AAPT UNITED STATES PHYSICS TEAM
AIP 2019

2019 $F = ma$ Exam

25 QUESTIONS - 75 MINUTES

INSTRUCTIONS

DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

- Use $g = 10$ N/kg throughout this contest.
- You may write in this booklet of questions. However, you will not receive any credit for anything written in this booklet. You may only use the scratch paper provided by the proctor.
- This test contains 25 multiple choice questions. Select the answer that provides the best response to each question. Please be sure to use a No.2 pencil and completely fill the box corresponding to your choice. If you change an answer, the previous mark must be completely erased. Only the boxes preceded by numbers 1 through 25 are to be used on the answer sheet.
- All questions are equally weighted, but are not necessarily of the same level of difficulty.
- Correct answers will be awarded one point; incorrect answers or leaving an answer blank will be awarded zero points. There is no additional penalty for incorrect answers.
- A hand-held calculator may be used. Its memory must be cleared of data and programs. You may use only the basic functions found on a simple scientific calculator. Calculators may not be shared. Cell phones may not be used during the exam or while the exam papers are present. You may not use any tables, books, or collections of formulas.
- The question booklet, the answer sheet and the scratch paper will be collected at the end of this exam.
- **In order to maintain exam security, do not communicate any information about the questions (or their answers or solutions) on this contest until after February 1, 2019.**

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We acknowledge the following people for their contributions to this year's exams (in alphabetical order):

JiaJia Dong, Mark Eichenlaub, Matthew Huang, David Jones, Abijith Krishnan, Daniel Longenecker, Marianna Mao, Joon Pahk, Kye Shi, Brian Skinner, Alex Small, Paul Stanley, Elena Yudovina, and Kevin Zhou.

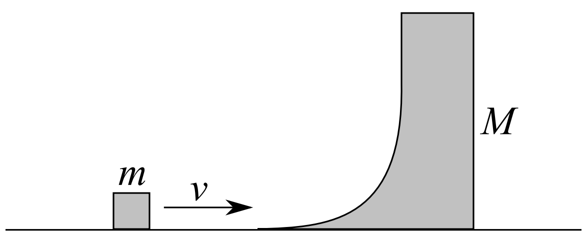
1. A coin of mass m is dropped straight down from the top of a very tall building. As the coin approaches terminal speed, which is true of the net force on the coin?

   (A) The net force on the coin is upward.
   (B) The net force on the coin is 0.
   (C) The net force on the coin is downward, with a magnitude less than mg.
   (D) The net force on the coin is downward, with a magnitude equal to mg.
   (E) The net force on the coin is downward, with a magnitude greater than mg.

2. A mass of $3M$ moving at a speed v collides with a mass of M moving directly toward it, also with a speed v. If the collision is completely elastic, the total kinetic energy after the collision is K_e. If the two masses stick together, the total kinetic energy after the collision is K_s. What is the ratio K_e/K_s?

   (A) 1/2
   (B) 1
   (C) $\sqrt{2}$
   (D) 2
   (E) 4

3. A block of mass m is launched horizontally onto a curved wedge of mass M at a velocity v. What is the maximum height reached by the block after it shoots off the vertical segment of the wedge? Assume all surfaces are frictionless; both the block and the curved wedge are free to move. The curved wedge does not tilt or topple.

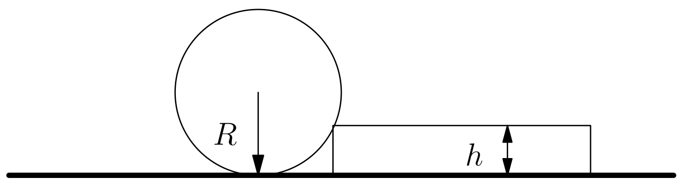


   (A) $\dfrac{v^2}{2g}$
   (B) $\left(\dfrac{m}{m+M}\right)^2 \cdot \dfrac{v^2}{2g}$
   (C) $\left(\dfrac{M}{m+M}\right)^2 \cdot \dfrac{v^2}{2g}$
   (D) $\dfrac{m}{m+M} \cdot \dfrac{v^2}{2g}$
   (E) $\dfrac{M}{m+M} \cdot \dfrac{v^2}{2g}$

4. A massless spring hangs from the ceiling, and a mass is hung from the bottom of it. The mass is supported so that initially the tension in the spring is zero. The mass is then suddenly released. At the bottom of its trajectory, the mass is 5 centimeters from its original position. Find its oscillation period.

