Acceleration of Light at Earth's Surface

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General relativity requires that light traveling upward or downward at the earth's surface has an acceleration equal to +2g.

The radial acceleration of an object at the earth's surface in a gravitational field $g = GM/r^2$ is equal to

$$a_r = - g(1 - 3\beta_r^2) \tag{1}$$

where β_r is the radial velocity v_r divided by c. When $\beta_r = 0$, the object will accelerate downward with acceleration g as expected. When the object in question is light, then $\beta_r = \pm 1$ and a_r is equal to $+2g$. If the object is traveling with a radial velocity $\beta_r = \pm 1/\sqrt{3}$, then it will not experience any radial acceleration at all.

We know that the velocity of light above the surface of the earth is greater than c relative to a surface observer, and it is less than c (in a matter free depression) below the surface. So when traveling from below to above the surface, light will accelerate upward; and when traveling from above to below the surface, it will also accelerate upward. Equation 1 gives the quantitative amount of that acceleration.

This equation is only valid when the gravitational potential ϕ is considerably less that c^2. More precisely, Eq. 1 is given by

$$a_r = - \sigma g(1 - 3\beta_r^2/\sigma^2) \tag{2}$$

where $\sigma = 1 + 2\phi/c^2$, with $\phi = - GM/r$ [Ref. 1, Eq. 12.37].

Equation 2 is derived from an even more general relationship in contravariant spatial coordinates $x^i = (x^1, x^2, x^3)$ given by

$$a^i = - \Gamma^i{}_{\mu\nu} v^\mu v^\nu + (v^i/c)\Gamma^4{}_{\mu\nu} v^\mu v^\nu \tag{3}$$

where $a^i = d^2x^i/dt^2$, $v^i = dx^i/dt$, and $v^\mu = (v^i, c)$ is not a four-vector. We call $x^4 = ct = \tau$. Greek indices sum over four coordinates. The acceleration a_r in Eq. 1 is a^1 in Eq. 3. Equation 3 is derived in Appendix D of Ref. 1.

To evaluate this equation for our case the Schwarzschild metric

$$ds^2 = \sigma^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2) - \sigma d\tau^2$$

is used to find the Christoffel symbols

$$\Gamma^1{}_{11} = -\mathrm{GM}/\sigma r^2c^2 \qquad \Gamma^1{}_{44} = \sigma \mathrm{GM}/r^2c^2$$
$$\Gamma^4{}_{14} = \Gamma^4{}_{41} = \mathrm{GM}/\sigma r^2c^2$$

Ref. 1. R. A. Mould, "Basic Relativity", *Springer* 2002