

INTRODUCTION - The video develops the concept that the gravitational force which exists between all masses is the same force responsible for the acceleration of a falling apple as well as the acceleration of the moon.

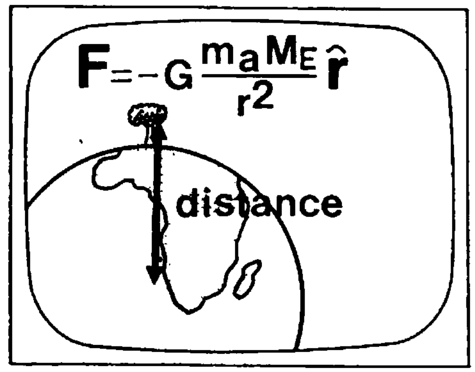
Terms Essential for Understanding the Video

inertial mass	gravitational mass
unit vectors	vacuum
weightlessness	gravitational force
Newton's Second Law	

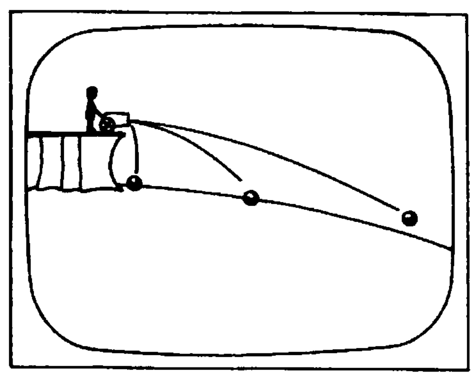
*** *NOTE: Parts of the video, especially mathematical equations, may go by quickly on the screen. If you have questions, you should ask your teacher to replay these sections.* ***

Points to Look for in the Video

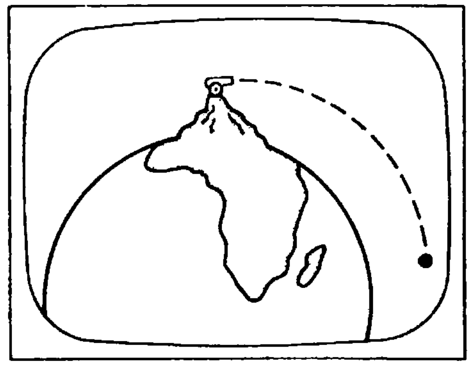
Newton assumed that every two bits of matter in the universe attract each other. The force of attraction is proportional to each of the masses and weakens, inversely, as the square of the distance between the two bodies. This law can be represented as a vector equation with a constant of proportionality called "G", which is the same for any two bodies in the universe. What does the negative sign in this frame of the video indicate?



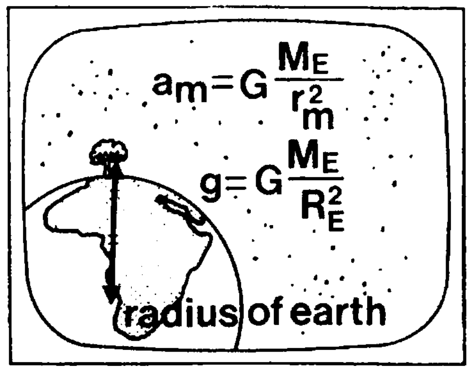
Newton could imagine a cannonball moving so fast, it would never strike the ground. It would just keep falling forever, while the earth curved away forever beneath it. In other words, it would be in orbit. Is the orbit a parabola or a circle?



Would the orbiting cannonball have the same acceleration toward the center of the earth as the moon would have?



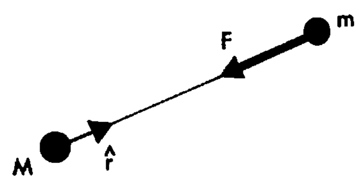
Since the moon is falling, why doesn't it hit the surface of the earth?



TEACHER RESOURCES

SUPPORTIVE BACKGROUND INFORMATION - Newton had struggled with questions on the nature of gravity in an effort to explain the basic rules of planetary motion that had been laid down by Johannes Kepler half a century earlier. In what way must gravity decrease to account for Kepler's third law, which relates the period and radius of a planet's orbit? What other physical quantities could this force depend on? And how is Galileo's law of falling bodies – gravity on the earth – related to gravity in the heavens?

From his study of Kepler's orbits, Newton had an inkling of what he needed. The force between any two bodies in the universe would have to diminish as the bodies moved farther apart. The force, he conjectured, would be inversely proportional to the square of the distance between the two bodies. To complete his law of universal gravitation, Newton further stated that the force of gravity is proportional to the mass of each of the two bodies involved. The law is expressed as follows:



$$\mathbf{F} = \frac{-GMm}{r^2}\,\hat{\mathbf{r}}\,.$$

$\mathbf{F}$	–	gravitational force
G	–	gravitational constant
M, m	–	masses
r	–	distance between masses
$\hat{\mathbf{r}}$	–	unit vector

Gravity tends to pull objects directly toward one another. The direction of the gravitational force between them is along the line which joins them. This direction is signified by using the unit vector $\hat{\mathbf{r}}$. A negative sign designates that the force is attractive.

The proportionality constant G is a universal constant; it has the same value for any two bodies in the universe. This constant should not be confused with g – the acceleration of a body on the surface of the earth due to the earth pulling on it. The constant G must be found from an experiment, but once it is determined, it is the same for *any* two bodies. That's why it is a *universal* constant. The value of G was first determined to be $6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$ by Henry Cavendish in 1798.

In applying the law of universal gravitation to objects such as the moon and the earth, Newton realized that he was dealing not with point sources, but with huge bodies containing infinite numbers of points. His solution to the problem required him to invent integral calculus, i.e., effectively apply the law an infinite number of times and sum all of the forces. He saw that the problem could be simplified by considering all the mass in the body to be mathematically concentrated at its center of gravity.

In Newton's second law, $F = ma$, the gravitational force can be substituted to explain the motion of all objects under the influence of gravity. This leads to

$$ma = F = GMm/r^2\,.$$

It is possible to eliminate the common factor of the mass of the object, m, and find the acceleration of gravity for some object a known distance from the center of mass of a body.

The cancellation of the mass of the object reveals something amazing. The gravitational acceleration does *not* depend on the mass of the falling object. Although a more massive object has greater inertia and is more difficult to accelerate, the gravitational force on it is correspondingly greater. This greater force is represented by m in $F = GMm/r^2$ and precisely compensates for the object's inertia. That, then, is the reason why all objects near the surface of the earth fall with the same constant acceleration – a fact Galileo had discerned a half century before Newton.

The acceleration of any mass m a distance r from the center of a mass M is given by

$$a = \frac{GM}{r^2} .$$

For an object falling near the surface of the earth, this acceleration is

$$a_{\text{earth}} = \frac{(6.67 \times 10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24}\text{kg})}{(6.37 \times 10^6\,\text{m})^2} = 9.80\,\text{m/s}^2 ,$$

whereas the acceleration of an object on the moon is

$$a_{\text{moon}} = \frac{(6.67 \times 10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2)(7.35 \times 10^{22}\text{kg})}{(1.74 \times 10^6\,\text{m})^2} = 1.63\,\text{m/s}^2 .$$

The video describes three such accelerations: (1) the acceleration of an object near the earth's surface (32 ft/s^2, or 9.8 m/s^2), (2) the acceleration of an object near the moon's surface (1/6 that of the earth – 5.4 ft/s^2, or 1.63 m/s^2), and (3) the acceleration of the moon itself toward the earth (1/10 in/s^2 or 0.25 cm/s^2). It will be important for student understanding to distinguish these roles of acceleration and to indicate the point of reference for each.

Once Newton understood that the moon is accelerated toward the earth, he could determine how far it should fall in each second compared to how far an apple falls in the first second near the surface of the earth. If the acceleration of one object toward another is given by $a = GM/r^2$, then

$$\frac{s_{\text{moon}}}{s_{\text{apple}}} = \frac{1/2\,at^2}{1/2\,gt^2} = \frac{GM_{\text{earth}}/r_m^2}{GM_{\text{earth}}/R_e^2} = \frac{R_e^2}{r_m^2} .$$

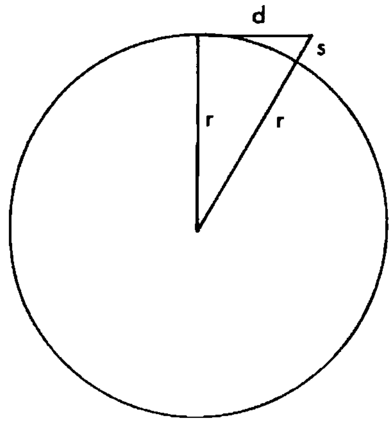
Ancient Greek mathematicians had figured out that the distance to the moon is about 60 times the radius of the earth. Therefore the ratio is

$$\frac{s_{\text{moon}}}{s_{\text{apple}}} = \frac{1^2}{60^2} = \frac{1}{3600} .$$

This means that if an apple falls 16 feet in its first second, the moon in that same second falls

$$s_{\text{moon}} = \frac{16\,\text{ft.}}{3600} = 0.05\,\text{in} = 1/20\,\text{in}.$$

Newton now had a prediction, but how could he check it and therefore show that gravity was correctly described by his law? In other words, how can we calculate how much the moon actually falls in one second? According to the law of inertia, the moon does not want to travel in a circle around the earth, but would rather fly tangentially out and keep moving in a straight line. Gravity from the earth always makes it fall, thereby causing it to move in a circle:



The distance d is the horizontal distance the moon tends to move in one second. It really falls distance s in that second. From the theorem of Pythagoras we have

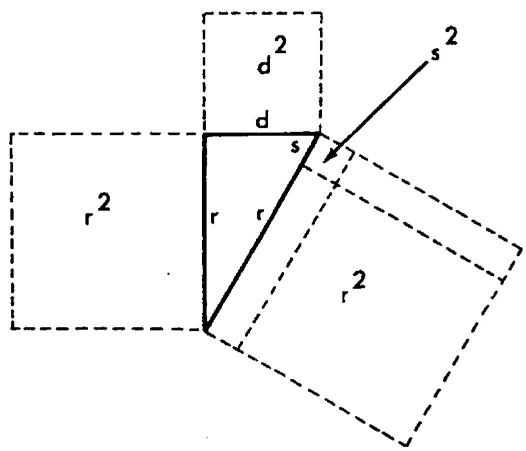
$$(r + s)^2 = r^2 + d^2,$$

$$r^2 + 2rs + s^2 = r^2 + d^2.$$

The r^2 term drops out and in a time of 1 second s^2 is insignificantly small (see diagram), so

$$2rs = d^2,$$

$$s = \frac{d^2}{2r}.$$



Because d is so much smaller than r, we can approximate d as part of the circle of the moon's orbit. Then the ratio of d to the circumference of the circle ($2\pi r$) is equal to the ratio of the time it takes the moon to travel the distance d (which is 1 second) to the time for the moon to travel once around the circle (27 1/3 days). This means that

$$\frac{d}{2\pi r} = \frac{1\,\text{s}}{27\ 1/3\ \text{days}} = \frac{1\,\text{s}}{2{,}358{,}720\,\text{s}} = 4.2 \times 10^{-7},$$

$$d = (2\pi r)(4.2 \times 10^{-7}) = 2\pi\ (240{,}000\ \text{mi})\ (4.2 \times 10^{-7}) = 0.63\ \text{mi}.$$

Substituting this value in our expression for s, we obtain

$$s = (0.63\ \text{mi})^2 / (2 \times 240{,}000\ \text{mi}) = 8.3 \times 10^{-7}\ \text{mi} = 0.05\ \text{in} = 1/20\ \text{in}.$$

This agrees with Newton's prediction, and in that 1/20 inch the physics of the earth and the heavens were united!

What Newton gave us was not just a series of scientific discoveries but, rather, a coherent view of how and why the universe works. That view has dominated all aspects of western thought from his time right down to our very own.

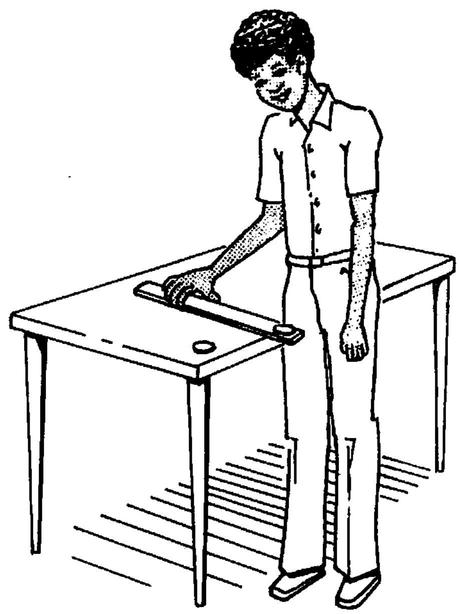
ADDITIONAL RESOURCES

Demonstration #1: Projectile Motion

Purpose: To demonstrate that two bodies dropped from the same height will strike the floor at the same time regardless of differences in their original horizontal motion.

Materials: Foot ruler, two coins, pieces of chalk, etc.

Procedure and Notes: Place a coin at the corner of a table. Hold a ruler as shown with a second coin on the end of the ruler over the edge of the table. Swivel the ruler quickly on the table to strike the coin on the corner of the table. Because of its inertia, the coin on the ruler is left behind and falls straight down. The coin on the table is given a large horizontal velocity. Both hit the floor simultaneously.



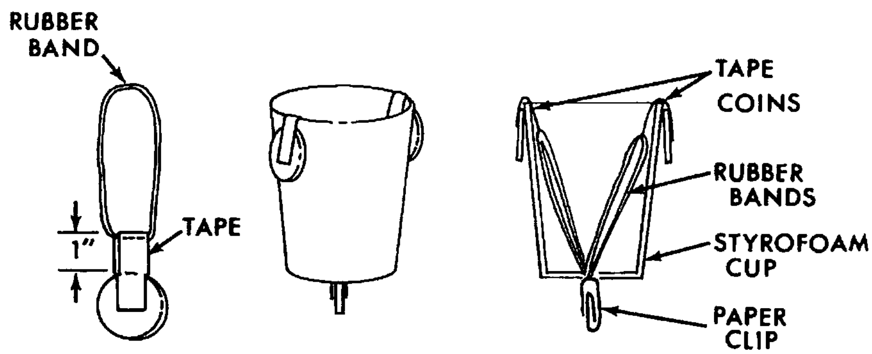
Explanation: Horizontal and vertical motion are independent of each other. Both coins fall with the same initial vertical motion: zero speed. Therefore, both travel the same vertical distance in the same time and strike the floor at the same instant.

Demonstration #2: Weightlessness

Purpose: To demonstrate effects of weightlessness.

Materials: Styrofoam cup, two coins or washers, two rubber bands, tape, a paper clip.

Procedure and Notes: Two coins are taped to rubber bands. The ends of the rubber bands are attached to a paper clip which keeps them under light tension as shown in the diagram below. When the system is allowed to fall, the coins hop into the cup.



Explanation: Initially the system of coins and rubber bands is in equilibrium: the force of gravity on a coin is balanced by tension in the rubber band. However, when the system is in free fall, each coin accelerates at g. Gravity provides this acceleration and so the tension in the rubber band must become zero, which only occurs when the rubber band is not stretched. When the system is allowed to fall, the rubber band shrinks to its natural length and in doing so pulls the coins into the cup. (Further analysis can be found in *The Physics Teacher*, November, 1983.)

Demonstration #3: Water Drop

Purpose: To demonstrate that in free fall objects fall with the same acceleration.

Materials: One aluminum can with the lid cut off and a hole near the bottom on the side.

Procedure and Notes: First fill the can with water and let some water run out of the hole into a wastebasket. Then cover the hole with thumb until ready to continue. Allow students to guess what will happen to the system when it is placed in free-fall. Release the can into the basket from a sufficient height to allow the students to see the effect.

Explanation: Since the can and the water are both falling at the same rate, they have the same velocities at each second and the water cannot precede the can. Therefore, no water will leave the can.

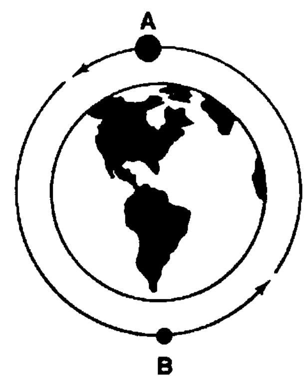
EVALUATION QUESTIONS

Two satellites, A and B, share a circular orbit of radius r_s about the center of the earth as shown in the sketch. Satellite A is *more massive* than Satellite B. The following questions refer to these satellites.

1. The magnitude of the earth's gravitational force on Satellite B is

A. equal to the force on Satellite A.
B. less than the force on Satellite A.
C. greater than the force on Satellite A.
D. zero.

mass A > mass B



2. The acceleration of Satellite B is

A. equal to that of Satellite A.
B. less than that of Satellite A.
C. greater than that of Satellite A.
D. zero.

3. If, in the middle of the night, the earth were suddenly to expand uniformly to a slightly larger radius without any increase in its mass, how would the motion of Satellites A and B be affected?

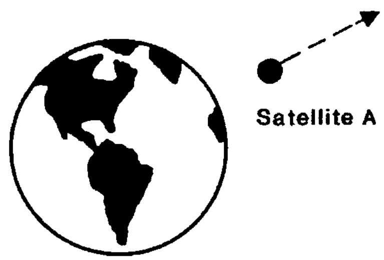
A. Both of the satellites would fall to the surface of the earth.
B. Neither of the satellites would travel in a circular orbit; instead, both would move away from the earth along a straight line.
C. The motions of the satellites would not change.
D. Satellite A would fall to the surface of the earth, whereas Satellite B would continue in its orbit.

4. If Newton were asked to explain why the two satellites from Questions 1, 2, and 3 don't fall to the surface of the earth, he would probably say that

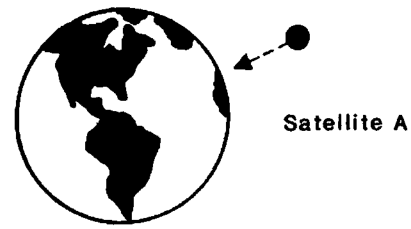
A. the inertia of the satellites keeps them moving in a circle.
B. the satellites are moving so fast that, as they continually fall, the surface of the earth curves away beneath them so they would never strike the ground.
C. the satellites are so far from the earth that the force of gravity has been weakened to the point that the satellites do not accelerate towards the earth.
D. circular motion is the natural motion of objects in the heavens; and, since the satellites are now in the heavens, they take on that characteristic.

5. **If the force of gravity from the earth could be "turned off" suddenly, which of the following paths indicates the subsequent motion of Satellite A?**

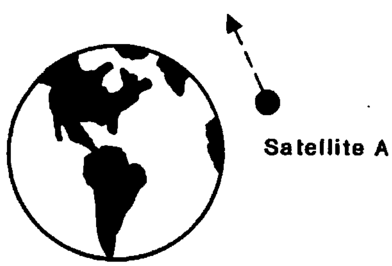
A.