   (A) 0.05 s
   (B) 0.07 s
   (C) 0.31 s
   (D) 0.44 s
   (E) Not enough information is given.

5. A cylinder has a radius R and weight G. You try to roll it over a step of height $h < R$. The minimum force needed to roll the cylinder over is:

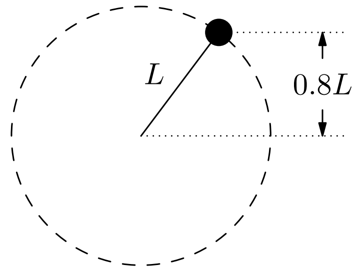


   (A) $\frac{\sqrt{2Rh - h^2}}{R - h}G$
   (B) $\frac{\sqrt{2Rh - h^2}}{2R - h}G$
   (C) $\frac{\sqrt{2Rh - h^2}}{2R}G$
   (D) $\frac{\sqrt{2Rh - h^2}}{R}G$
   (E) $\frac{\sqrt{Rh - h^2}}{2R}G$

6. A ball is released from rest above an inclined plane and bounces elastically down the plane. As the ball progresses down the plane, the time and the distance between each collision will:

   (A) remain the same, and increase.
   (B) increase, and remain the same.
   (C) decrease, and increase.
   (D) decrease, and remain the same.
   (E) both remain the same.

7. An object of mass m is attached to the end of a massless rod of length L. The other end of the rod is attached to a frictionless pivot. The object is raised so that its height is $0.8L$ above the pivot, as shown in the figure. After the object is released from rest, what is the tension in the rod when it is horizontal?



(A) 0.6 mg

(B) 1.6 mg

(C) 2.6 mg

(D) 3.6 mg

(E) 5.36 mg

8. The mass and the radius of the Earth are M and R. If an object starting at a distance of R from the Earth's surface is moving at a velocity $v_0 = \sqrt{\frac{2GM}{3R}}$ tangentially, what is its trajectory?

(A) A parabola or a hyperbola

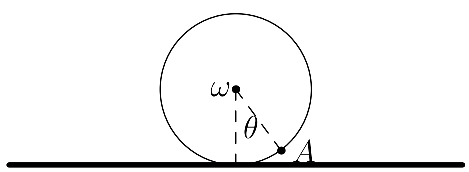
(B) A circle around Earth

(C) An ellipse whose minimum distance from the Earth's surface is R

(D) An ellipse whose maximum distance from the Earth's surface is R

(E) A straight line along the direction of the initial velocity

9. A wheel of radius R is rolling without slipping with angular velocity ω.



For point A on the wheel at an angle θ with respect to the vertical, shown in the figure, what is the magnitude of its velocity with respect to the ground?

(A) ωR

(B) $\omega R \sin(|\theta|/2)$

(C) $\sqrt{2}\omega R \sin(|\theta|/2)$

(D) $2\omega R \sin(|\theta|)$

(E) $2\omega R \sin(|\theta|/2)$

10. A flat uniform disk of radius $2R$ has a hole of radius R removed from the center. The resulting annulus is then cut in half along the diameter. The remaining shape has mass M. What is the moment of inertia of this shape, about the axis of rotational symmetry of the original disk?

    (A) $\frac{45}{32}MR^2$

    (B) $\frac{7}{6}MR^2$

    (C) $\frac{8}{5}MR^2$

    (D) $\frac{5}{2}MR^2$

    (E) $\frac{15}{8}MR^2$

11. To test the speed of a model car, you time the car with a stopwatch as it travels a distance of 100 m. You record a time of 5.0 s, and your measurement has an uncertainty of 0.2 s. What is the uncertainty in your estimate of the car's speed? Assume that the car travels at a constant speed and the distance of 100 m is known very precisely.

