General relativity requires that light traveling upward or downward at the earth’s surface has an acceleration equal to +2g.

The radial acceleration of an object at the earth’s surface in a gravitational field $g = GM/r^2$ is equal to

$$a_r = -g(1 - 3\beta_r^2) \quad (1)$$

where $\beta_r$ is the radial velocity $v_r$ divided by $c$. When $\beta_r = 0$, the object will accelerate downward with acceleration $g$ as expected. When the object in question is light, then $\beta_r = \pm 1$ and $a_r$ is equal to $+2g$. If the object is traveling with a radial velocity $\beta_r = \pm 1/\sqrt{3}$, then it will not experience any radial acceleration at all.

We know that the velocity of light above the surface of the earth is greater than $c$ relative to a surface observer, and it is less than $c$ (in a matter free depression) below the surface. So when traveling from below to above the surface, light will accelerate upward; and when traveling from above to below the surface, it will also accelerate upward. Equation 1 gives the quantitative amount of that acceleration.

This equation is only valid when the gravitational potential $\phi$ is considerably less than $c^2$. More precisely, Eq. 1 is given by

$$a_r = -\sigma g(1 - 3\beta_r^2/\sigma^2) \quad (2)$$

where $\sigma = 1 + 2\phi/c^2$, with $\phi = -GM/r$ [Ref. 1, Eq. 12.37].

Equation 2 is derived from an even more general relationship in contravariant spatial coordinates $x^1, x^2, x^3$ given by

$$a^1 = -\Gamma_{\mu\nu}^{\alpha} v^\mu v^\nu + (v^4/c^4)\Gamma_{\mu\nu}^{4} v^\mu v^\nu \quad (3)$$

where $a^1 = d^2x^1/dt^2$, $v^1 = dx^1/dt$, and $v^\mu = (v^1, c)$ is not a four-vector. We call $x^4 = ct = \tau$. Greek indices sum over four coordinates. The acceleration $a_r$ in Eq. 1 is $a^1$ in Eq. 3. Equation 3 is derived in Appendix D of Ref. 1.

To evaluate this equation for our case the Schwarzschild metric

$$ds^2 = \sigma^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) - \sigma dt^2$$

is used to find the Christoffel symbols

$$\Gamma_{11}^1 = -GM/\sigma r^2 c^2 \quad \Gamma_{14}^4 = \sigma GM/r^2 c^2$$

$$\Gamma_{41}^1 = \Gamma_{44}^4 = GM/\sigma r^2 c^2$$