C.



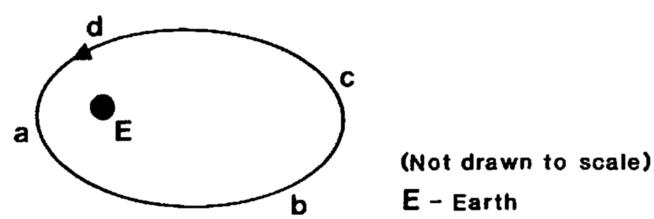
B.



D.

6. **A satellite is in an elliptical path about the earth. Where is the force greater on the satellite?**

A. at point a.
B. at point b.
C. at point c.
D. at point d.



7. **If the distance of a satellite from the earth at Position c is 4 times the distance when it is at Position a, the force on the satellite at c will be**

A. 4 times greater.
B. 1/4 as great.
C. 16 times greater.
D. 1/16 as great.

8. **A satellite is at a particular distance above the earth's surface. Doubling its mass and halving its distance from earth's center would cause the gravitational force to**

A. increase by 4.
B. increase by 8.
C. decrease to 1/4 its original value.
D. decrease to 1/8 its original value.

9. Two objects fall toward the earth. The first mass has an acceleration quadruple that of the second. What could account for this?

A. The mass of the 1st object is 4 times greater than the 2nd.
B. The mass of the 1st object is half that of the second.
C. The 2nd object is 4 times as far from the earth's center.
D. The 1st object is half as far from the earth's center.

10. A short range projectile is fired horizontally. Neglecting air friction, the time for it to impact the earth's surface

A. depends on the mass of the projectile.
B. increases as the initial height of the projectile increases.
C. decreases as the initial horizontal velocity decreases.
D. depends on none of the above statements.

ESSAY QUESTIONS

11. A cannon fires a projectile parallel to the earth's surface. Describe the possible paths of the projectile for different velocities of the projectile as it leaves the cannon.

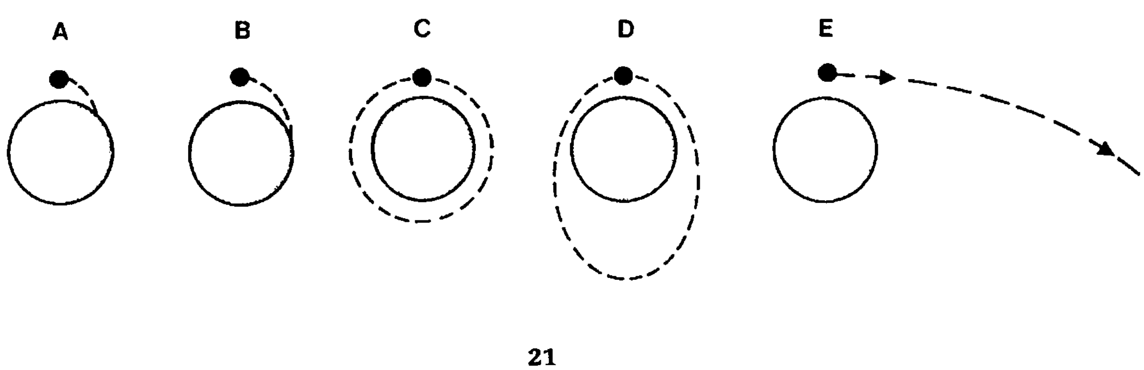
12. Discuss how an apple's falling is similar to the moon's motion.

KEY

1. **B**
2. **A**
3. **C**
4. **B**
5. **B**
6. **A**
7. **D**
8. **B**
9. **D**
10. **B**

SUGGESTED ESSAY RESPONSES

11. The diagrams below represent possible projectile paths for different initial velocities. The smallest initial velocity is represented by diagram a and the greatest by diagram e.



12. Both the apple and the moon are falling. However, the moon also has a velocity perpendicular to its falling path. If it had no such velocity, it, like the apple, would fall to the earth. But the moon's velocity is so fast that, as it falls forever, the earth forever curves away beneath it.

TEACHER'S GUIDE TO HARMONIC MOTION

CONTENT AND USE OF THE VIDEO - In the video simple harmonic motion is examined as an application of Newton's second law. Consequently, it assumes that students have already studied dynamics and have worked with Newton's laws of motion. The video also assumes an understanding of (1) the kinematics of falling bodies; (2) inertia; (3) uniform circular motion; and (4) conservation of energy. Resonance is introduced as an extension of harmonic motion.

TERMS ESSENTIAL FOR UNDERSTANDING THE VIDEO - Terms are introduced in the video which students might have heard before but possibly do not understand thoroughly. Therefore, it would be helpful to discuss these terms briefly prior to viewing.

periodic motion--any motion that repeats itself in equal periods of time.

oscillatory motion--any periodic motion in which the object moves back and forth over the same path.

Hooke's law--a force law which describes the force exerted by a stretched spring; it states that the force exerted by the spring is directly proportional to the amount the spring is stretched (compressed) and in a direction opposite to that stretching (compression). This law is usually written as $F = -kx$, where F is the force exerted by the spring, x is the displacement of the spring from equilibrium, and k is the constant of proportionality, known as the spring constant, which measures the stiffness of the spring.

simple harmonic motion--any periodic motion in which the restoring force is directly proportional to the displacement from equilibrium.

equilibrium position--position at which the net force is zero; also, the position toward which the restoring force is always directed.

frequency--the number of vibrations a body makes per unit time.

natural frequency--the frequency with which a system displaced from its equilibrium position vibrates.

amplitude--the maximum displacement a body in simple harmonic motion makes on either side of its equilibrium position.

angular frequency (ω)--the quantity 2π multiplied by the prequency of the oscillator. ($\omega = 2\pi f$)

elastic potential energy--the energy that can be stored as potential energy in a spring; it is proportional to the square of the amplitude of oscillations, A: $U = \frac{1}{2}kA^2$, where k is the spring constant.

resonance--the phenomenon of abnormally large oscillations of a system occurring when a force is applied to the system at a frequency equal to the natural frequency of the system.

WHAT TO DO AND HOW TO DO IT - Time and its measurement have occupied man throughout history. Up until two centuries ago, no one was able to design a clock that could keep time accurately at sea. The rolling motion of the ship, temperature-induced expansion and contraction, and the corrosive salt spray presented an insurmountable challenge to even the best clockmakers.

But it was Galileo who laid the groundwork for improved timekeeping nearly 400 years ago. He made the crucial observation that a pendulum takes almost the same time per swing, even as its motion dies down. Once it was realized that a simple harmonic oscillator could be used as a timing device, it led almost immediately to the invention of the first accurate clocks, at least on dry land.

Although early clocks used pendulums, most clocks today do not. Further investigation revealed that the pendulum is not quite a harmonic oscillator because its motion is along a circular path rather than a parabolic one. However, the principle underlying modern timepieces is the same as that governing the early pendulum clocks--harmonic motion. Whether we record time with pendulums, springs, or quartz crystals, we record it as a consequence of harmonic motion.

Through an analysis of harmonic motion, the video develops the concept that simple harmonic motion occurs when an object experiences a force that is directed toward the object's equilibrium position and is directly proportional to the object's displacement.

Objective 1: Identify examples and characteristics of simple harmonic motion.

DEMONSTRATION #1 suggests a number of illustrations of simple harmonic motion. Through these examples encourage your students to begin thinking about the characteristics of simple harmonic motion, thereby establishing a focus for the video. A common misconception is that *any* repeated or periodic motion is simple harmonic motion. The table below lists examples of repetitive motion, only some of which are simple harmonic motion. Ask your students how these motions differ from one another.

Examples of Repetitive Motion

Periodic Motion	Simple Harmonic Motion	Strongly Damped Simple Harmonic Motion	Resonance
The seasons	Vibrating spring	Door closers	Radio tuning
Migratory movement of animals	Simple pendulum	Shock absorbers	Breaking crystal with sound
Heartbeat	Swings	Laboratory balance	
Circular motion	Torsion pendulum		Swinging higher and higher
Lunar calendar	Tuning fork		Pipe organ
	Metronome		Automobile rattles at specific speeds
	Object floating - displaced downward and released		
	Liquid in a U-tube set into oscillation		
	An inductance-capacitance electrical circuit		
	Musical instruments		

Objective 2: Analyze the direction of displacement and force on a simple harmonic oscillator.

The video points out the directions of displacement and force on an oscillating springs. Stop the video on this spot to reinforce the concept with direction vectors on the blackboard. A pendulum--such as a child's swing--might also be shown. Again, emphasize the direction of the displacement from equilibrium and the direction of the restoring force.

Objective 3: Recognize the factors that determine the frequency of a simple harmonic oscillator.

For a spring the frequency depends on the ratio of the spring constant to the mass ($\omega = \sqrt{k/m}$). These factors are cleverly shown in the video by "enlarging" or "shrinking" the symbols for the spring constant and the mass. Instruct your students to pay careful attention to these transitions.

DEMONSTRATION #1 can be used to illustrate factors which influence the frequency for a number of systems vibrating in simple harmonic motion.

If students have difficulty recognizing that the time needed to make a complete cycle does not depend on the size of the oscillations, then demonstrate the independence of period and amplitude using DEMONSTRATION #2 from ADDITIONAL RESOURCES. Follow the demonstrations with questions which firmly establish student understanding that the frequency of a harmonic oscillator depends on the physical characteristics of the oscillator. In the case of a mass on a spring, these characteristics are the stiffness of the spring (k) and the mass (m).

Objective 4: Recognize factors that are necessary to establish resonance.

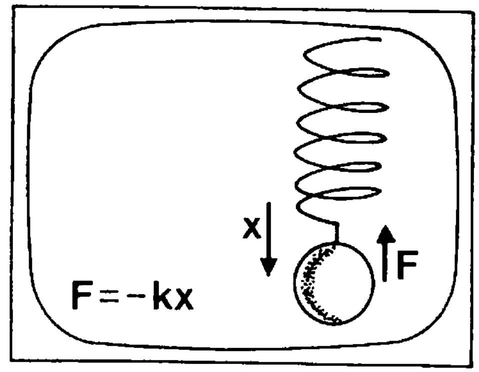
Resonance occurs when a system is driven by a periodic applied force which oscillates with a frequency equaling the natural frequency of the system. DEMONSTRATIONS #3, #4, and #5 illustrate resonance. Since the video spends only a short time on this concept, it is important to utilize these demonstrations to help students realize the importance of a *periodic* force applied with just the right frequency to establish resonance. Students are easily confused if they have not experienced concrete illustrations of resonance. The demonstrations should help students note that in simple harmonic motion an oscillator is often set into vibration by one initial force. The period remains the same while the amplitude of the vibrations decreases due to frictional forces. In the case of resonance, however, the system vibrates with large amplitude when a periodic force is applied at its natural frequency.

Resonance can lead to very destructive results as shown in the video by the Tacoma Narrows Bridge. The video also mentions that the height of buildings is a factor in their ability to withstand resonance effects during earthquakes. Buildings between five and forty stories tall are typically resonant at earthquake frequencies. Only by strict adherence to earthquake building codes can a structure resist being ripped apart at its very foundations.

POINTS TO LOOK FOR IN THE VIDEO - Several questions are posed in the STUDENT'S GUIDE. Here are those questions along with suggested responses and selected frames from the video.

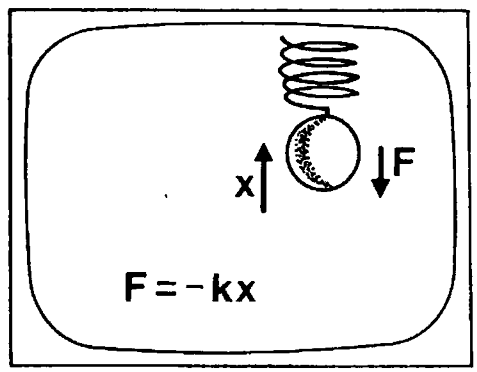
What is the significance of the negative sign in F = -*k*x, Hooke's law for the force that a spring exerts on an object?

The negative sign indicates that the force exerted by the spring on the object is in a direction opposite to the direction in which the spring is stretched or compressed. This sign mathematically reflects the fact that a compressed spring pushes outward whereas a stretched spring pulls inward toward the equilibrium position.



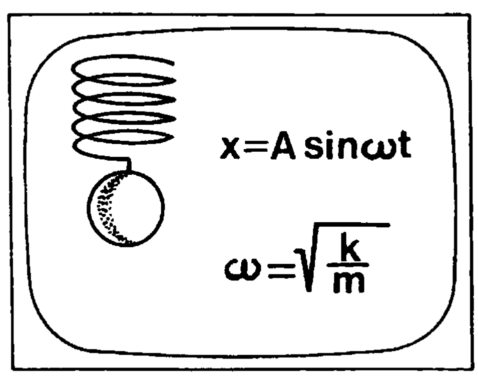
When a spring with a mass attached to it is stretched and released, why does the mass-spring system oscillate?

Initially the mass is at rest when it is released. It accelerates in response to the spring force acting on it. However, its inertia carries it past the equilibrium position of the spring. Oscillation results from this interplay between the spring force and inertia of the mass.

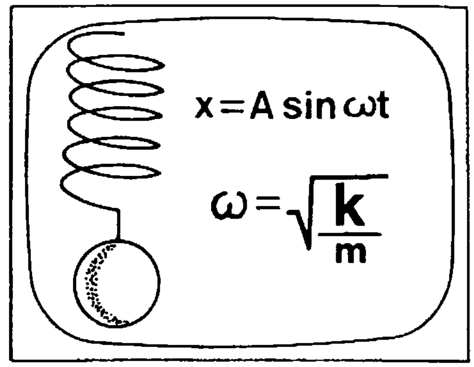


What physical quantities do the symbols shown in this video frame represent?

x--displacement
A--amplitude
ω--angular frequency
t--time
k--spring constant
m--mass of the object

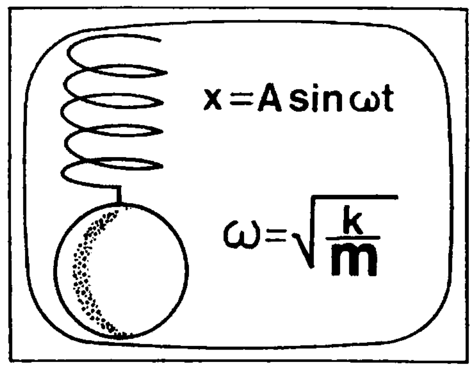


The stiffer the spring the faster the oscillations. Why do you suppose this is true?



The oscillations of a mass-spring system result from the interplay between the inertia of the mass and the spring force. A stiffer spring exerts a greater force on the mass according to Hooke's law, F = -kx. This greater force would accelerate the mass toward the equilibrium position more. Therefore, the oscillations would occur more rapidly. Mathematically, as shown in this video frame, the natural frequency is increased if the spring constant k is increased.

The greater the mass, the slower the oscillations. Explain why.



The greater the mass, the greater the inertia of the system and so the greater the resistance to changes in motion. Therefore, the oscillations would be slower. Mathematically, as shown in this video frame, if the mass m increases, the natural frequency decreases.

EVERYDAY CONNECTIONS AND OTHER THINGS TO DISCUSS - To reinforce further the concepts presented in the video, you might pose the following questions to your students.

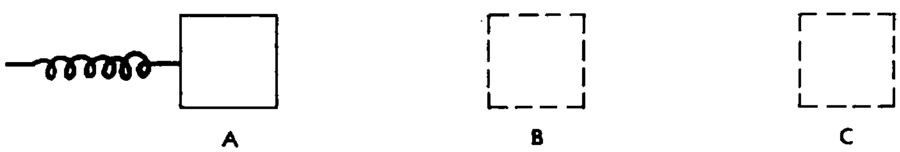
1. **If a mass on the end of a spring is set into simple harmonic motion, does the maximum value of (a) force acting on it, (b) speed, and (c) acceleration occur at the equilibrium point or endpoints?**

   The acting force and acceleration are greatest at the endpoints while speed is greatest at the equilibrium point.

2. **What change could you make in the mass-spring system to double its (a) maximum speed and (b) maximum acceleration?**

   Both will be increased if the amplitude of oscillation is increased by pulling the spring farther out.

3. **A mass is attached to a spring and rests on a frictionless surface. The system is set into vibration so that the mass oscillates between Points A and C and passes through the equilibrium Point B as shown below.**



Draw arrows to represent the velocity vector at each point as the mass moves from A to C.

Velocity	•	•	•
	A	B	C

Draw arrows to represent the force vector at each point as the mass moves from A to C.

Force	•	•	•
	A	B	C

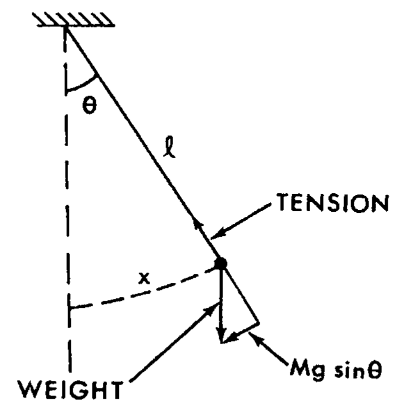
	A	B	C
VELOCITY	Ø	⟶	Ø
FORCE	⟶	Ø	⟵

4. How will the period of a pendulum change if the length is increased? What if the length is decreased?

The period is directly proportional to the square root of the length of the pendulum. Therefore, increasing the length will increase the period, whereas decreasing the length will decrease the period.

5. Why is the frequency of a pendulum independent of its mass?

The answer is the same as the answer to why all bodies in a vacuum fall with the same acceleration. The mass in $F = mg$ that acts on the pendulum cancels when F is inserted into $F = ma$. If a more massive pendulum is used, gravity is greater but so is the pendulum's inertia. The two compensate, giving a frequency that is independent of the mass.



6. How does the period of a pendulum change if moved from earth to a planet with four times the earth's gravitational acceleration?

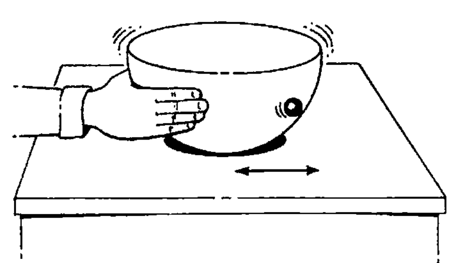
Since the period is inversely proportional to the square root of gravitational acceleration, if the gravitational acceleration is increased fourfold, the period will be one-half as great.

7. How can you use a simple pendulum to determine the local acceleration due to gravity?

Using a pendulum of known length, determine the period, solve for the value of g in the equation $T = 2\pi\sqrt{l/g}$, substitute for T to get the value of g.