    (A) $v = 20 \pm 0.16$ m/s

    (B) $v = 20 \pm 0.8$ m/s

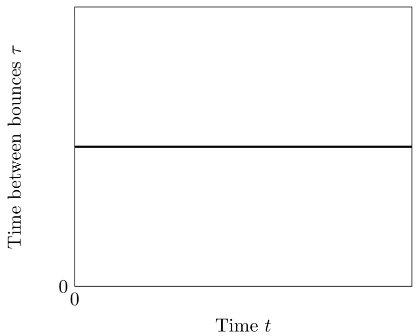
    (C) $v = 20 \pm 1.0$ m/s

    (D) $v = 20 \pm 1.25$ m/s

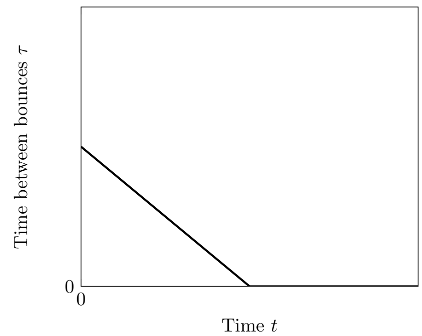
    (E) $v = 20 \pm 4.0$ m/s

12. A steel ball bearing bounces vertically on a steel plate. If the speed of the ball just before a bounce is v_i, the speed of the ball immediately afterward is $v_f = \alpha v_i$, with $\alpha < 1$. Which one of the following graphs best shows the time between successive bounces, τ, as a function of time?

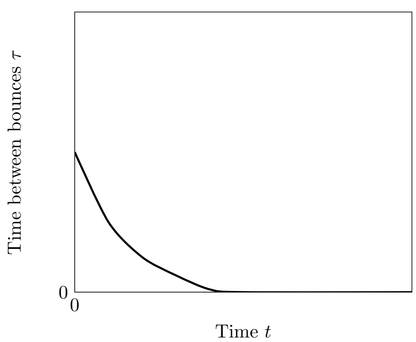
(A)



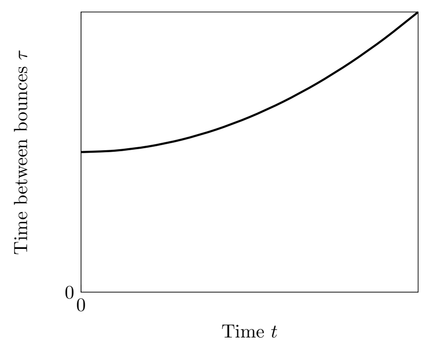
(B)



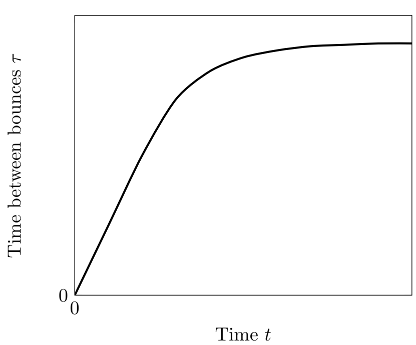
(C)



(D)



(E)



13. A juggler juggles N identical balls, catching and tossing one ball at a time. Assuming that the juggler requires a minimum time T between ball tosses, the minimum possible power required for the juggler to continue juggling is proportional to

    (A) N^0
    (B) N^1
    (C) N^2
    (D) N^3
    (E) N^4

14. A man standing at 30° latitude fires a bullet northward at a speed of 200 m/s. The radius of the Earth is 6371 km. What is the sideway deflection of the bullet after traveling 100 m?

    (A) 3.1 mm west
    (B) 1.8 mm west
    (C) 0 mm
    (D) 1.8 mm east
    (E) 3.1 mm east

15. An upright rod of length ℓ is launched into the air with vertical velocity v_y. It is given enough angular momentum so that the rod rotates by an angle of 2π before landing. Find the initial horizontal velocity of the bottom of the rod.

    (A) $\dfrac{2\ell g}{v_y^2}$
    (B) $\dfrac{\pi \ell g}{2v_y}$
    (C) $\sqrt{\ell g}$
    (D) $\dfrac{2v_y}{\ell g}$
    (E) $\dfrac{2v_y^2}{\pi\sqrt{\ell g}}$

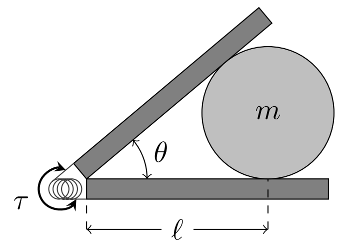
16. The depth of a well, d, is measured by dropping a stone into it and measuring the time t until the splash is heard at the bottom. What is the smallest value of d for which ignoring the time for the sound to travel gives less than a 5% error in the depth measurement? The speed of sound in air is 330 m/s.

