Using a Modified Boyle's Law Experiment to Estimate the Density of Salts*

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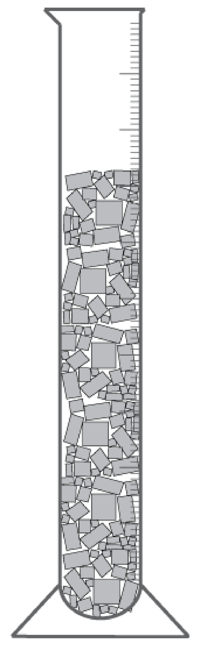
Abstract

Physics and chemistry students learn several methods to determine the density of materials. While measuring the mass of materials is usually simple, volume measurements are more complex.

Common methods to measure the volume of materials:
- Measure the material geometry and calculate volume
- Archimedes' principle

Difficulties measuring density of salt crystals include
- Small size and irregular shape of salt crystals
- Air between crystals while making a bulk density measurement
- Salt crystals dissociate in water

Here, we modify Boyle's law experiment to find the volume of salt crystals and thus accurately determine their density.



Introduction

Boyle's law describes the relationship between pressure P and volume V of an ideal gas:

$$P = \frac{A}{V},$$

with A some constant that depends on the absolute temperature and number of moles in the gas.

This relationship is easily obtained by changing the volume of some object (often a syringe) and measuring the pressure.

However, there is a systematic error associated with most Boyle's law experiments: the volume of the headspace between the syringe and the pressure sensor is typically ignored. Rupright recommends correcting for the headspace volume error by fitting data to a new equation:

$$P = \frac{A}{V + V_h} \qquad (1)$$

where V_h is a fitting parameter equal to the headspace volume.[1]

We extend this method first to calculate the volume of an empty attached flask and then to find the volume of air in a flask filled with a given salt. The difference in these volumes represents the salt volume. Thus, we are able to determine the crystal density of salt using this modification to Boyle's law.

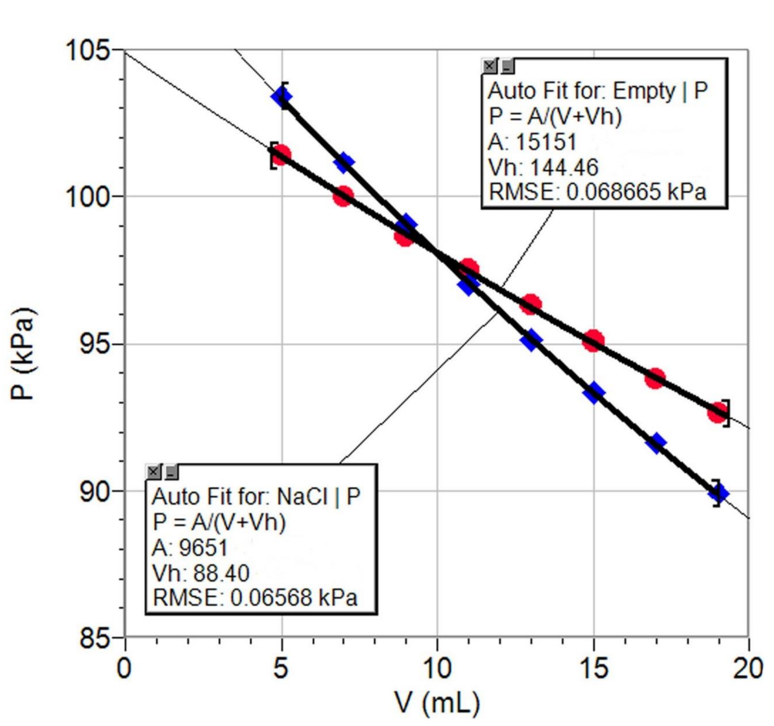


Figure 1. Pressure-volume curve for a single trial with an empty flask trial (●) and a single trial with a flask with NaCl (◆). Best-fit lines show V_h, the headspace volume.

Methodology

We tested five different salts in this experiment:

- NaCl, NaOH, $CaSO_4 \cdot 2H_2O$, $BaCl_2 \cdot H_2O$, $Ba(NO_3)_2$.

These salts were selected since they were readily available and have accepted densities that vary considerably[2] from 2.165 g/ml for NaCl to 3.24 g/ml for $Ba(NO_3)_2$.

Steps:
1. Connect pressure sensor to empty flask
2. Take Volume-Pressure data, varying syringe volume from 5-19 ml
3. Fit curve with equation (1) to find headspace volume V_1
4. Fill flask with salt and record mass of salt (m_s)
5. Repeat steps 2-3 to find headspace volume V_2
6. Calculate volume of salt $V_s = V_1 - V_2$
7. Record density $\rho = m/V_s$
8. Repeat 4 more times for each sample
9. Calculate average density for each salt

A representative graph in Fig. 1 shows pressure and volume measurements for an empty flask and a flask filled with NaCl. The data were fit to Eq. (1) to find the headspace volumes. We note the significant difference in the slope of these curves depending on the volume of the headspace.