8. Suppose you have a marble which is free to roll inside a shallow bowl. Describe how by *horizontal* motion of the bowl alone, you can cause the marble to roll over the edge of the bowl.

The rolling motion of the marble in the bowl has a certain natural frequency. By rocking the bowl back and forth horizontally at the natural frequency of the marble's rolling motion, resonance can occur. In that way the amplitude of the marble's motion up and down the sides of the bowl will become large enough so that the marble will leap over the edge of the bowl.



9. Similar to a pendulum, a ship has a natural frequency corresponding to a rocking motion of the entire ship. What happens if the frequency of ocean waves matches the natural frequency of a ship?

The amplitude of the ship's motion could be increased to the point where the ship would tip over.

10. **Your arm is a type of pendulum, which according to its shape and length has a natural frequency of swinging. What happens if you are walking with a stride that matches the natural frequency of your arms?**

    Resonance could cause the amplitude of the arm's swing to become large. Otherwise the amplitude of the arm's swing rapidly decreases due to friction.

11. **How can a soprano break a wine glass using only her voice?**

    If the soprano sings a note which has a frequency equal to a natural frequency of the wine glass, resonance will occur. The amplitude of molecular vibrations may then become so large that the glass will break. The power of the soprano's voice alone is usually inadequate and needs to be amplified. A baritone can't break a glass because he can't reach the natural frequency of the wine glass; it's too high a note for his voice range.

12. **Discuss the following applications of simple harmonic motion and resonance.**

    (a) Pendulum clocks rely on the unchanging frequency of a simple harmonic oscillator. Such clocks can be calibrated to a desired frequency by adjusting their lengths and thus keep accurate time.

    (b) A child's swing will vibrate in simple harmonic motion if pulled back and released. The swing will exhibit resonance if pushed at the natural frequency of the swing.

    (c) Metronomes keep accurate time by relying on the constant frequency of a pendulum of certain length. The inverted pendulum at the heart of the metronome is usually driven by a spring.

    (d) Most musical instruments (violins, guitars, drums, etc.) operate with vibrating strings, surfaces, or air columns (clarinets, flutes, organs, etc.). These instruments illustrate simple harmonic motion. Often the vibrating component drives a larger structure and, thus, a larger air mass in resonance. For example, a vibrating string on a guitar causes the guitar to vibrate, which in turn vibrates the air outside for sound louder than the string alone.

    (e) The use of tuning forks to tune instruments relies on the characteristic, reproducible frequency of a simple harmonic oscillator.

    (f) The natural frequency of buildings having 5 to 40 stories is close to the frequency of earthquake waves. During an earthquake these buildings vibrate wildly in resonance but don't usually fall apart because of friction in joints.

    (g) Engineers must attend carefully to mechanical resonance in their design of aeronautics (propellers, helicopter rotary blades, jet engines), bridges, plumbing systems, steam generators, etc. If not properly designed and balanced, these systems may set up potentially destructive resonant vibrations.

13. How is the angular frequency of an object in circular motion related to the number of oscillations a corresponding object in harmonic motion makes per second?

An object in circular motion moves through 2π radians before it returns to where it started. In other words, it moves through 2π radians in one cycle. The frequency is the number of cycles an object in harmonic motion completes per second. Since 2π radians corresponds to one cycle,

$$f = \omega/2\pi .$$

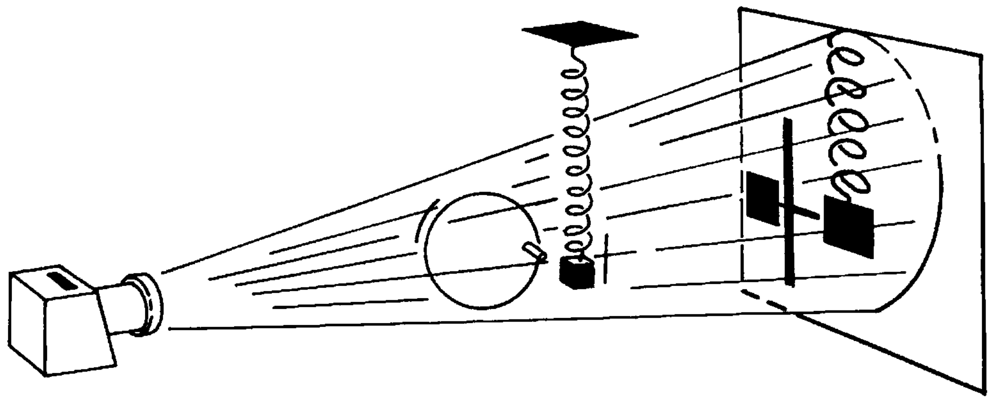


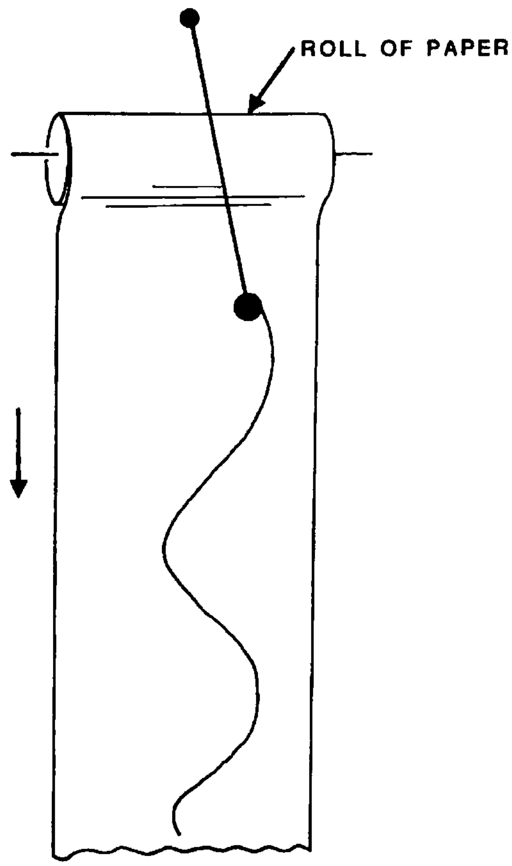
Figure 6. The shadow of a peg on an object executing uniform circular motion exhibits simple harmonic motion.

14. The motion of a pendulum is often described in terms of energy conservation. How could it be described in terms of inertia?

Initially the pendulum is at rest. When disturbed from equilibrium by an unbalanced force, the pendulum tends to move toward its equilibrium point. Since an object in motion remains in motion, the pendulum's inertia carries it past its equilibrium point. Then the restoring force acts in the opposite direction, trying to return the system to equilibrium. The result is that the system oscillates back and forth.

15. How can a pendulum be used to trace out a sinusoidal curve?

Attach a pen to a pendulum, and as the pendulum moves to and fro, move a piece of paper downward at right angles to the pendulum motion to obtain a sinusoidal curve.



16. As you are emptying a jug of water, does the frequency of the gurgles increase, decrease, or remain the same? In other words, does the sound change from a deep, bass sound to a higher pitch or vice versa?

As the liquid runs out, the size of the air space inside the jug becomes bigger. The air inside the jug will have a resonant frequency at which it will oscillate. For a mass-and-spring system, the square of natural frequency is inversely proportional to the mass; the more massive the system, the greater its inertia, and the more sluggishly it vibrates. Similarly, the natural frequency of the air inside the jug depends on the mass of the air. Therefore, as the space becomes larger and increases in mass, the natural frequency decreases because it is more difficult to accelerate the larger mass. You hear the pitch becoming deeper.

SUMMARY - Any vibrating object that experiences a restoring force proportional to its displacement is called a simple harmonic oscillator. This phenomenon can be used to explain musical instruments, vibrating springs, and other oscillating objects. When an object is driven to oscillate at its natural frequency, resonance occurs. Resonance can be destructive as well as useful.

NOTE OF EXPLANATION REGARDING THE STUDENT'S GUIDE - The following two pages of the STUDENT'S GUIDE should be duplicated and distributed to the students for use in preparation for viewing the video.

In general, the STUDENT'S GUIDE lists topics, terms, and questions, and the TEACHER'S GUIDE provides definitions, discussion, and answers to the questions. It is very important to have the students receive an appropriate "preparatory set" for viewing the VIDEO and also, following the showing of the VIDEO, to have a systematic discussion, analysis, and summarization of the objectives of the module.

The students should be informed that the INTRODUCTION, TERMS ESSENTIAL FOR UNDERSTANDING THE VIDEO, and POINTS TO LOOK FOR IN THE VIDEO should be read and discussed *prior* to viewing the VIDEO. These should also be rediscussed following the viewing.

Answers to the questions listed in the STUDENT'S GUIDE have been included under POINTS TO LOOK FOR IN THE VIDEO in the Teacher's Guide. The questions which follow this section of the Teacher's Guide and deal with EVERYDAY CONNECTIONS AND OTHER THINGS TO DISCUSS as well as the SUMMARY should be discussed as a part of the activities that *follow* the viewing(s) of the VIDEO and give closure to the lesson.

STUDENT'S GUIDE TO HARMONIC MOTION

INTRODUCTION - This video introduces the topic of simple harmonic motion. While many objects oscillate or vibrate, not all vibrating objects exhibit simple harmonic motion.

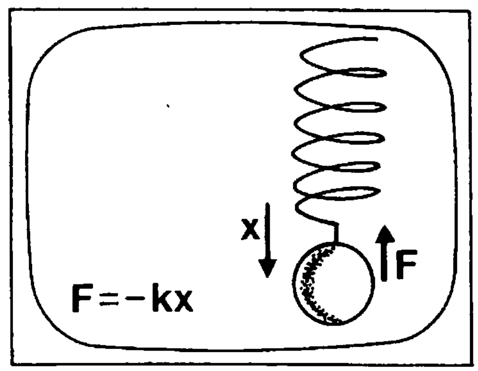
Terms Essential for Understanding the Video

periodic motion
oscillatory motion
Hooke's Law
simple harmonic motion
equilibrium position
frequency
natural frequency
amplitude
angular frequency
elastic potential energy
resonance

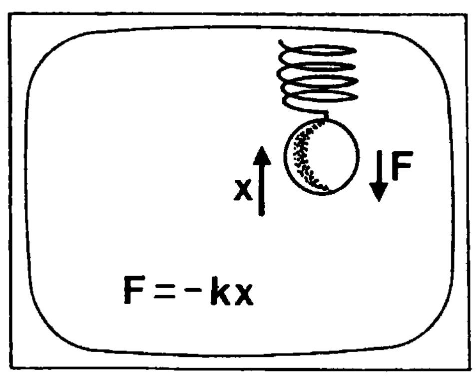
**** NOTE: Parts of the video, especially mathematical equations, may go by quickly on the screen. If you have questions, you should ask your teacher to replay these sections. ****

Points to Look for in the Video

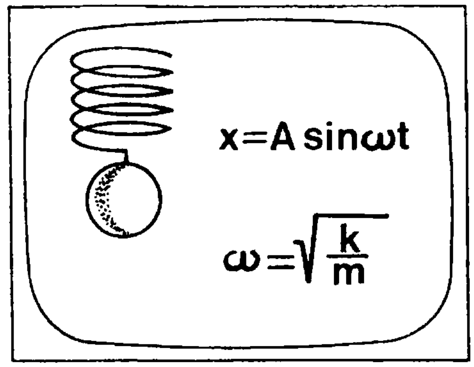
What is the significance of the negative sign in $F = -kx$, Hooke's law for the force that a spring exerts on an object?



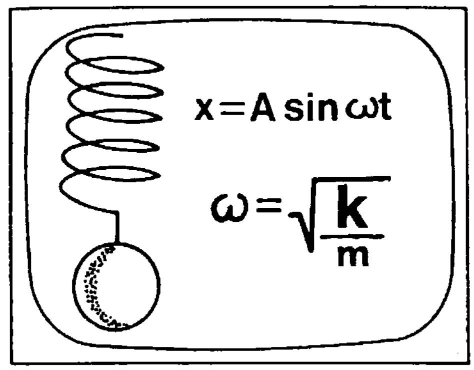
When a spring with a mass attached to it is stretch and released, why does the mass-spring system oscillate?



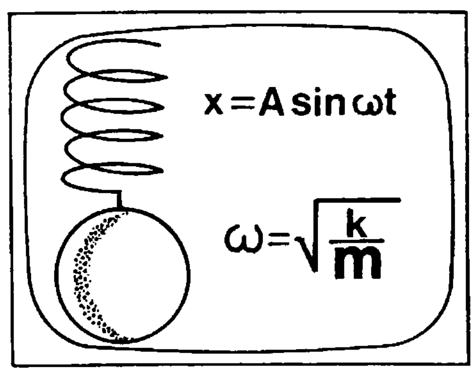
What physical quantities do the symbols shown in this video frame represent?



The stiffer the spring the faster the oscillations. Why do you suppose this is true?



The greater the mass, the slower the oscillations. Explain why.



TEACHER RESOURCES

SUPPORTIVE BACKGROUND INFORMATION - The to-and-fro motion of a simple pendulum, the vibrations of a tuning fork, the rhythm of a heartbeat, and the lunar cycles are but a few examples of periodic motion – motion that repeats itself. If the force moving an object is always directed toward the equilibrium position, and if the force is proportional to the displacement from the equilibrium position, the motion is called simple harmonic motion. Simple harmonic motion is a special case of periodic motion. The force producing simple harmonic motion is given by

$$\mathbf{F} = -k\mathbf{x} \ . \tag{1}$$

The minus sign indicates that the force is always toward the equilibrium position being opposite to the displacement.

As an example of simple harmonic motion, consider an object of mass m oscillating on the end of a spring of stiffness k as in **Figure 1**. Substituting the force causing simple harmonic motion into Newton's second law, $\mathbf{F} = m\mathbf{a}$, gives

$$-k\mathbf{x} = m\mathbf{a} \ ,$$

or

$$\mathbf{a} = -(k/m)\,\mathbf{x} \ . \tag{2}$$

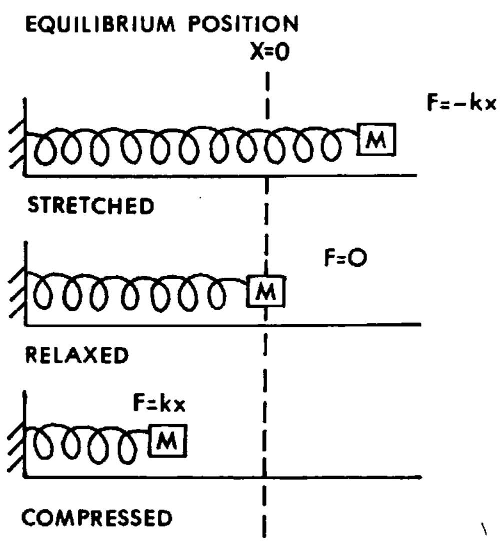


Figure 1. A mass-and-spring system.

Eq. (2) shows that the acceleration of the mass is directly proportional to the displacement x and is opposite in direction to the displacement. This application of Newton's second law is important for two reasons. First, nature provides a multitude of examples of simple harmonic motion in all areas of physics. Second, as shown in **Eq. (2)**, simple harmonic motion is radically different from situations where the acceleration is constant.

Students may feel it helpful to see simple harmonic motion contrasted with free fall, as shown in **Table 1**, because free fall is a situation where acceleration is constant.

Table 1: A Comparison of Free-Fall With Simple Harmonic Motion

	Acceleration	Velocity v(t)	Position x(t)
Free Fall	$a = g = \text{constant}$	$v = gt$	$x = 1/2\, gt^2$
Simple Harmonic Motion	$a = -k/m\, x$ $\neq \text{constant}$	$v = A\omega \cos \omega t$	$x = A \sin \omega t$

As shown in the table, the position as a function of time is given by

$$x = A \sin \omega t \ , \tag{3}$$

where A is the amplitude of the motion and ω is the angular frequency. [Starting with the acceleration in **Eq. (2)**, **Eq. (3)** can be derived using calculus. Algebra alone is not sufficient.]

The amplitude A is the maximum displacement of the mass from the equilibrium position and is a constant independent of both time and the frequency of oscillation. The angular frequency ω depends only on the physical characteristics of the system, i.e., the spring constant and the mass, and is given by

$$\omega = \sqrt{\frac{k}{m}} \ . \tag{4}$$

This is the angular frequency at which the system will naturally oscillate and is called the *natural frequency*. The units of ω are radians per second. The angular frequency is intimately related to the number of oscillations per second the object makes. The connection is

$$\omega = 2\pi f \ , \tag{5}$$

where f is the frequency of vibration and is typically measured in vibrations per second or cycles per second (1 cycle per second = 1 Hertz). The time required to complete one oscillation, known as the period T, is the reciprocal of the frequency,

$$T = \frac{1}{f} \ . \tag{6}$$

For a mass and spring system the period of oscillations is given by

$$T = 2\pi\sqrt{\frac{m}{k}} \; . \tag{7}$$

All these relations seem at first too numerous to remember. Perhaps students should be reminded that the physics is expressed in **Eq. (4)**; **Eqs. (5)**, **(6)**, **(7)** are simply different quantities that are sometimes easier to measure directly but are only versions of **Eq. (4)**.

The shadow of an object undergoing uniform circular motion executes exactly the same motion as the oscillating mass and spring system. (**See Figure 2.**) The motion of the oscillating mass is one component of uniform circular motion and the video brilliantly illustrates this connection by a demonstration and a computer animation. Since circular motion should already be familiar to students, a comparison such as that in **Table 2** between circular motion and simple harmonic motion might be helpful.

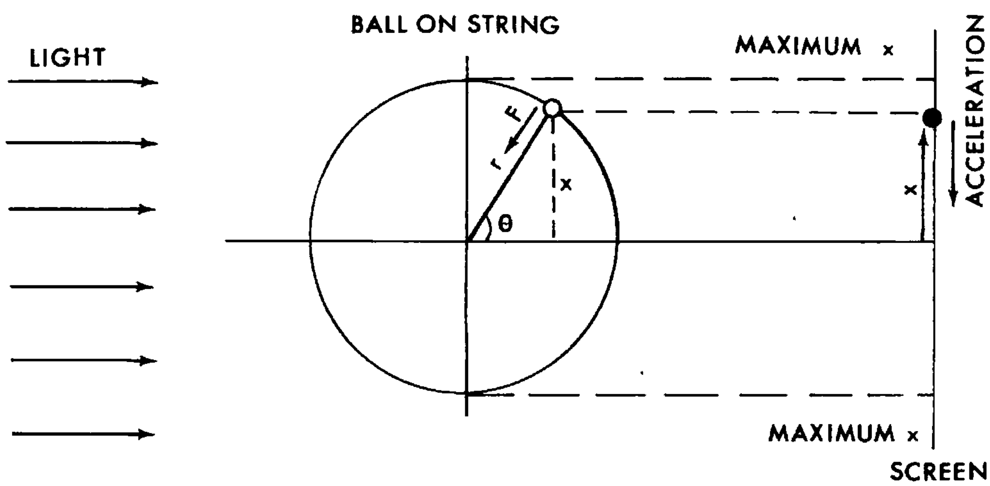


Figure 2. The projection of circular motion along a diameter.