    (A) 3.5 m
    (B) 7 m
    (C) 14 m
    (D) 54 m
    (E) 330 m

The following information applies to questions 17 and 18.

A launcher is designed to shoot objects horizontally across an ice rink. It consists of two boards of negligible mass connected via a spring-loaded hinge, which exerts a constant torque τ on each board to keep them together. For both problems, neglect friction with either the ice or the launch boards.

17. A hard disc is pushed into the launcher between the boards until the boards make contact with it a distance ℓ from the hinge and are open to an angle θ, as shown in the figure. What is the minimum force necessary to hold the disc in this position?



(A) $2\tau\sin(\theta/2)/\ell$

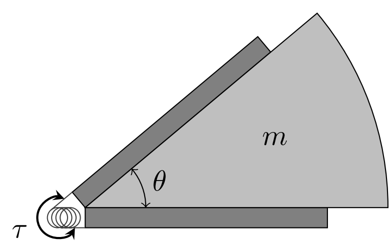
(B) $2\tau\cos(\theta/2)/\ell$

(C) $\tau\cos(\theta)/\ell$

(D) $\tau\tan(\theta)/\ell$

(E) $2\tau\tan(\theta)/\ell$

18. The disc is removed and replaced with a pie-shaped wedge of the same mass m, so that the hinge is still initially held open at an angle θ, as shown in the following figure. If the wedge is released from rest, what is its speed after it exits the launcher?



(A) $\sqrt{\dfrac{\tau}{2m\theta}}$

(B) $\sqrt{\dfrac{4\tau\theta}{m}}$

(C) $\sqrt{\dfrac{\tau\theta^2}{2m}}$

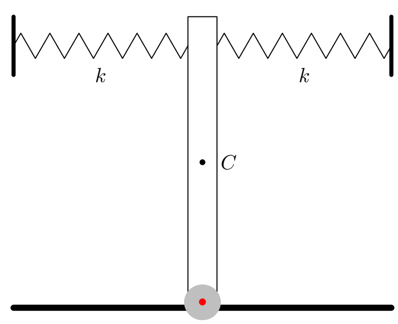
(D) $\sqrt{\dfrac{2\tau\theta}{m}}$

(E) $\sqrt{\dfrac{2\tau}{m\theta}}$

19. A small rock is tied to a massless string of length 5 m. The density of the rock is twice the density of the water. The rock is lowered into the water, while the other end of the string is attached to a pivot. Neglect any resistive forces from the water. The rock oscillates like a pendulum with angular frequency of which of the following?

    (A) 1 rad/s
    (B) 0.7 rad/s
    (C) 0.5 rad/s
    (D) 1.4 rad/s
    (E) 2 rad/s

20. A uniform rod of mass M and length L is hinged on a horizontal surface at the bottom. Its top end is connected to two springs, both with spring constant k. What relation must M, k, and L satisfy such that the position shown in the figure is a stable equilibrium?



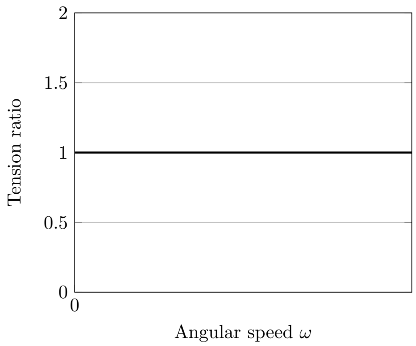
    (A) $Mg < 4kL$
    (B) $Mg < 2kL$
    (C) $2kL < Mg < 4kL$
    (D) $kL/2 < Mg < kL$
    (E) $Mg < kL$

21. A spherical cloud of dust has uniform mass density ρ and radius R. Satellite A of negligible mass is orbiting the cloud at its edge, in a circular orbit of radius R, and satellite B is orbiting the cloud just inside the cloud, in a circular orbit of radius r, with $r < R$. If v_i is the speed of satellite i and T_i is the period of satellite i, which of the following is true? Neglect any drag forces from the dust.