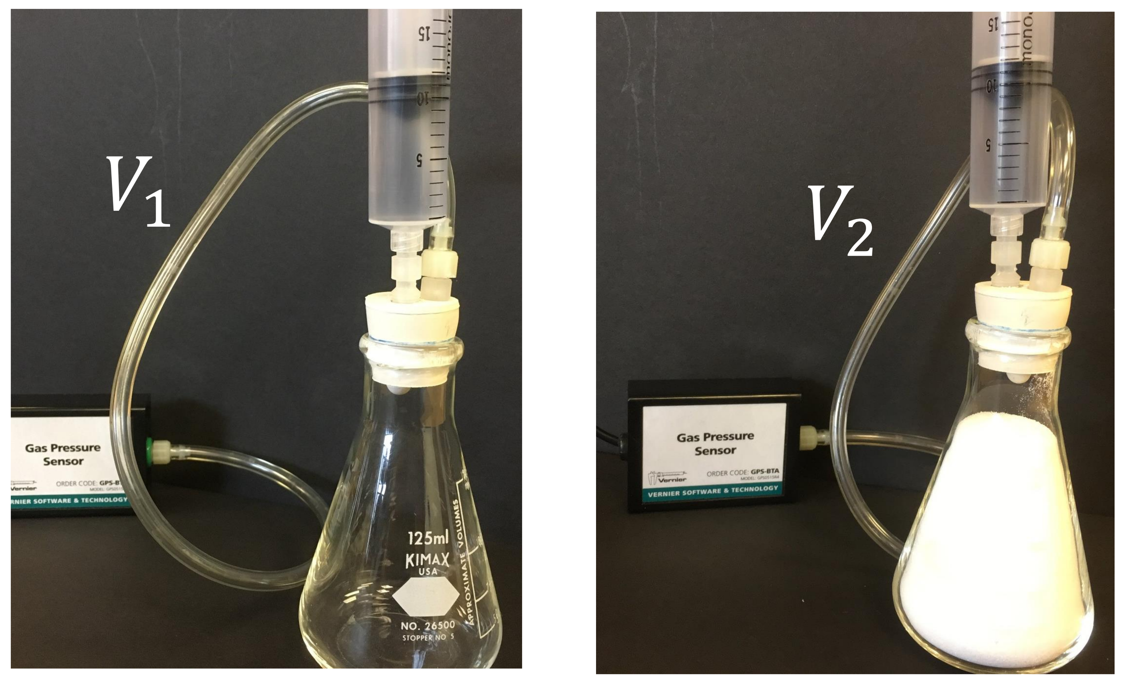


Figure 2. Experimental setup of (left) an empty flask and (right) a flask filled with salt.

Table 1. Method of calculating density with NaCl.

	Mean Value	Probable Error	Probable Error (%)
V_1 (ml)	144.62	0.69	0.48
V_2 (ml)	88.38	0.13	0.14
V_s (ml)	56.24	0.70	1.25
m (g)	122.7	0.1	0.08
Experimental ρ (g/ml)	2.182	0.029	1.33
Accepted ρ (g/ml)	2.165		
Percent Error	0.766%		