Table 2: Comparison of Circular Motion and Simple Harmonic Motion

	Circular Motion	Simple Harmonic Motion
Position	$r^2 = x^2 + y^2$ $x = \sqrt{r^2 - y^2}$	$x = r \sin\theta$ (x is the projection along the diameter of the circle.)
Force	$F = -m\omega^2 r$ (centripetal force)	$F = -m\omega^2 r \sin\theta$ (component of centripetal force along the diameter of the circle) Substituting $x = r\sin\theta$, $F = -m\omega^2 x$ Force equation for simple harmonic motion: $F = -kx$ Matching coefficients of x $k = m\omega^2$ $\omega = \sqrt{k/m}$
Period of Motion	$T = \frac{2\pi}{\omega}$	$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

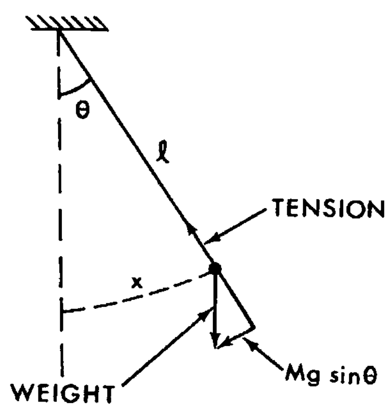


Figure 3. Simple pendulum.

Although the physics of the pendulum is not explicitly addressed in the video, a simple pendulum is another common example of simple harmonic motion. (**See Figure 3.**) The pendulum bob has two forces acting on it: the tension of the string and the weight. In the direction of motion the force (the magnitude of the vector sum of the string tension and the weight) is

$$F = -mg\sin\theta .$$

The negative sign indicates a restoring force. For small angles $\sin\theta \approx \theta$, so $F \approx -mg\theta$. The approximation is good to 1% for angles less than 20°. In the above approximations θ is measured in radians, where an angle in radians is defined as the arc length divided by the radius. Here $\theta = x/l$. Substituting this into the force equation, we get $F = -(mg/l)x$, and we see that the force on the pendulum is of the form $-kx$. By comparison to the case of a mass and spring, we see that for the pendulum the force constant is $k = mg/l$ and so the period of oscillations for the pendulum is

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{l}{g}} .$$

The motion of a simple harmonic oscillator can also be described in terms of energy. Consider again the motion of a mass attached to a vibrating spring (**Figure 1**). The kinetic energy of the body is

$$K = 1/2\, mv^2 = 1/2\, m A^2\omega^2 \cos^2 \omega t .$$

The potential energy of this system is

$$U_{spring} = 1/2\, kx^2 ,$$

$$U_{spring} = 1/2\, kA^2 \sin^2 \omega t .$$

Thus the total energy of the spring-mass system is

$$E = K + U_{spring} ,$$

$$E = 1/2\, mA^2\omega^2 \cos^2 \omega t + 1/2 kA^2 \sin^2 \omega t .$$

Since $k = m\omega^2$ the kinetic energy term can be rewritten and we get

$$E = 1/2\, kA^2 \cos^2 \omega t + 1/2\, kA^2 \sin^2 \omega t ,$$

$$E = 1/2\, kA^2 (\cos^2 \omega t + \sin^2 \omega t) ,$$

$$E = 1/2\, kA^2 .$$

The last equation follows from the trigonometric identity $\cos^2 \omega t + \sin^2 \omega t = 1$. Thus, the total energy remains constant even though there is a continual transfer between energy of motion (kinetic) and energy of position (potential). Without dissipative (i.e., frictional) forces acting, the system would vibrate forever.

Damped Simple Harmonic Motion

The video briefly mentions the effects of friction in simple harmonic motion. In a system like that in **Figure 1**, frictional effects are often small (at least over the time required for a few oscillations). This system can thus be treated as an "ideal" harmonic oscillator. The graph in **Figure 4A** shows the displacement vs. time for this oscillator.

However, there are many situations where frictional or damping effects must be considered. For these situations the amplitude of oscillations decreases with time, but the frequency remains constant as illustrated in **Figure 4B**. The oscillations eventually stop because energy is transformed by friction into heat energy. Students have difficulty accepting that the frequency remains constant as the amplitude decreases; therefore, demonstrating this might be helpful (see DEMONSTRATION #2 in ADDITIONAL RESOURCES).

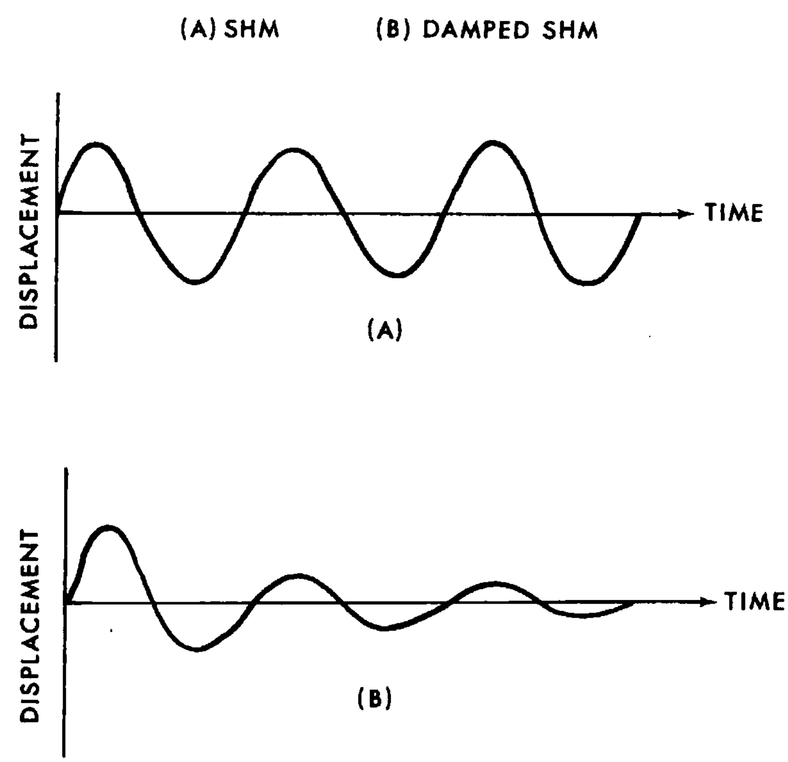


Figure 4. Displacement versus time graphs for (A) simple harmonic motion and (B) damped simple harmonic motion.

Resonance

The concept of simple harmonic motion is extended in the video by a discussion of resonance. If a periodic push – even a quite small one – is applied to a vibrating object, some startling effects may take place. When the driving frequency of the applied force approaches the natural frequency of the system, the amplitude of oscillations becomes huge. That is the phenomenon of resonance. The graph of amplitude versus driving frequency in **Figure 5** illustrates this phenomenon. Pushing a child on a swing is a common example of this. If the pushes are timed just right, small pushes result in the child and swing reaching great heights. Random pushes result in irregular and smaller excursions. A far more dramatic example is shown in the video with the resonance vibrations of the Tacoma Narrows Bridge.

The sounding box of a guitar is another example of resonance. A string vibrating in air produces only a faint sound, but when the box is set into resonance vibrations a larger mass of air is moved and louder sounds are produced than by the string alone.

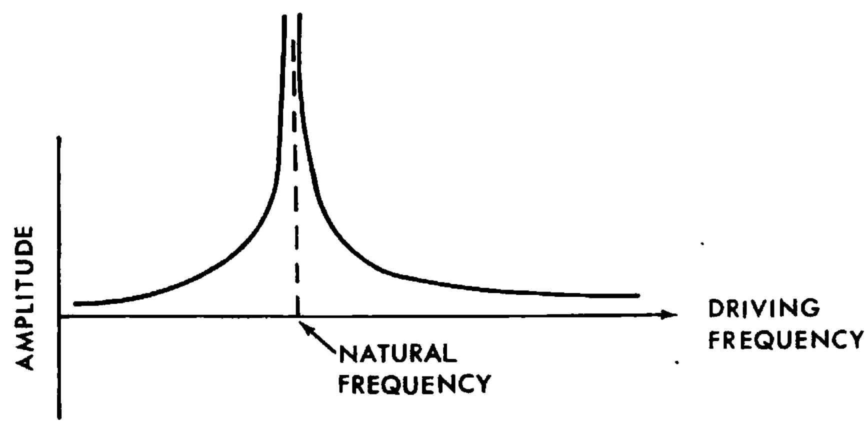


Figure 5. Amplitude plotted as a function of driving frequency shows a dramatic increase when the driving frequency equals the natural (resonant) frequency.

ADDITIONAL RESOURCES

Demonstration #1: Examples of Simple Harmonic Motion

Purpose: To introduce the topic of simple harmonic motion by allowing students to observe various examples.

Materials Two or three pendulums of different shapes or lengths; a mass on a spring; a guitar or other stringed instrument; a marble in a round bowl; a torsion balance; a bathtub rubber duck bobbing on water; a sine generator attached to a speaker. (Although the above are recommended, if preparation time is at a premium, pendulums of the same length with different size bobs could be used.)

Procedure and Notes: Set each apparatus into oscillation. It is important to have two or three different apparatuses with nearly the same period of oscillation.

Ask students to compare the similarities of all the motions being demonstrated. Possible answers are:

1. They go back and forth.
2. They vibrate.
3. The vibrations decrease in size.

Some students might mention:

1. Some seem to vibrate together.
2. The vibrations always seem to be the same.

Write the students' answers on the board. When finished, mention that all of these objects are undergoing a special kind of motion called simple harmonic motion. Then show the video.

Explanation: Each object listed undergoes simple harmonic motion for small displacements from its equilibrium position.

Demonstration #2: Period vs. Amplitude in Simple Harmonic Motion

Purpose: To demonstrate that the period of a body in simple harmonic motion is independent of the amplitude of oscillation.

Materials: A long pendulum over one meter in length and a mass on a spring.

Procedure and Notes: Set the pendulum in motion with a very small amplitude. Time the pendulum for 10 complete cycles (to and fro). Write the period of vibration on the board. Ask students what they predict will happen if you repeat the experiment with a larger amplitude. Repeat the experiment, giving the pendulum a larger amplitude. The time for 10 complete cycles and, therefore, for one period should be very close to the original value.

Set the mass on the spring into small oscillation and time it for 10 complete oscillations. Write the average period for one cycle on the board. Ask the students what they predict will happen this time when you give the mass a larger amplitude. Repeat the experiment with an amplitude about three times the original. Time the vibrations for 10 complete cycles and write the period on the board. Students should be able to see that the periods are independent of the amplitude of oscillation.

Explanation: In simple harmonic motion, frequency and period are independent of the amplitude of oscillation. Since both the pendulum and spring bob approximate simple harmonic motion, their frequencies and periods are independent of amplitude.

Note: In these procedures, do not get carried away with the amplitudes. A pendulum only approximates simple harmonic motion, and these approximations break down at large amplitudes. Keep the maximum amplitude to 15-20°. The maximum amplitude of the spring bob should be kept less than the equilibrium stretch of the spring.

Demonstration #3: Puff Pendulum (Resonance)

Purpose:

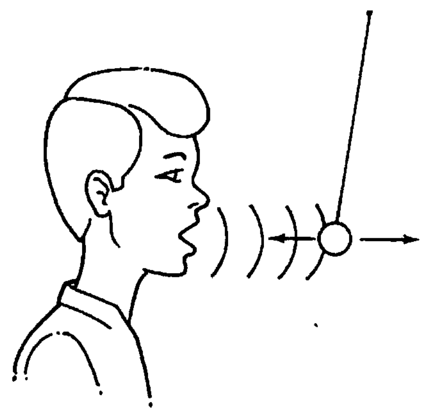
To demonstrate that a large object can be set into oscillation by small, properly timed forces.

Materials:

A long pendulum over one meter in length and a heavy bob of one or two kilograms.

Procedure and Notes:

Before class, the pendulum should be set in motion so you are familiar with the approximate period of the system. Blow on the bob with a series of strong puffs at intervals approximately equal to the natural period of the system. With luck, the pendulum will be moving through a noticeable angle before you hyperventilate!



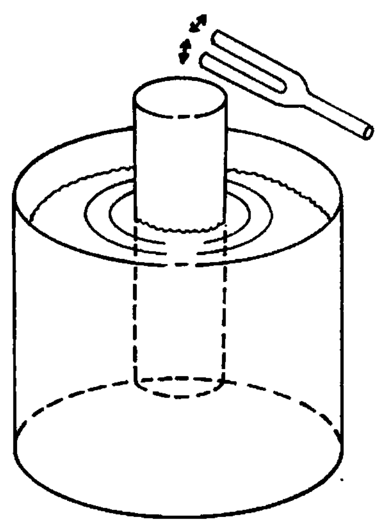
Explanation:

Resonance occurs when a system which can oscillate in simple harmonic motion is acted upon by a periodic force that has a period close to the natural period of the system.

Demonstration #4: Sound Column (Resonance)

Purpose: To illustrate sound reinforcement by superimposing compressions and rarefactions.

Materials: A hollow tube 0.3 to 1 m in length and 2 to 5 cm in diameter; a container of water as tall as the tube; a tuning fork (the higher the frequency, the shorter the tube can be).



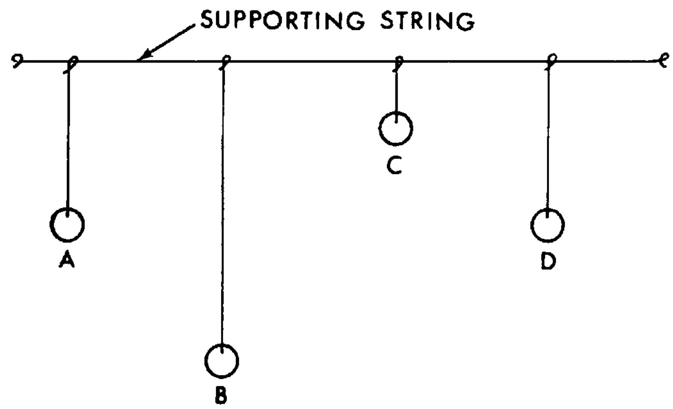
Procedure and Notes: Set up the demonstration as in the figure. Strike the tuning fork and hold it at the top end of the tube. Slowly immerse the tube into the water. As the tube descends, the loudness of sound will markedly increase at certain lengths of the air column in the tube.

Explanation: The tuning-fork-generated compressions and rarefactions travel down the tube and reflect off the water (180° out of phase). If the air column above the water is the proper length, the reflected wave will travel back just in time (the natural frequency) to encounter the next generated similar wave. Constructive interference will occur, and the tuning fork's sound will be amplified. Other points of amplification result from successive similar wave encounters.

Demonstration #5: Coupled Pendulums (Resonance)

Purpose: To demonstrate that resonance occurs when coupled systems have about the same natural frequency.

Materials: A string is tightly strung between two supports. Four pendulums of length L, 2 L, 1/2 L, and L are hung from the horizontal string as shown below.

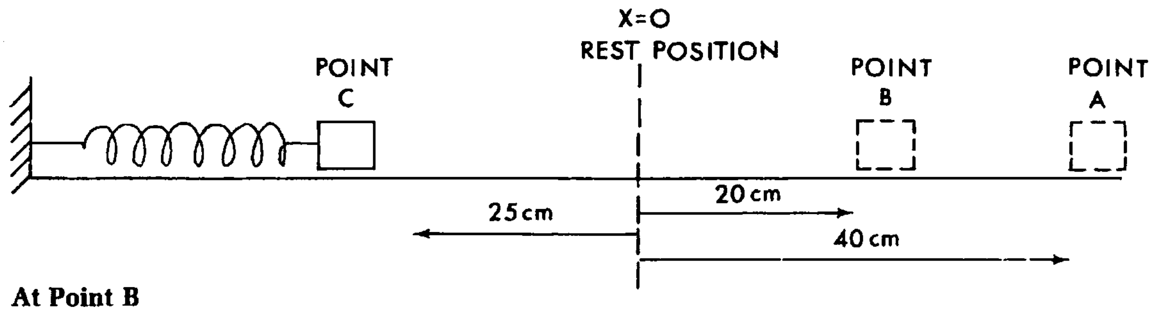


Procedure and Notes: The students should know that the length of a pendulum determines its period. With all four pendulums originally at rest, pull back pendulum A and set it oscillating. Although B and C may move slightly, pendulum D will start moving to and fro with the same period as A but about 180° out of phase. Depending on the system, A may give all its energy to D and stop, then start up again as D starts to give energy back to A.

Explanation: As pendulum A swings back and forth, it produces small tugs on the support string, which in turn applies small forces on B, C, and D. Since D has the same natural frequency as A, it is driven to greater and greater amplitudes while A loses energy and swings with smaller and smaller amplitudes. It is also interesting to note that in resonance the two pendulums vibrate out of phase by almost exactly 180°. This is the relationship between the driving and the driven oscillators at resonance.

EVALUATION QUESTIONS

A mass, M, on a frictionless table is connected to a spring having a spring constant k. The mass is displaced from its equilibrium position at x = 0 to Point A and released. Points B and C are positions through which the mass passes as the system oscillates. The following questions refer to this situation.



1. **At Point B**

   A. the spring force is pushing to the right.
   B. the spring force and the displacement are in the same direction.
   C. the spring force and the displacement are in opposite directions
   D. the displacement is to the right and the spring exerts no force.

2. **The spring force on the mass at Point B is**

   A. one-forth the force at Point A.
   B. one-half the force at Point A.
   C. the same as the force at Point A.
   D. twice the force at Point A.

3. **The direction of the acceleration of *M* at Point B is**

   A. the same as the direction of acceleration at Point A but opposite the direction of acceleration at Point C.
   B. the same as the direction of acceleration at Point C but opposite the direction of acceleration at Point A.
   C. in the same direction as the acceleration at both Point A and Point C.
   D. opposite the direction of acceleration at both Point A and Point C.

4. **If friction were present and the mass *M* were displaced to Point A, then released,**

   A. both the frequency and the amplitude of the oscillations would gradually decrease to zero.
   B. the frequency of the oscillations would gradually decrease to zero while the amplitude would remain constant.
   C. the amplitude of the oscillations would gradually decrease to zero while the frequency would remain constant.
   D. both the frequency and amplitude would be constant, having the same values as if no friction were present.

5. **Resonance occurs when**

   A. a force is applied periodically with just the right frequency to a vibrating system.
   B. a vibrating system is subject to a constant force.
   C. a frictionless system is displaced from rest and released.
   D. a strong, steady breeze blows across a bridge.