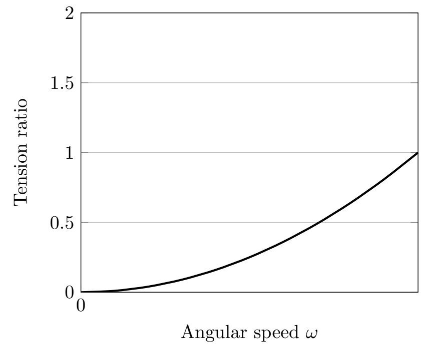
    (A) $T_A > T_B$ and $v_A > v_B$
    (B) $T_A > T_B$ and $v_A < v_B$
    (C) $T_A < T_B$ and $v_A > v_B$
    (D) $T_A < T_B$ and $v_A < v_B$
    (E) $T_A = T_B$ and $v_A > v_B$

22. A vertical pole has two massless strings, both of length L, attached a distance L apart. The other ends of the strings are attached to a mass M. The mass is rotated around the pole with an angular speed ω. Which of the following graphs best gives the ratio of the tension in the bottom string to the tension in the top string as a function of ω?

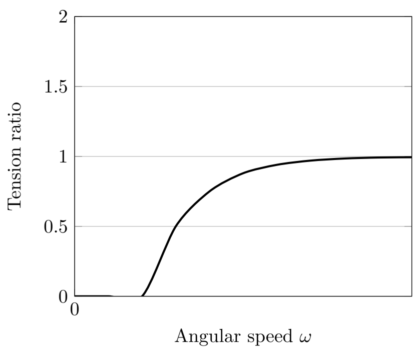
(A)



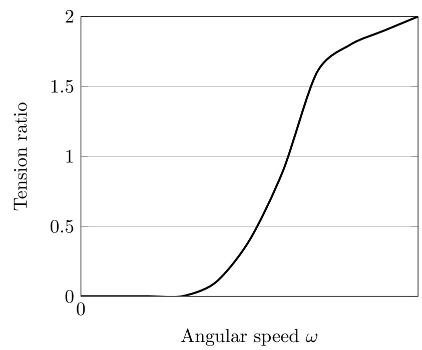
(B)



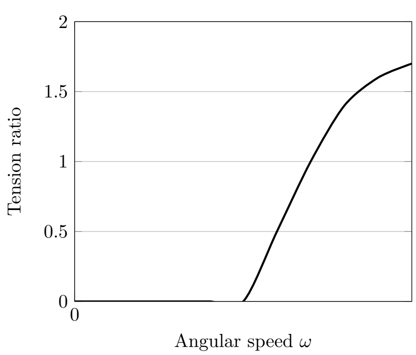
(C)



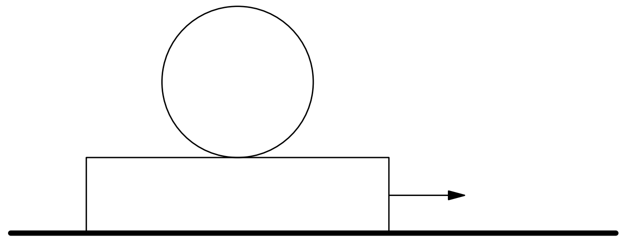
(D)



(E)



23. A rectangular slab sits on a frictionless surface. A sphere sits on the slab. There is sufficient friction between the sphere and the slab such that the sphere will not slip relative to the slab. A force to the right is applied to the slab, with both the slab and the sphere initially at rest.



The sphere will then:

(A) begin spinning clockwise while its center of mass accelerates to the right.
(B) begin spinning counterclockwise while its center of mass accelerates to the left.
(C) begin spinning clockwise while its center of mass accelerates to the left.
(D) begin spinning counterclockwise while its center of mass accelerates to the right.
(E) not spin, while its center of mass accelerates to the right.

24. A particle moves in the xy plane with the potential energy

$$U(x, y) = 9kx^2 + 16ky^2.$$

The particle can perform several different types of periodic motion. The ratio between the maximum and minimum possible periods is

(A) $2/\sqrt{3}$
(B) $4/3$
(C) $\sqrt{5}$
(D) 4
(E) 5

25. A car is turning left along a circular track of radius r at a constant speed v. A cylindrical beaker is placed vertically inside the car. The beaker has a small hole on its right side. If the water's highest point in the beaker is a height h above the hole, at what instantaneous speed does water escape the hole, from a passenger's perspective?

(A) $\sqrt{2gh}$
(B) $\sqrt{(v^2/r)h}$
(C) $\sqrt{gh}$
(D) $\sqrt{h\sqrt{(v^2/r)^2 + g^2}}$
(E) $\sqrt{2h\sqrt{(v^2/r)^2 + g^2}}$