Results

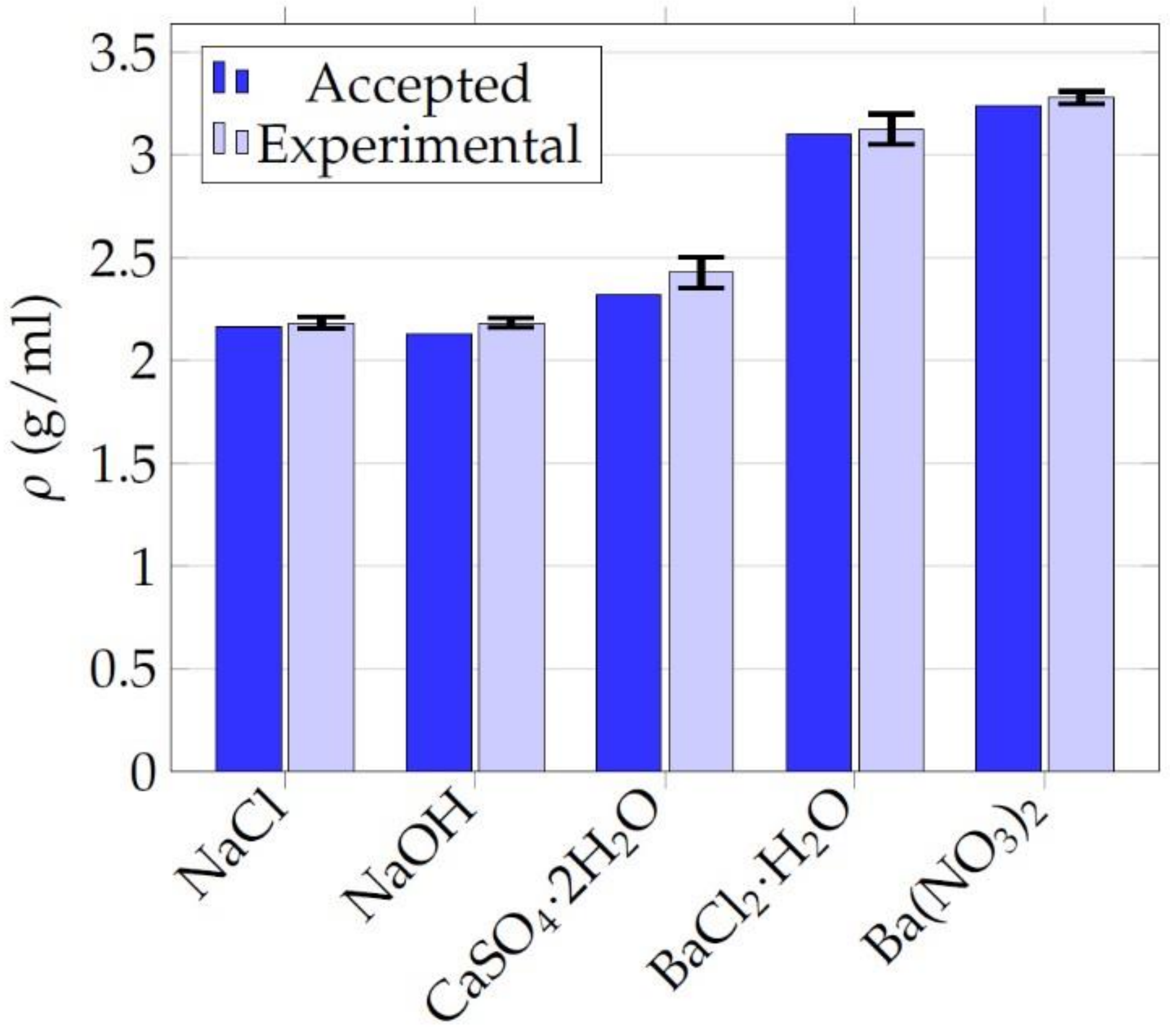


Fig. 3. Density measurements of various materials using the modified Boyle's law and comparisons with accepted values.[5] Error bars represent the 50% confidence interval (probable error).

Table I shows the values used to calculate the density of table salt (NaCl), and similar measurements were made for the other salts. The 50% confidence interval is the probable error for the average. The probable error of the empty flask headspace (V_1) is only 0.70 ml (or 1.25%), suggesting that this method is repeatable within 1 ml.

For NaCl the density measurements showed:
Experimental error of 1.33%, suggesting reasonable precision.
Percent error of 0.766%, indicating high accuracy as well.

Figure 3 shows the average and probable error of density measurements for all the salts tested. Comparisons are made with the accepted values. In all cases, the percent error was less than 4.6%, and the **average percent error was 1.96%.**

Sources of Error:

- Amount of salt used
  - More salt increases mass
  - More salt reduces headspace volume
- Reading the volumes on the syringe
  - Markings every 1.0 ml
  - Could be off by 0.2 ml
- Pushing the stopper in too far or not enough
- Variable temperature during experiment

Extensions

Estimate density of zinc and water

- Percent error below 2.5% for both materials
- Process should work well for solids
- Low vapor pressure of certain liquids may cause inaccurate density measurements

Calculate the packing density (bulk density divided by crystal density)

- We poured NaCl into a graduated cylinder until it reached the 100-ml line
- Mass was 140.5 g, so bulk density was 1.405 g/ml
- Packing density $= \frac{1.405}{2.182} = 64.4\%$
- Theoretical packing density[3] of random-sized spheres: 63.4%

Conclusion

This modification to Boyle's law experiment provides physics and chemistry teachers with **a simple way to estimate volumes of oddly shaped materials,** which are then used to find the density of these materials. It is also a good way to discuss systematic errors in experiments.

Extensions:
- Estimate void volumes,
- Calculate volumes of oddly shaped containers
- Determine packing density

References

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3. C. Song, P. Wang, and H. A. Makse, "A phase diagram for jammed matter," *Nature* **453**, 629–632 (May 29, 2008).

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