6. **Which of the following are *not* good examples of simple harmonic motion?**

   A. a metronome and a tuning fork.
   B. a pendulum and a playground swing.
   C. a vibrating spring and a rotating (torsional) pendulum.
   D. heart beats and shock absorbers.

7. **Which of the systems below exhibits resonance?**

   A. A child swinging.
   B. A skier speeding downhill.
   C. A heart beating.
   D. A pendulum at rest.

8. **A mass at the end of a spring is vibrating in simple harmonic motion. The frequency of the vibration is**

   A. directly proportional to the mass.
   B. dependent on the mass and the spring constant.
   C. dependent on the mass and the amplitude of the motion.
   D. independent of the mass, amplitude, and spring constant.

9. **How could you speed up a slow pendulum clock?**

   A. Use a larger mass.
   B. Use a smaller mass.
   C. Shorten the pendulum.
   D. Lengthen the pendulum.

10. **If the stiffness of a spring is increased,**

    A. the amplitude of the oscillations will increase.
    B. the amplitude of the oscillations will decrease.
    C. the natural frequency will increase.
    D. the natural frequency will decrease.

ESSAY QUESTIONS

11. **How can a soprano break a wine glass using only her voice?**

12. **Since friction eventually will cause a pendulum to eventually stop swinging, how can a pendulum be used as a clock?**

KEY

1. C
2. B
3. A
4. C
5. A
6. D
7. A
8. B
9. C
10. C

SUGGESTED ESSAY RESPONSES

11. If the soprano sings a note which has a frequency equal to the natural frequency of the wine glass, resonance will occur. The amplitude of glass's vibration may then become so large that it will break.

12. The frequency of simple harmonic motion is independent of amplitude. Thus, even though friction reduces the amplitude, the period of the motion remains constant.

TEACHER'S GUIDE TO NAVIGATING IN SPACE

CONTENT AND USE OF THE VIDEO - The video assumes that students have studied Newton's law of universal gravitation, Kepler's three laws of planetary motion, circular motion, and conservation of energy. They should be familiar with the mathematics of the ellipse.

The topic of navigation in space is not typically in high school physics textbooks. If considerable time is spent in a study of Kepler's laws, then the video might be most effectively used in conjunction with that study. Otherwise, the video serves well as a culmination of the study of mechanics.

TERMS ESSENTIAL FOR UNDERSTANDING THE VIDEO - The video introduces several terms with which students may not be familiar. It is strongly suggested that these be discussed in detail:

ellipse--a geometric shape resembling a flattened circle; the major axis is the longest "diameter" and the minor axis is the shortest "diameter"; the eccentricity describes how much the ellipse is distorted from being circular.

parking orbit--a temporary orbit around the earth prior to launch toward another astronomical body; the direction of the orbital path is in the direction of the earth's rotation.

transfer orbit--the path of a spacecraft from one planet to another; the most energy-efficient transfer orbits are called Hohman transfer orbits.

launch opportunity--the time when the angular alignment of the earth, sun, and planet permits a transfer orbit.

launch window--the time of day or night when the spacecraft is in the proper position to leave its temporary orbit and be inserted into a transfer orbit.

apolapsis velocity--the orbital velocity associated with the outer of two planets.

peripolapsis velocity--the orbital velocity associated with the inner of two planets.

gravity assist--the use of a planet's gravitational pull to change the speed and direction of the flight of a spacecraft.

astronomical unit (AU)--the average radius of the earth's orbit around the sun; i.e., 1 AU = 1.496×10^{11} m.

WHAT TO EMPHASIZE AND HOW TO DO IT - How do you get from one planet to another? Do you just aim your spacecraft at it and blast off? How do you navigate across the vast oceans of the solar system without the aid of so much as a lighthouse?

There are no free rides in space. Every trip requires some expenditure of energy. But there are ways to go that minimize the cost. Brute force is not one of those ways. The video discusses the most elegant and practical method, which is to make use of the sun's gravitational pull and of Kepler's ellipses. This method launched *Mariner 10* to Venus and to Mercury, the *Viking* probes to Mars, and *Voyager 1* and *2* toward the giant planets--Jupiter, Saturn, Uranus, and Neptune.

The video provides an excellent opportunity to summarize and correlate previously studied concepts within the framework of their application to the problems of navigating in space. It should be especially exciting to students to realize that the celestial mechanics that Kepler and Newton used to compute the positions of bodies in the heavens provides the operating principles which today launch new bodies into those same heavens. The video develops the concept that **efficient navigation in space is made feasible by using the gravitational pull of the sun and other planets.**

Objective 1: Explain and justify the use of transfer orbits in interplanetary travel.

Before showing the video, ask students to respond to the following question and to include diagrams that illustrate their ideas: **What is the easiest and most energy-efficient way of traveling from earth to Mars?** Most students will draw the straight-line path from earth to Mars. An in-depth discussion is not necessary at this time. You simply want to engage the students in the question. The sun's gravitational pull can be used to establish an elliptical transfer orbit for the spacecraft from the parking orbit around earth to the target planet. Point out that this is the most energy-efficient method of interplanetary travel since the craft's rockets need only to be fired for placement into the transfer orbit and not again until it reaches the target planet. Remind students that the gravitational force from the sun (described by Newton's law of universal gravitation) keeps the craft coasting around the sun in an elliptical orbit (described by Kepler's laws of planetary motion).

DEMONSTRATION #3 provides a swimsuit model for elliptical orbits in the solar system. Following discussion of the demonstration, present the students with the related situation described below.

You are 100 m behind in the same orbit as your partner's spacecraft. Suddenly that craft develops mechanical difficulties and you wish to go to its assistance.

(a) According to Newton's third law, throwing a wrench backwards along the orbital path should increase your velocity. But you find yourself in worse shape than before. Why?

As indicated by Newton's third law, your spacecraft's velocity would initially increase. But this would give a velocity higher than that required by its orbit radius. Therefore, your craft would move out into a larger radius orbit.

(b) What would happen to your own craft if you threw the wrench forward to your space partner?

Your spacecraft's velocity would initially decrease. Its orbit radius must then decrease accordingly.

(c) Would the wrench in (b) get to your partner?

No, for the same reason as in Part (a).

Objective 2: Discuss the relationship of launch opportunities to planetary geometry.

Before showing the video, you might elicit tentative answers from your students for the following question:

Do you think a planet has to be in any certain position for launching a spacecraft to it?

Students might be expected to identify the varying orbital velocities of the planets as a significant factor. They might also point out that the initial launch point must be tangential to the parking orbit of the craft and directed toward the target planet. Then perform DEMONSTRATION #1, Launch-Window Analogy on a Spinning Stool, and again discuss the previous question.

The video presents a conceptual approach to space navigation. If the level of the class is appropriate for a more quantitative treatment of the topic, refer to SUPPORTIVE BACKGROUND INFORMATION for launch opportunity calculations and examples. It will be helpful to use the "pause" or "freeze frame" capability of the video recorder in segments showing the angular planetary alignment for launch opportunities.

Objective 3: Distinguish between launch windows to inner and outer planets.

Before showing the video, ask your students the following question:

Do you think there is any difference between launching to an inner or an outer planet?

Students would not be expected to introduce a difference in speeds at this point. However, they should recognize that the directions of the launch are different for an inner and outer planet.

The launch window deals with the few fleeting moments during the day (or night) when the launch must take place. Launch to an outer planet must occur during the time when a small velocity added to the orbital parking velocity will cause the craft to move in a large elliptical orbit with the sun as a focus. This must take place, then, when the craft is on the dark side of the earth.

Launch to an inner planet requires an elliptical orbit smaller than the earth orbit. The craft already has a velocity in its parking orbit. This velocity must be decreased. Therefore, the craft's added velocity must occur on the sunlit side of the earth, with the craft facing in the opposite direction to the earth's velocity around the sun. The craft's net velocity with respect to the sun is initially decreased, thereby allowing it to escape from the earth's orbit. DEMONSTRATION #1 further clarifies this idea. Perform DEMONSTRATION #2, a Rubber-Band Launch Window Analogy, to help clarify the difference in launch windows to inner and outer planets.

Objective 4: Describe the effect of a gravity assist on a satellite and on the boosting planet.

The video mentions that Jupiter dragged *Voyager* along, giving the craft some extra kinetic energy. The energy *Voyager* gained is equal to the gravitational potential energy sacrificed by Jupiter, thus minutely decreasing the size of Jupiter's orbit around the sun.

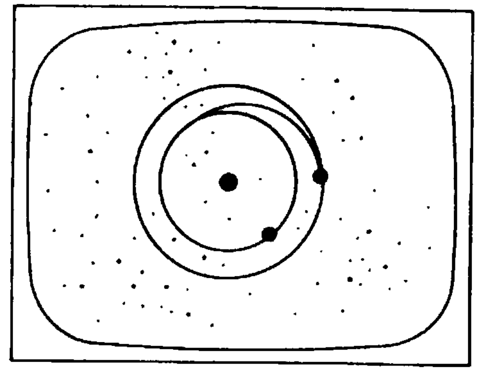
DEMONSTRATION #4, Gravity Assist Analogy, and #5, a Superball Boost, illustrate the transfer of energy in gravity assists.

You may encourage further discussion about gravity boost by pointing out that the following situations are analogous to gravity boost:

(a) a water skier going faster than the towboat;
(b) an ice skater "slinging" his partner past him.

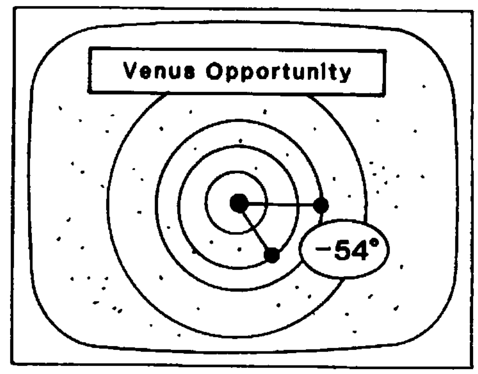
POINTS TO LOOK FOR IN THE VIDEO - Several questions are posed in the STUDENT'S GUIDE. Those questions along with suggested responses and frames from the video are presented here.

To travel between two points in space, a craft coasts to its destination in orbit around the sun, just as if it were an orbiting planet. The path of a spacecraft from one planet to another is called a transfer orbit. What governs the path that a spacecraft follows in a transfer orbit?



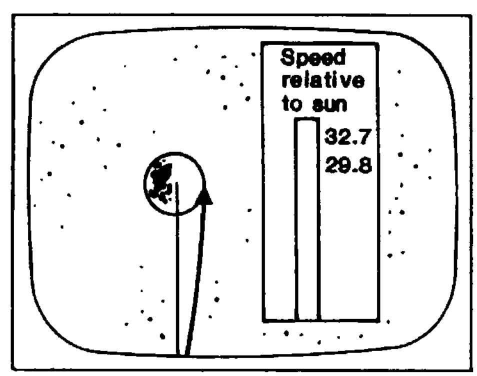
The gravitational pull of the sun governs the path.

What is the "Venus Opportunity"?



In order for Venus to be waiting when the craft arrives, the craft must be launched when Venus is just 54 degrees behind the earth. That alignment is called an "opportunity."

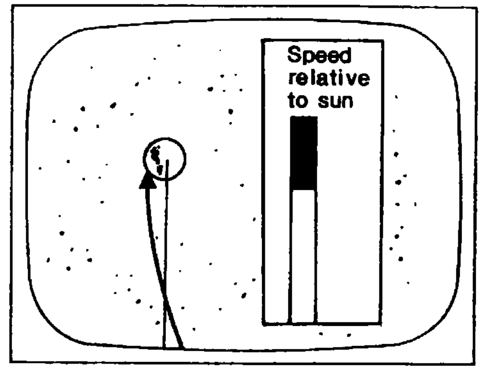
How must the velocity of a spacecraft change to enter a transfer orbit from the earth to an outer planet?



Relative to the sun, the velocity of the spacecraft must be increased to enter a transfer orbit to an outer planet. To go from an earth orbit into a Mars "transfer orbit", for example, requires a boost of exactly 2.9 km/s. The rocket thrust will boost the craft into a "hyperbola" that drifts to infinity. That moment is 7:56 p.m. local time. The sun does the rest of the job, bending the trajectory into a Mars transfer orbit.

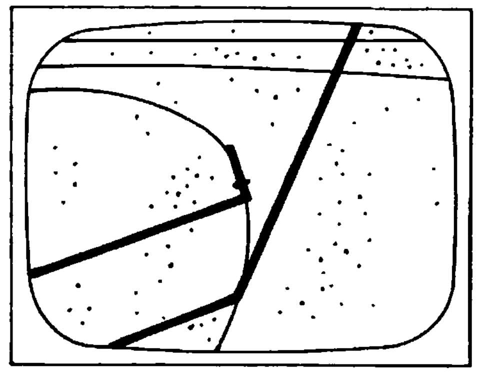
How must the velocity of a spacecraft change to enter a transfer orbit from the earth to an inner planet?

Relative to the sun, velocity of the spacecraft must be decreased to enter a transfer orbit to an inner planet. The basic idea is the same as for a journey to an outer planet. However, the spacecraft takes off from its parking orbit in the opposite direction of that in which the earth is moving, in order to slow down relative to the sun. If, for example, the journey is toward Venus, the craft must be going backward--relative to the earth's orbit, losing speed and falling inward toward Venus.



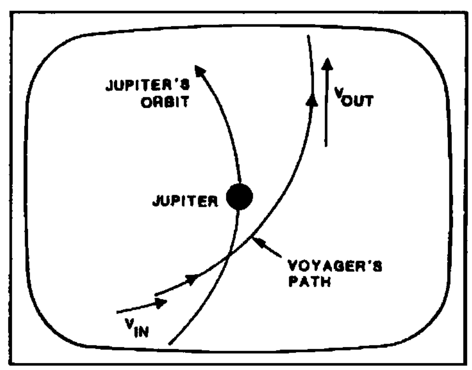
What are the consequences of a gravitational boost to the boosting planet and to a boosted spacecraft?

The spacecraft receives energy from the planet's motion and then continues with an increased added speed and a new direction as a result of the boost from the planet. The planet loses the same amount of energy and moves ever so slightly closer to the sun.



Why is the velocity vector v_{out} drawn longer than the velocity vector v_{in}?

Through its gravitational force, Jupiter pulled the spacecraft along with it in its orbit about the sun. In doing so, Jupiter increased the kinetic energy of the spacecraft. Since the kinetic energy was increased ($\frac{1}{2}mv^2$), the velocity also increased.



The following table provides data for problems that you might assign to the students:

Table 1: Planetary Parameters

Planet	Sun-planet distance r (AU)	Period of revolution, T (years)	Orbital speed V_0 (km/s)	Launch opportunity period T_L
Mercury	0.387	0.241	47.87	116 days
Venus	0.723	0.615	35.02	583 days
Earth	1.000	1.000	29.78	——
Mars	1.524	1.881	24.13	780 days
Jupiter	5.203	11.867	13.06	1.09 years
Saturn	9.538	29.461	9.64	1.04 years

$G = 6.67 \times 10^{-11}\ N \cdot m^2 / kg^2$
$M_{sun} = 2.00 \times 10^{30}\ kg$
Earth-Sun distance $R_E = 1.496 \times 10^{11}\ m = 1.0\ AU$

EVERYDAY CONNECTIONS AND OTHER THINGS TO DISCUSS - To reinforce further the concepts presented in the video, you might pose the following questions to your students.

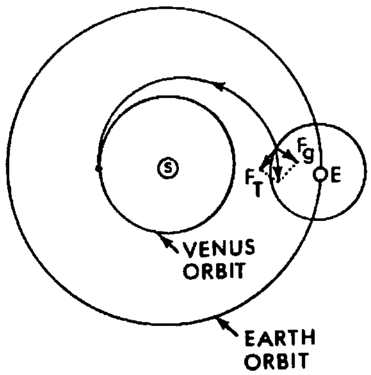
1. **Why is "brute force" an ineffective way to launch a spacecraft from a parking orbit to its target planet?**

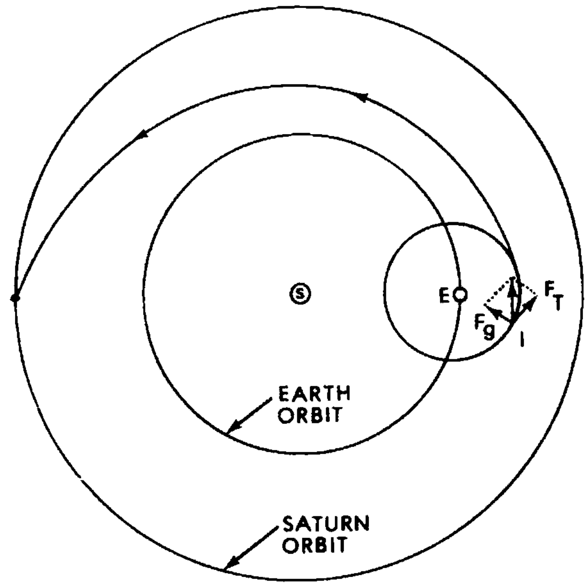
   Brute force implies the shortest route or a straight line shot. This would require exorbitant amounts of fuel, more than present technology can accommodate.

2. **Ask students to contrast trips to an outer planet and to an inner planet by examining the following conditions at firing time (relative to the sun):**

   (a) **Position:** *For a trip to an outer planet, a spacecraft is launched from the dark side of the earth, whereas to an inner planet a spacecraft is launched from the sunlit side.*

   (b) **Direction:** *For a trip to an outer planet the spacecraft's rockets are fired in a direction such that the rocket thrust and the centripetal force toward the earth result in motion in the same direction as the earth's orbital motion; to an inner planet the rockets are fired such that the resultant motion is opposite to the earth's orbital motion. See the following figure in which* F_g *is the force of the earth's gravity and* F_T *is the rocket thrust.*





(c) **Orbit:** *A spacecraft travels along a semi-elliptical orbit in the same direction as the earth's orbital motion for journeys to either an inner or an outer planet.*

(d) **Velocity:** *To an outer planet, the velocity of a spacecraft relative to the sun needs to be increased; to an inner planet, the velocity must be decreased.*

3. **Suppose you are on a carousel (merry-go-round) and a friend is standing alongside on the ground, how would you toss a ball to your friend?**

During each revolution of the carousel you would have only a short range of time when you could see your friend to toss the ball. The toss would be influenced by the movement of the carousel as well as the movement of your hand, as effectively shown with DEMONSTRATION #1.

4. **If you were riding on a carousel, would there be a difference between throwing the ball to someone standing on the ground alongside the carousel versus throwing it to someone standing in the middle of the carousel?**

Throwing it to someone off the carousel involves throwing it ahead of time. If, while facing the center, your movement is to the right, then you must toss it left of center to get it to the person in the middle.

5. **What is the difference in bunting a ball versus trying to hit a home run? Do you think a planet could act like a bat to a spacecraft?**

A bunt simply changes the direction of the flight of the ball. If the batter is swinging, the moving bat has energy which it then imparts to the ball. The motion of a large planet combined with its gravitational pull can impart energy to a spacecraft in much the same way.

6. **If you were planning a trip from earth to Mars in a spacecraft, what is the most efficient route you could take to get there?**

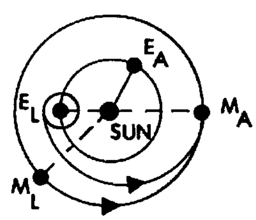
A straight line route is not the most efficient because of the energy expenditure required. The most efficient route is called a transfer orbit which is a semi-elliptical orbit that is tangent to the earth's orbit and the orbit of the target planet.

7 **How is the force of gravity used in interplanetary travel?**

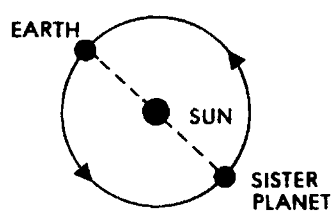
Once the spacecraft has blasted free of the "home" planet, the sun's gravitational force shapes its path and controls its velocity.

8. **Construct an orbit diagram illustrating the launch opportunity in a trip from earth to Mars.**

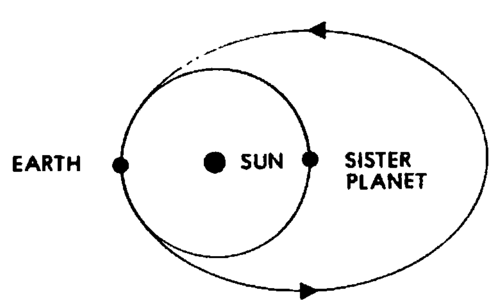
The positions of earth at launch (E_L) and Mars (M_A) at arrival are all that are really expected of the students here. You may wish to go further in pointing out the positions of earth at arrival (E_A) and Mars at launch (M_L). The background information shows examples of how to calculate the angles.



9. **Some science-fiction writers have suggested a sister planet to earth, as shown. It shares the same orbit but is always opposite the sun from earth. Suggest how a spacecraft could be sent from earth to its sister planet.**



One method is to send the spacecraft into an elliptical orbit of 1.5-year period. The sister planet is then located in the same position as the earth was when the craft was launched.



10. Why do we launch spacecraft into parking orbits from Florida and not Alaska?

The spacecraft uses the velocity of the rotating earth, which is greater nearer the equator.

SUMMARY - Journeys to other planets along straight paths are impractical. They are too expensive and not enough energy is available. The most efficient means of space travel is to put the spacecraft into orbit (the Hohman transfer orbit) about the sun. The sun's gravitational field directs the craft. Launch, however must take place when the earth and target planet are in the correct positions in space. This is called the launch opportunity. Additionally, the launch must occur at the right time of day, called the launch window. Gravity assists from other planets also aid navigating in space.

NOTE OF EXPLANATION REGARDING THE STUDENT'S GUIDE - The following two pages of the STUDENT'S GUIDE should be duplicated and distributed to the students for use in preparation for viewing the video.

In general, the STUDENT'S GUIDE lists topics, terms, and questions, and the TEACHER'S GUIDE provides definitions, discussion, and answers to the questions. It is very important to have the students receive an appropriate "preparatory set" for viewing the VIDEO and also, following the showing of the VIDEO, to have a systematic discussion, analysis, and summarization of the objectives of the module.

The students should be informed that the INTRODUCTION, TERMS ESSENTIAL FOR UNDERSTANDING THE VIDEO, and POINTS TO LOOK FOR IN THE VIDEO should be read and discussed *prior* to viewing the VIDEO. These should also be rediscussed following the viewing.

Answers to the questions listed in the STUDENT'S GUIDE have been included under POINTS TO LOOK FOR IN THE VIDEO in the Teacher's Guide. The questions which follow this section of the Teacher's Guide and deal with EVERYDAY CONNECTIONS AND OTHER THINGS TO DISCUSS as well as the SUMMARY should be discussed as a part of the activities that *follow* the viewing(s) of the VIDEO and give closure to the lesson.

STUDENT'S GUIDE TO NAVIGATING IN SPACE

INTRODUCTION - This video introduces the most efficient means of interplanetary travel--the transfer orbit. The sun's gravitational field directs the spacecraft in its elliptical orbit. The gravitational field of the planets also provide extra energy to propel spacecraft.

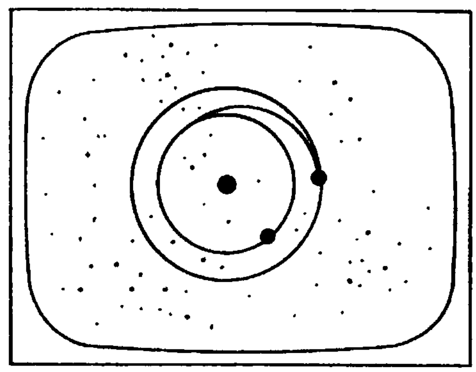
Terms Essential for Understanding the Video

- ellipse
- parking orbit
- transfer orbit
- launch opportunity
- launch window
- apolapsis velocity
- peripolapsis velocity
- gravity assist
- astronomical unit (AU)

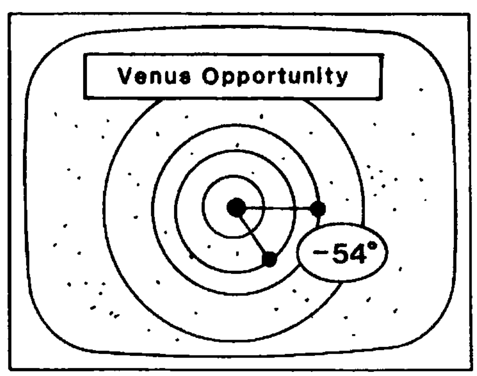
****** NOTE: Parts of the video, especially mathematical equations, may go by quickly on the screen. If you have questions, you should ask your teacher to replay these sections. ******

Points to Look for in the Video

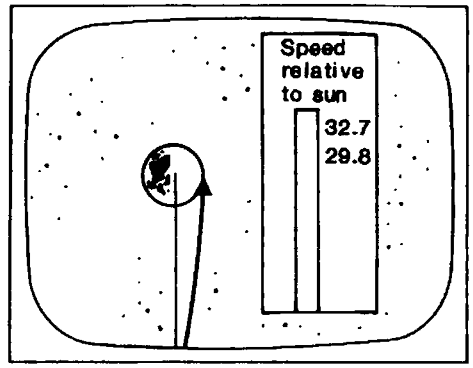
To travel between two points in space, a craft coasts to its destination in orbit around the sun, just as if it were an orbiting planet. The path of a spacecraft from one planet to another is called a transfer orbit. What governs the path that a spacecraft follows in a transfer orbit?



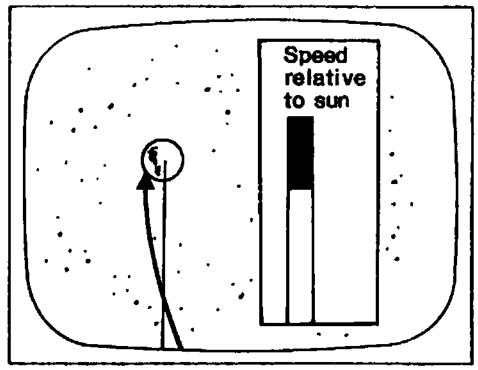
What is the "Venus Opportunity"?



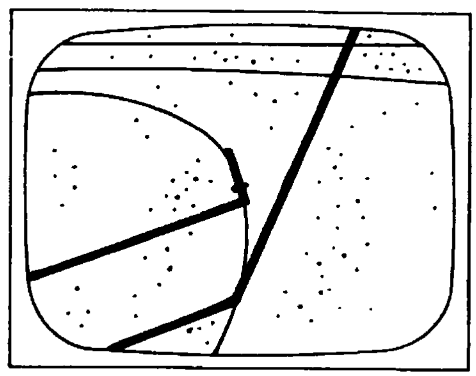
How must the velocity of a spacecraft change to enter a transfer orbit from the earth to an outer planet?



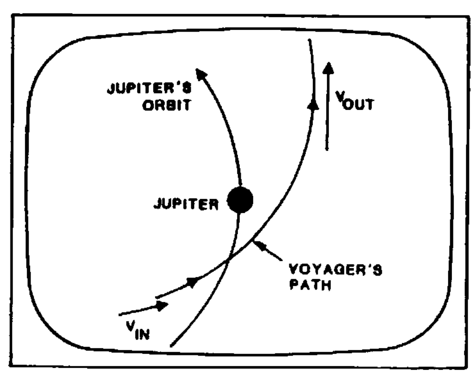
How must the velocity of a spacecraft change to enter a transfer orbit from the earth to an inner planet?



What are the consequences of a gravitational boost to the boosting planet and to a boosted spacecraft?



Why is the velocity vector v_{out} drawn longer than the velocity vector v_{in}?



The following table provides data for problems that your teacher might assign.

Table 1: Planetary Parameters

Planet	Sun-planet distance r (AU)	Period of revolution, T (years)	Orbital speed V_0 (km/s)	Launch opportunity period T_L
Mercury	0.387	0.241	47.87	116 days
Venus	0.723	0.615	35.02	583 days
Earth	1.000	1.000	29.78	—
Mars	1.524	1.881	24.13	780 days
Jupiter	5.203	11.867	13.06	1.09 years
Saturn	9.538	29.461	9.64	1.04 years

$G = 6.67 \times 10^{-11}\ N \cdot m^2 / kg^2$
$M_{sun} = 2.00 \times 10^{30}$ kg
Earth-Sun distance $R_E = 1.496 \times 10^{11}$ m $= 1.0$ AU

TEACHER RESOURCES

SUPPORTIVE BACKGROUND INFORMATION - Frequently a student's first thought is that the most direct, straightline route of a spacecraft from earth to Mars is the best route. While this so called "brute force" method is possible, it requires more energy than can be supplied by present-day technology. The most energy-efficient means of travel is a Hohman transfer orbit. This is a semi-elliptical orbit about the sun which is tangent to both the earth's orbit and the orbit of the target planet, as shown in **Figure 1**. Once a spacecraft is in a transfer orbit, its rockets need not be fired until it reaches the orbit of the target planet. The spacecraft coasts around the sun while obeying Kepler's laws of planetary motion. The gravitational force from the sun, which is described by Newton's law of universal gravitation, keeps the spacecraft in the transfer orbit.

Transfer orbits put constraints on space travel. A probe can't be sent out to some destination at just any time. Instead, the launch must take place when the earth and target are in the correct relative positions. This occurrence is known as *launch opportunity*. The appropriate positions are shown in **Figure 1**. The spacecraft must be launched when the earth is at one end of the major axis of the transfer orbit (peripolapsis) E_L; the target planet must arrive at the other end (apolapsis) P_A simultaneously with the spacecraft. Of course the earth will have moved by the time of arrival of the craft to position E_A, and the target planet will have been in a different location at the time of the launch. All these positions are shown in **Figure 1**. The methods of calculation of the launch opportunities are discussed later.

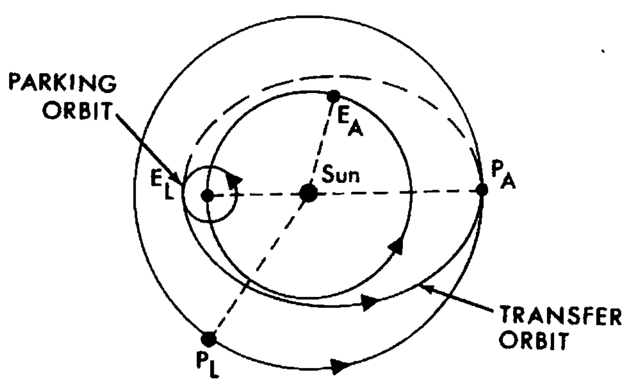


Figure 1

To put a spacecraft into a transfer orbit one has somehow to get it out of its earth orbit by supplying it with enough energy to meet or exceed the escape velocity of the earth. This is accomplished by rocket thrusts.

During each launch opportunity there is a short time each day to launch during which a least-energy orbit (transfer) can be achieved. Those few hours (sometimes minutes) are called *launch windows*. To launch to an *outer* planet, a craft should start from its parking orbit around the earth when it is on the dark side of the earth as shown in **Figure 2**. In this way the initial velocity of the craft will include a contribution due to the earth's orbital velocity in addition to the craft's velocity relative to the earth. A coordinated rocket blast gives the craft an additional contribution enabling it to exceed the escape velocity of earth. In the sun's reference frame it enters an elliptical oribt with the sun at one focus.

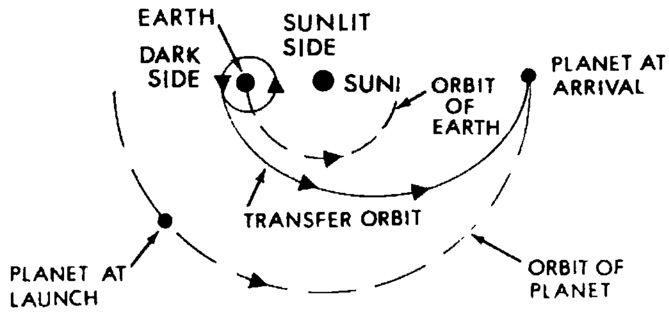


Figure 2

The launch window to an *inner* planet occurs when the spacecraft is in a parking orbit and is on the sunlit side of the earth. In this case, again, the initial velocity of the craft relative to the sun is the sum of velocities due to both the earth's motion about the sun and the craft's motion around the earth. However, on the sunlit side of the earth, these velocities are in opposite directions. A coordinated rocket firing in the direction of the craft's orbital motion around the earth is in a direction opposite to the direction of the earth's motion about the sun and slows down the spacecraft, forcing it into an orbit closer to the sun as shown in **Figure 3**.

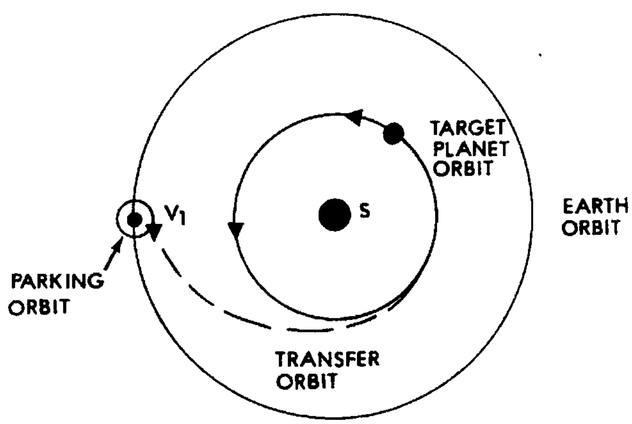


Figure 3

In the early days of the space program many parts of orbits were computed by hand. It was a complicated business to switch from the earth orbit to the transfer orbit to the target-planet orbit. People doing the calculations thought of the target planet's gravitational field as an annoyance. But later it was realized that the gravitational field was a gift of energy. When *Voyager 2* was launched in 1977 for a grand tour of Jupiter, Saturn, Uranus, and Neptune, mission navigators used the gravitational fields of these planets to provide extra boosts to the craft. This technique is known as *gravity assist*. *Voyager* probably would never have made it to Saturn without a gravity boost from Jupiter. **Figure 4** facilitates an understanding of the gravity assist of Jupiter on *Voyager*.

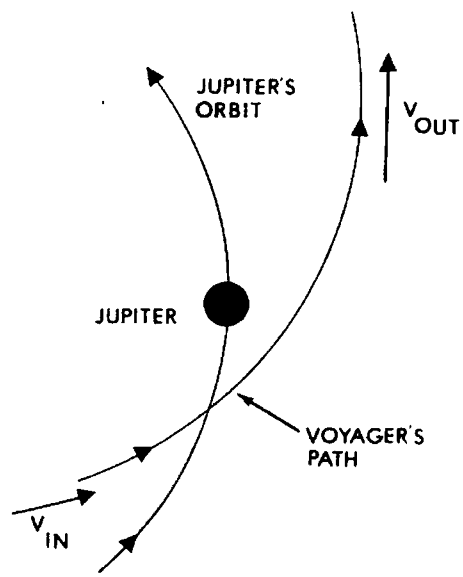


Figure 4

Viewed from the sun, Jupiter moved in its orbit and dragged the craft along with it, thus giving it some extra energy. (The video makes reference to a baseball and bat analogy.) Because energy is conserved, Jupiter must lose some energy. This energy comes from the gravitational potential energy of the planet in the field of the sun, forcing Jupiter in an orbit a hair closer to the sun than before.

Launch-Opportunity Calculations

The frequency with which the launch opportunity occurs equals the difference between the frequencies of the two planets (f_e for the earth, f_p for the planet) orbiting around the sun:

$$f_L = |f_e - f_p|.$$

Since the period is reciprocal of the frequency,

$$\frac{1}{T_L} = \left|\frac{1}{T_e} - \frac{1}{T_p}\right|,$$

so that

$$T_L = \left|\frac{T_e \times T_p}{T_e - T_p}\right|,$$

where T_L is the number of days between launch opportunities for any planet, T_e (T_p) is the time that it takes the earth (the other planet) to revolve once around the sun. All periods are to be measured in earth years.

EXAMPLE 1 - Calculate the launch-opportunity period for the earth and Venus. Earth has a revolution period of one year, Venus has a revolution period of 0.615 year. (See **Table 1** in the POINTS TO LOOK FOR IN THE VIDEO for T.) Thus

$$T_L = \left| \frac{1.00 \times 0.615}{1.00 - 0.615} \right| = \frac{0.615}{0.385} = 1.60\,\text{years} = 583\,\text{days}.$$

EXAMPLE 2 - Calculate the launch-opportunity period for the earth and Saturn. Earth has a revolution period of 1.00 year; Saturn has a revolution period of 29.461 years. (See **Table 1** in the POINTS TO LOOK FOR IN THE VIDEO for T.) Thus

$$T_L = \left| \frac{1.00 \times 29.461}{1.00 - 29.461} \right| = \frac{29.461}{28.461} = 1.04\,\text{years} = 378\,\text{days}.$$

Transfer-Orbit Calculations

The total energy of a spacecraft in a circular orbit is given by the expression

$$E_t = K.E. + P.E. = 1/2\,mV_0^2 - \frac{GmM}{r}, \quad (1)$$

where

$K.E.$ = kinetic energy
$P.E.$ = potential energy (zero at infinity)
m = spacecraft mass
M = mass of sun
V_0 = orbital spacecraft velocity
r = spacecraft-to-sun distance.

For a spacecraft to be in a stable circular orbit the centripetal force on it must be equal to the gravitational force:

$$F_c = F_g \text{ , or } \frac{mV_0^2}{r} = \frac{GmM}{r^2}.$$

Multiplying each side by r/m gives

$$V_0 = \sqrt{\frac{GM}{r}}. \quad (2)$$

To find the speed of any planet in its orbit we can use this result as well because the orbits of the planets are very nearly circular. See **Table 1** in the POINTS TO LOOK FOR IN THE VIDEO for the orbital velocities of the six innermost planets of the solar system. We'll denote the orbital velocity of the earth by V_e.

Substituting the expression for V_0 in **Eq. (2)** back into **Eq. (1)**, we see that the total energy in a circular orbit can also be expressed as

$$E_t = 1/2 \frac{GmM}{r} - \frac{GmM}{r},$$

$$E_t = \frac{-GmM}{2r}.$$

It can be shown that for any object in an elliptical orbit, whether it be a planet or a spacecraft, the total energy can be expressed as

$$E_t = \frac{-GmM}{2a}.$$

The denominator $2a$ is the major axis of the ellipse and is analogous to $2r$, the diameter of a circle (see **Figure 5**).

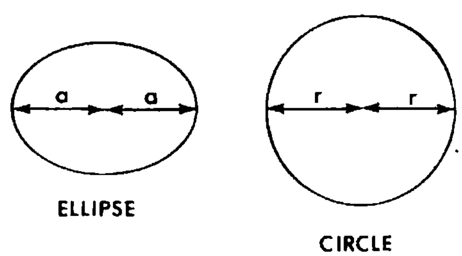


Figure 5

Calculation of Peripolapsis and Apolapsis Velocities

To inject a spacecraft into an elliptical orbit around the sun from the earth's almost circular orbit (**Figure 6**), a change in the spacecraft's velocity must be made. The magnitude of this change can be calculated once the velocity V_1 that the spacecraft needs to have to enter a transfer orbit is known. Since the distance $2a$, the major axis of the elliptical orbit, is specified, we can use the equation for the total energy of a spacecraft in an elliptical orbit to find V_1. By setting this total energy equal to the sum of its *K.E.* and *P.E.*, we get

$$\frac{-GmM}{2a} = 1/2mV_1^2 + \frac{GmM}{r_e}, \text{ or } V_1 = \sqrt{\frac{2GM}{r_e}\left(1 - \frac{r_e}{2a}\right)}. \quad (3)$$

Since $V_e = \sqrt{\frac{GM}{r_e}}$, we can write $V_1 = V_e \sqrt{2(1 - \frac{r_e}{2a})}$.

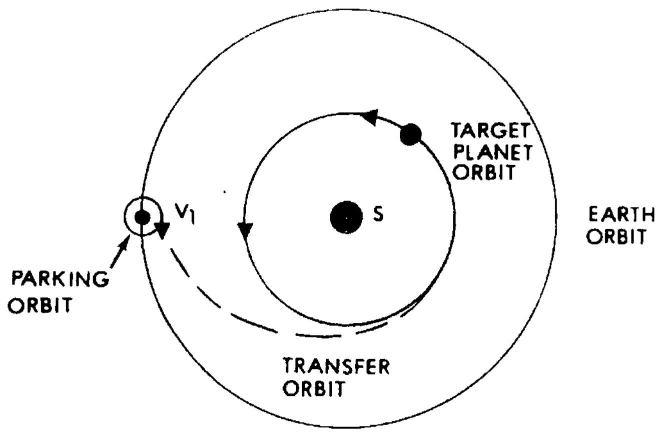


Figure 6

The difference between V_e and V_1 represents the change provided by the rocket thrust needed to accomplish the injection. Once the spacecraft is in the transfer orbit, no additional thrusts are needed; the sun's gravitational force provides a free ride.

At the opposite end of the transfer orbit, the spacecraft's velocity must also be changed to match that of the target planet. The velocity associated with the inner of the two planets involved is called the peripolapsis velocity, whereas that associated with the outer planet is called the apolapsis velocity. To find V_2, the velocity of the spacecraft when it arrives at the target planet, we use the law of conservation of angular momentum:

$$m V_1 r_e = m V_2 r_p ,$$

$$\text{or} \quad V_2 = \frac{V_1 r_e}{r_p} . \tag{4}$$

Kepler's third law of planetary motion determines the period of time, T, that is required for a spacecraft in an elliptical orbit to make one complete orbit:

$$\frac{T_e^2}{T^2} = \frac{r_e^3}{a^3} . \tag{5}$$

T_e is 1.00 year, r_e is 1.00 AU, and a is the semimajor axis of the transfer orbit.

Since the spacecraft will complete only one-half an orbit before it arrives at the target planet, $T/2$ will give us the time of flight.

EXAMPLE 1: Earth to Venus

If a spacecraft is to be launched from earth to Venus via a minimum-energy Hohman transfer orbit, we need to find the change of velocity necessary to inject the spacecraft into the transfer orbit. From **Eq. (2)** we find that the earth's orbital velocity is

$$V_e = \sqrt{\frac{GM}{r_e}} = \sqrt{\frac{(6.67 \times 10^{-11}\,\mathrm{N \cdot m^2/kg^2})(2.00 \times 10^{30}\,\mathrm{kg})}{1.5 \times 10^{14}\,\mathrm{m}}} = 29.8 \ \mathrm{km/s}.$$

The major axis of the transfer orbit is $2a = 0.72r_e + 1.00r_e = 1.72r_e$. According to **Eq. (3)**,

$$V_1 = V_e \sqrt{2\left(1 - \frac{r_e}{2a}\right)} = 29.8\,\mathrm{km/s} \sqrt{2\left(1 - \frac{1.00}{1.72}\right)} = 27.2 \ \mathrm{km/s}.$$

Therefore, $V_e - V_1 = -2.6$ km/s. The spacecraft must be slowed down by 2.6 km/s *with respect to the sun* in order to put it into the transfer orbit to Venus. To accomplish this, a rocket thrust is given the spacecraft when it is traveling in the direction opposite to the earth's motion around the sun, that is, on the daylight side of the earth. Once slowed down in its motion with respect to the sun, it falls inward along an elliptical orbit toward its rendezvous with Venus. (See **Figure 7**.) The time T for the spacecraft to complete one period using **Eq. (5)** is

$$\frac{(1\,\mathrm{yr})^2}{T^2} = \frac{1^3}{0.86^3}, \qquad T = 0.8\,\mathrm{yr},$$

and $t = T/2 = 0.4$ yr for a trip to Venus.

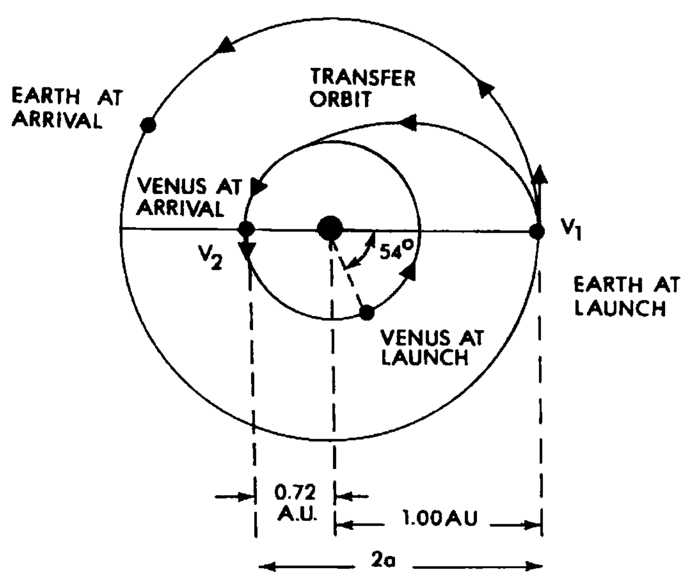


Figure 7

After arriving at Venus via the transfer orbit, the spacecraft must again use its rocket thrust to match its speed to that of Venus. By **Eq. (2)** we can calculate the orbital velocity of Venus; the result is listed in **Table 1** as 35.0 km/s. Using **Eq. (4)** we can determine the spacecraft's velocity:

$$V_2 = V_1 \frac{r_e}{r_p} = (27.2\,\text{km/s}) \frac{r_e}{0.723\, r_e} \,,$$

$$V_2 = 37.8 \text{ km/s}.$$

The rockets must fire to slow down the spacecraft by (37.8 − 35.0) km/s, or 2.8 km/s.

During the time the spacecraft travels in the transfer orbit (0.4 year, or 146 days), both Venus and the earth continue to move in their orbits around the sun. Venus moved 146 days out of a 225-day orbital period, that is, through (146/225) (360°) = 234°. The earth has moved (146/365) (360°) = 144° during the same time. Anticipating these angular changes, a similar transfer orbit between these two planets is possible every 583 days. As shown in **Figure 7**, the earth-sun-Venus-launch opportunity angle is 234° − 180° = 54°.

EXAMPLE 2: Earth to Saturn

If a spacecraft is sent to Saturn along a single transfer orbit without the benefit of a gravitational boost from another planet, what injection velocity $V_1 - V_e$, rendezvous velocity $V_p - V_2$, time of flight, and angular change in the positions of the planets would be required? From **Figure 8** it can be seen that $2a = 10.538\, r_e$, so

$$V_1 = V_e \sqrt{2\left(1 - \frac{r_e}{2a}\right)} = 40.1\,\text{km/s},$$

$$V_1 - V_e = (40.1 - 29.8)\,\text{km/s} = 10.3\,\text{km/s}.$$

This is the additional velocity that must be given to the spacecraft to inject it into an elliptical orbit away from the sun. The rocket is fired when it is on the dark side of earth in order to use both the rocket's orbital motion around the earth and the earth's orbital motion around the sun to assist in this task. The rendezvous velocity with Saturn can be computed by applying conservation of angular momentum:

$$V_2 = V_1 \frac{r_e}{r_p} = (40.1\,\text{km/s}) \frac{r_e}{9.538 r_e} \,,$$

$$V_2 = 4.2 \text{ km/s}.$$

From **Table 1** we know Saturn's orbital velocity is 9.6 km/s, so

$$V_p - V_2 = 5.4 \text{ km/s}.$$

The time of flight can be found from Kepler's third law.

$$\frac{1^2}{T^2} = \frac{1^3}{(5.27)^3},$$

$$T = 12.1 \text{ years}.$$

One-half of this time, or 6.05 years would be needed. The spacecraft would complete its path along the transfer orbit while Saturn moved along its orbit by (6.05/29.46) (360°) = 74°. The launch-opportunity period is

$$T_L = \left| \frac{1.00 \times 29.46}{1.00 - 29.46} \right|,$$

$$T_L = 1.04 \text{ years} = 378 \text{ days}.$$

As shown in **Figure 8**, the earth-sun-Saturn launch-opportunity angle is 180° − 74°, or 106°.

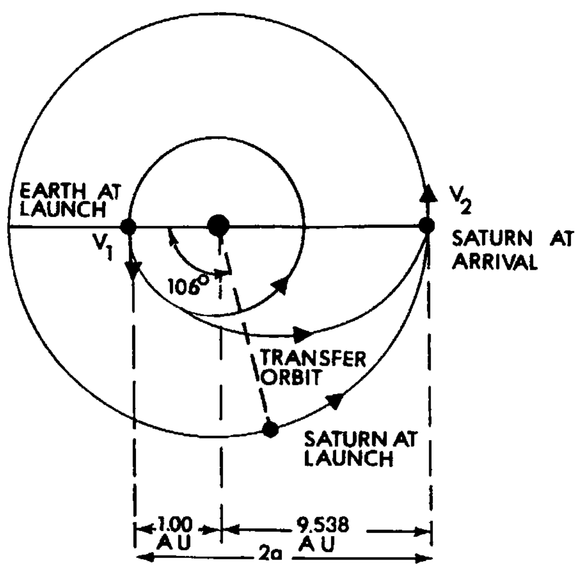


Figure 8

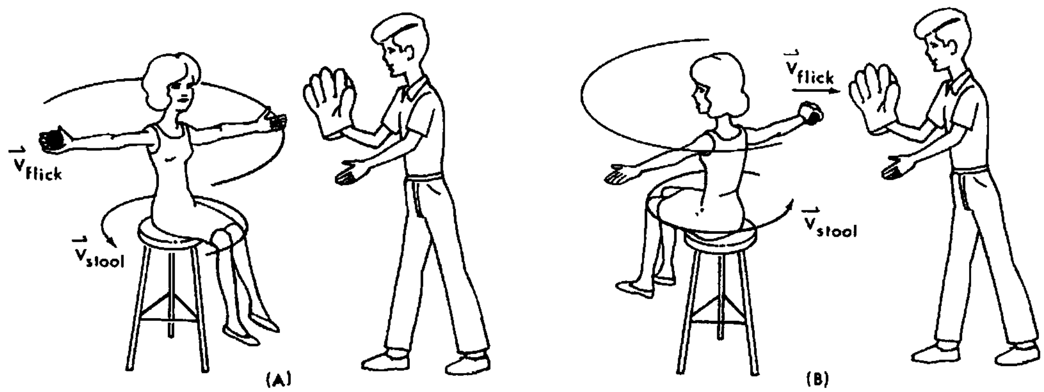
ADDITIONAL RESOURCES

Demonstration #1: Launch-Window Analogy on a Spinning Stool

Purpose: To illustrate the effect of orbital velocity and thrust direction on an escaping satellite.

Materials: Two students, a tennis ball, and a stool or chair that rotates.

Procedure and Notes: Have a student sit on the stool with arms rigidly outstretched. Place the tennis ball in one hand. Ask the student to flick the ball to another student as you rapidly rotate the stool. The flick should occur around the fourth rotation. Now ask the student to flick the ball to the other student when they are not facing each other. [See situations (A) and (B) below.]



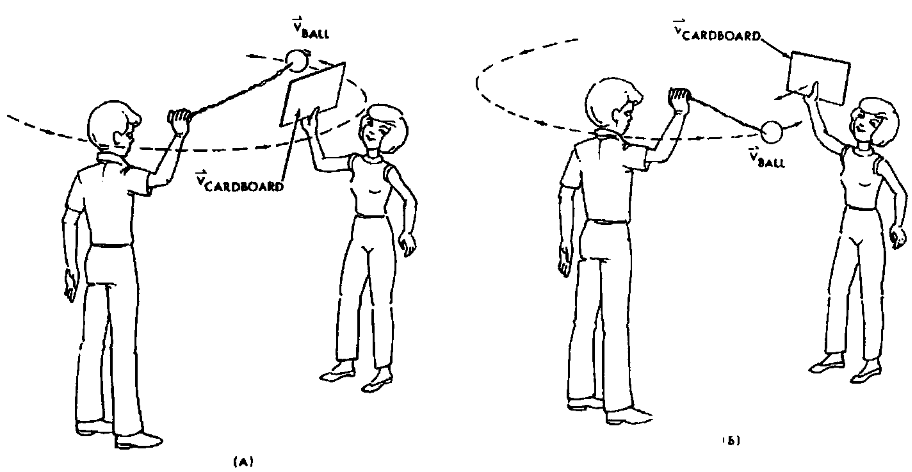
Explanation: In situation (A), the student on the stool will be able to throw the ball far, since the launch velocity will be great (V due to the stool plus the V due to the flick). In situation (B), the launch velocity will not be as great (V due to flick minus V due to stool), and the ball will not go as far. Similarly, with added velocity, as in situation (A), a satellite *could* escape earth's orbit. A satellite in situation (B) would not have sufficient velocity to escape earth's orbit.

Demonstration #2: A Rubber-Band Launch-Window Analogy

Purpose: To illustrate the use of launch velocities to achieve larger or smaller orbit radii relative to the sun.

Materials: Several rubber bands linked together, a stiff piece of cardboard, and a nerf ball (or any other soft object).

Procedure and Notes: Attach the rubber-band chain to the object, and have a student swing it overhead in about a 1 meter radius circle. Take the cardboard and hit the object so that it speeds up. Observe its new orbit. Now tap the object so that it slows down and observe the resulting orbit. [See situations (A) and (B) below.]



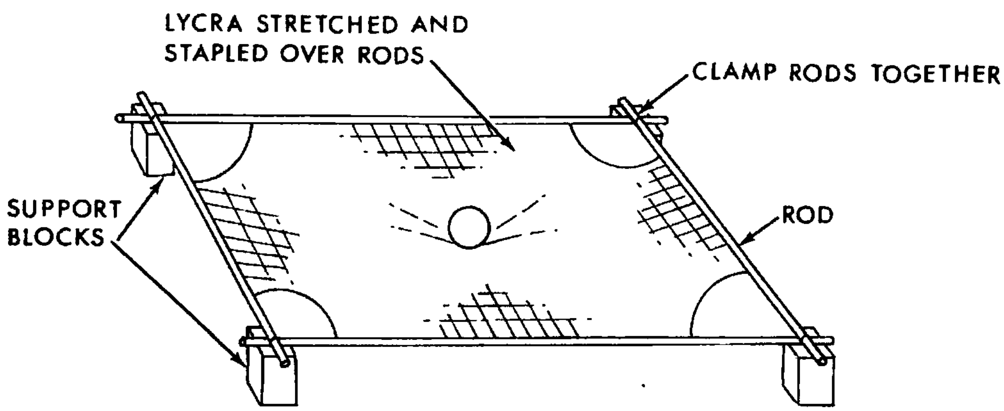
Explanation: The rubber-band chain provides a continuous inward pull analogous to the sun's gravitational pull. The swinging cardboard provides a force analogous to the thrust of a satellite's engine. Together, these illustrate the satellite's motion relative to the sun. Situation (A) demonstrates a night-side launch window for a satellite. The velocity of the orbiting body increases, and an orbit of larger radius will result. Situation (B) illustrates a daytime launch window. The orbit velocity decreases, and an orbit of smaller radius results.

Demonstration #3: A Swimsuit Model of the Solar System

Purpose: To illustrate the elliptical orbits of planets.

Materials: Four aluminum rods (about 1 m long); Lycra (swimsuit material); heavy ball; several small steel balls.

Procedure and Notes: Set up the material as shown below. Place the heavy ball in the center. Roll the steel balls at different angles to a radius originating at the heavy ball. Repeat the rolls using different speeds.



Explanation: The depression caused by the heavy ball will simulate a "gravity" well or a centrally directed force on the rolling balls. The varying speeds and angles will result in various shaped orbits. The shape and path of the orbits will depend on the initial velocity of the balls.

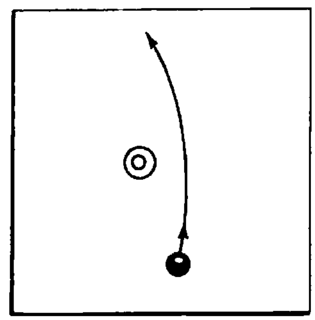
Demonstration #4: Gravity Assist Analogy

Purpose: To illustrate the transfer of energy from one object to another and to provide an analogy to gravity assists.

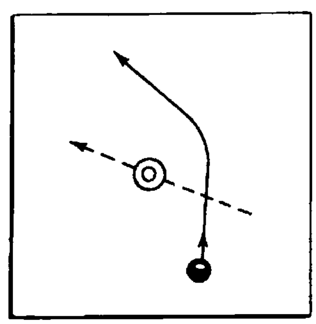
Materials: Button magnet, steel balls, plexiglass sheet.

Procedure and Notes:

1. Place a plexiglass sheet horizontally with the button magnet underneath. Roll steel balls past the magnet, noting the change in direction and the apparent identical speed.



2. Move the magnet at an angle to the path of the approaching steel ball such that it passes directly in front of the moving ball. Note the change in both the direction and speed of the ball.



Explanation:

Case 1. The magnetic field deflected the ball's path. Nothing was present which should change the ball's speed.

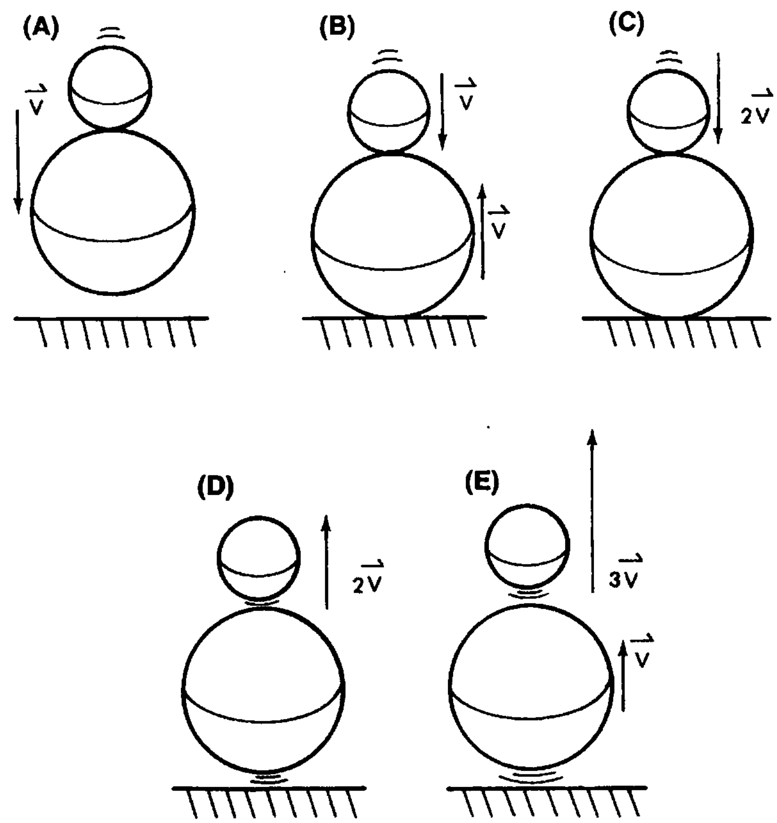
Case 2. The moving magnet has kinetic energy as it passes in front of the rolling ball; some of that energy is transferred to the ball, whipping it up to a higher speed.

Demonstration #5: Superball Boost

Purpose: To illustrate the gravity-assist principle with two superballs.

Materials: Two different-size superballs.

Procedure and Notes: Drop the two balls with the smaller one on top of the larger one. The small ball will rebound with much larger velocity than before the impact.

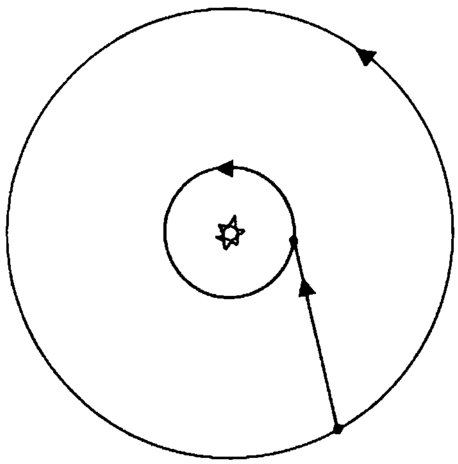


Explanation: Assume that the small ball has so much less mass than the large one that its effect on the large one's speed can be neglected. Situation (A) above represents the pair just before they collide with the floor. Just after the collision (B) the big ball starts up with velocity V. In the frame of reference of the big ball the little ball is continuing downward with velocity $2V$ as in (C). After an elastic collision (D) the little ball proceeds up with a velocity of $2V$. In the frame of reference of the floor, the big ball is also rebounding with a velocity V. The little ball, therefore, is moving up with velocity $3V$. This situation is similar to a satellite having an "elastic collision" with a planet.

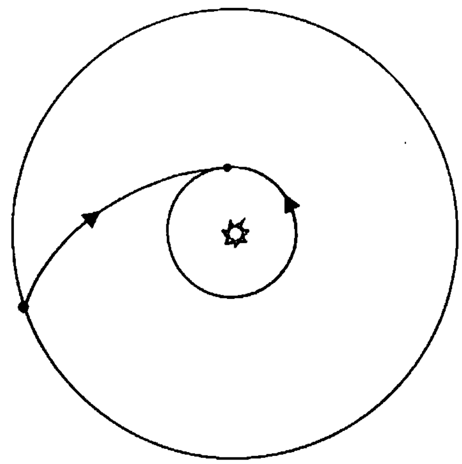
EVALUATION QUESTIONS

1. Which diagram represents a Hohman (most efficient) transfer orbit?

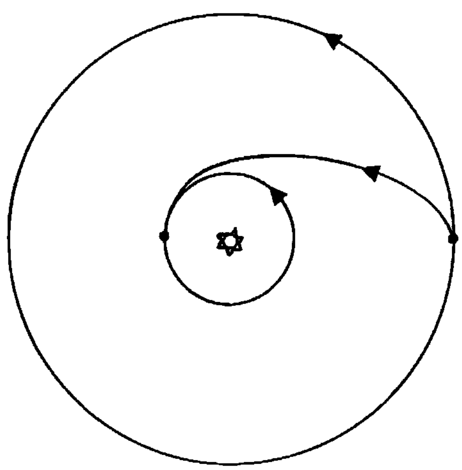
A



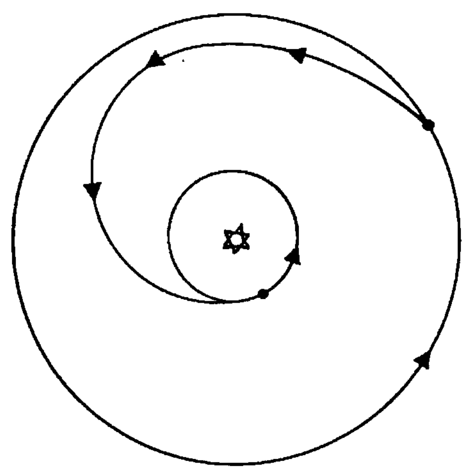
C



B



D

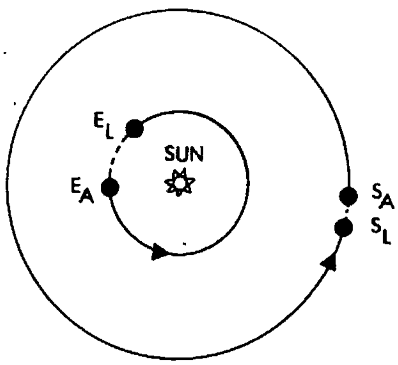


2. **Which of the four diagrams below shows the correct planetary alignment for earth, sun and Saturn for a launch opportunity for a flight from Saturn to earth via a Hohman transfer orbit?**

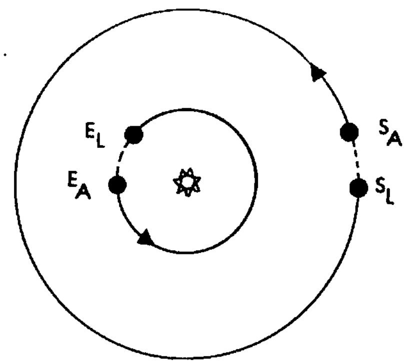
Key:

S_L = *position of Saturn at time of launch.*
E_L = *position of earth at time of launch.*
S_A = *position of Saturn by time of arrival.*
E_A = *position of earth by time of arrival.*

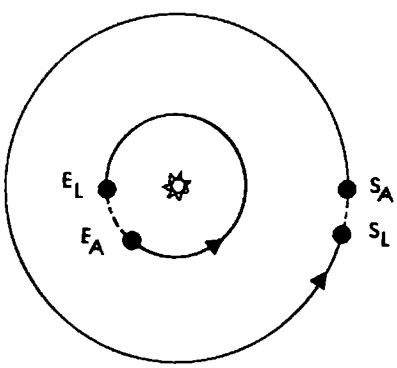
A.



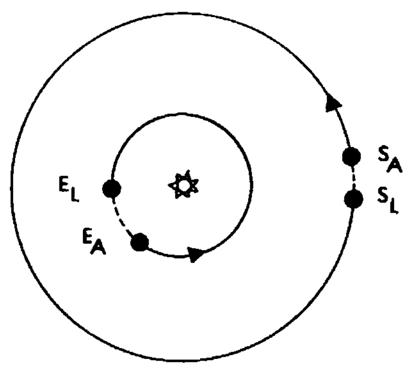
C.



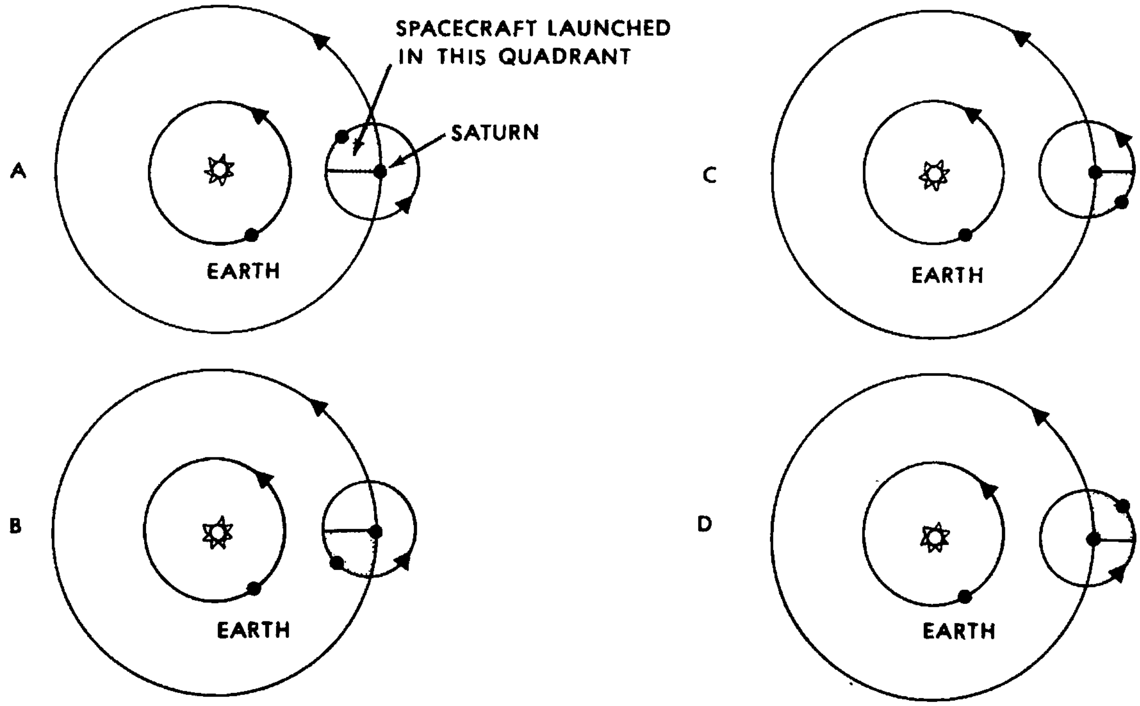
B.



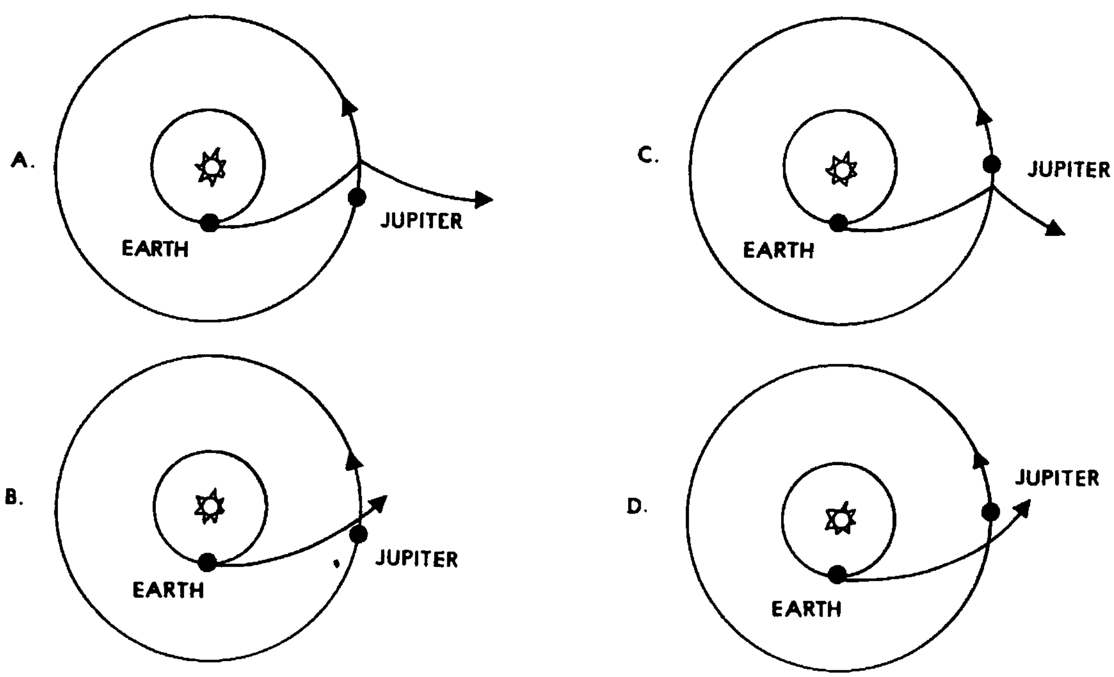
D.



3. If a spacecraft were in orbit around Saturn and were going to leave on a transfer orbit toward earth, which of the following quadrants should it be in before it should fire its rocket and leave orbit?



4. In which of the cases diagrammed below would a spacecraft receive the greatest gravity boost from Jupiter?



5. **If a spacecraft were in orbit around the sun--first at the distance of the planet Mercury, then at the distance of the planet Venus, and then at the distance of each of the other planets in succession--in which orbit would it have the greatest speed?**

   A. The speed would be the same in each of the orbits.
   B. The speed would be greatest in the orbit of Mercury; less in each succeedingly distant orbit.
   C. The speed would be greatest in the orbit of Pluto; less in each closer orbit.
   D. There is no constant relationship between speed and the distance from the sun. The speed, therefore, would vary according to other factors.

6. **Relative to the sun, which of the following is *not* changed for a spacecraft that receives a gravity assist?**

   A. speed
   B. direction
   C. momentum
   D. mass

7. **What is the effect of a gravity assist on the assisting planet?**

   A. The planet is pushed farther from the sun.
   B. The planet's speed is increased.
   C. The planet's speed is decreased.
   D. There is no effect on the planet.

8. **A spacecraft traveling from earth to another planet uses the gravitational attraction of the sun to**

   A. go into orbit about the sun.
   B. speed up.
   C. lose speed.
   D. sling shot its trajectory.

9. **The Hohman transfer orbit to Neptune is**

   A. the only orbit possible.
   B. the least-energy orbit.
   C. the brute-force method.
   D. possible all the time.

10. **Relative to the sun, a spacecraft in a parking orbit around the earth travels**

    A. faster on the night side of earth.
    B. faster on the day side of earth.
    C. with constant velocity.
    D. with constant speed.

ESSAY QUESTIONS

11. Make a diagram showing a Hohman transfer orbit from Neptune to Venus. On the diagram include approximate positions of the planets at launch time and at arrival time.

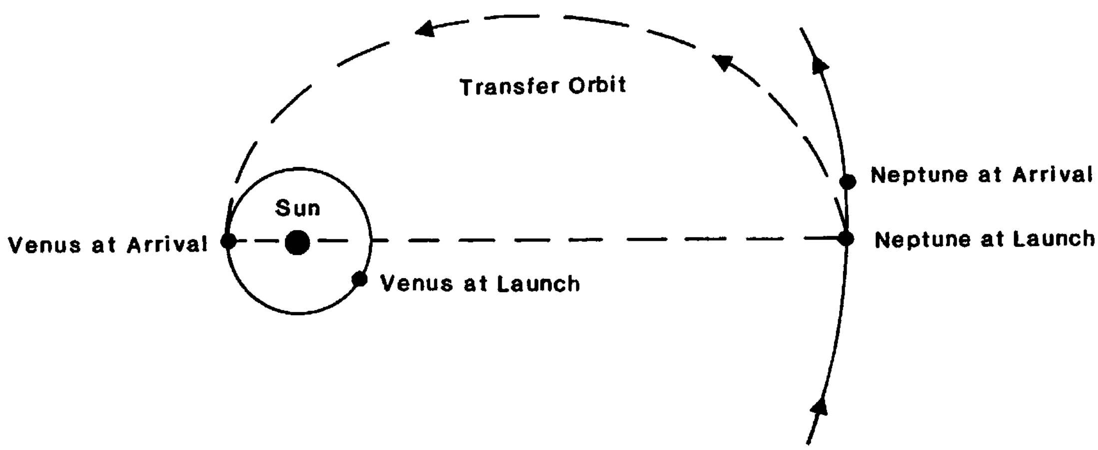
12. Using the idea of "falling" in the sun's gravitational field, explain how a spacecraft (a) gains speed traveling along a transfer orbit to an inner planet, and (b) loses speed traveling to an outer planet.

KEY

1. B
2. C
3. A
4. D
5. B
6. C
7. C
8. A
9. B
10. A

SUGGESTED ESSAY RESPONSES

11. The diagram of a Hohman transfer orbit from Neptune to Venus is shown below.



12. (a) Once the satellite escapes from its parking orbit about the earth, it enters an elliptical orbit about the sun. As the satellite moves closer to the sun, it experiences a stronger gravitational pull, and consequently it will speed up.

    (b) As the satellite travels farther from the sun, the gravitational force on it from the sun diminishes. Consequently, the spacecraft will slow down.