AAPT UNITED STATES PHYSICS TEAM
AIP 2022

2022 $F = ma$ Exam B

25 QUESTIONS - 75 MINUTES

INSTRUCTIONS

DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

- Use $g = 10$ N/kg throughout this contest.
- You may write in this booklet of questions. However, you will not receive any credit for anything written in this booklet. You may only use the scratch paper provided by the proctor.
- This test contains 25 multiple choice questions. Select the answer that provides the best response to each question. Please be sure to use a No.2 pencil and completely fill the box corresponding to your choice. If you change an answer, the previous mark must be completely erased. Only the boxes preceded by numbers 1 through 25 are to be used on the answer sheet.
- All questions are equally weighted, but are not necessarily of the same level of difficulty.
- Correct answers will be awarded one point; incorrect answers or leaving an answer blank will be awarded zero points. There is no additional penalty for incorrect answers.
- A hand-held calculator may be used. Its memory must be cleared of data and programs. You may use only the basic functions found on a simple scientific calculator. Calculators may not be shared. Cell phones may not be used during the exam or while the exam papers are present. You may not use any tables, books, or collections of formulas.
- The question booklet, answer sheet and scratch paper will be collected at the end of this exam.
- **In order to maintain exam security, do not communicate any information about the questions (or their answers or solutions) on this contest until after February 25, 2022.**

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We acknowledge the following people for their contributions to this year's exams (in alphabetical order):

Tengiz Bibilashvili, Abi Krishnan, Andrew Lin, Kris Lui, Kye Shi, Brian Skinner, Mike Winer and Kevin Zhou

1. A ball is held a height h above a slope, which is at an angle 45° from the horizontal. The ball is dropped from rest. Assume the ball bounces off the slope perfectly elastically. What is the distance between its first and second impact points?

(A) $2h$ (B) $2\sqrt{2}\,h$ (C) $4h$ **D** $4\sqrt{2}\,h$ (E) $8h$

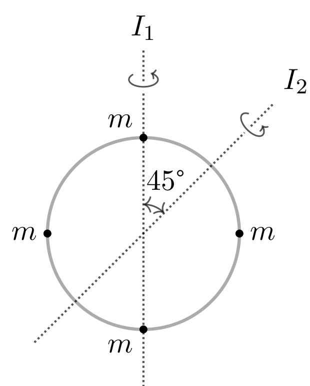
By conservation of energy, the ball has speed $v = \sqrt{2gh}$ when it hits the slope the first time. After the first impact, this velocity is redirected horizontally. Let Δx and Δy be the displacement from the first bounce to the second. Then we have

$$\Delta x = vt, \quad \Delta y = -\frac{1}{2}gt^2$$

and the second bounce obeys $\Delta x = -\Delta y$. Solving gives $t = 2v/g$ and distance

$$\sqrt{2}\,\Delta x = 2\sqrt{2}\,\frac{v^2}{g} = 4\sqrt{2}\,h.$$

2. A massless wheel of radius R has four small masses M placed evenly along its rim. Let the moment of inertia for rotations about the center of wheel, in the plane of the wheel, be I_0.

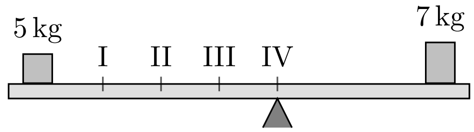


Now consider rotations about the two axes shown, with corresponding moments of inertia I_1 and I_2. Which of the following is true?

A $I_1 = I_2 = I_0/2$ (B) $I_1 = I_2 = I_0/\sqrt{2}$ (C) $I_1 = I_2 = I_0$
(D) $I_1 = I_0/2, I_2 = I_0/\sqrt{2}$ (E) $I_1 = I_0, I_2 = I_0/\sqrt{2}$

By the definition of the moment of inertia, $I_0 = 4MR^2$, $I_1 = 2MR^2$, and $I_2 = 4M(R/\sqrt{2})^2 = 2MR^2$. Therefore, $I_1 = I_2 = I_0/2$.

3. Four evenly spaced points are marked on a massless seesaw, as shown.



Two blocks, of masses 5 kg and 7 kg, are placed on the seesaw, so that it balances when the fulcrum is at point IV. (The diagram is not drawn to scale.) Now suppose the fulcrum is moved to point II. How much mass should be placed at point I so that the seesaw again balances?

(A) 12 kg

(B) 18 kg

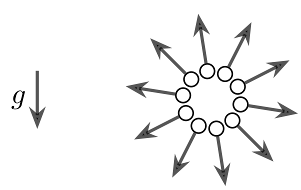
C 24 kg

(D) 36 kg

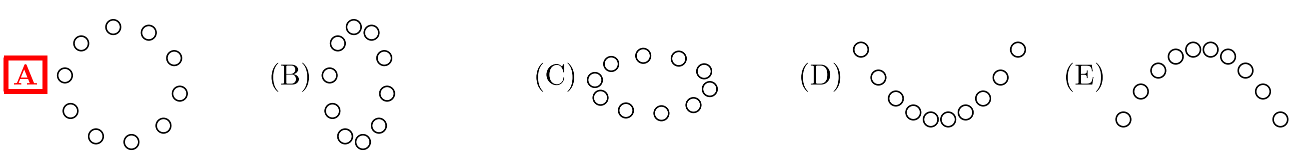
(E) There is not enough information to decide.

At first, it might seem like there isn't enough information to decide, because the locations of the masses aren't given. However, this turns out to not affect the answer. If the seesaw was initially balanced, its center of mass must have been directly over the fulcrum. Therefore, the total weight force of 120 N can be regarded as acting at point IV. After the fulcrum is moved, balancing torques about point II implies that the new mass should have a weight of 240 N, and therefore a mass 24 kg.

4. A firework explodes, sending shells in all directions in a vertical plane, as shown.



Suppose the shells are all launched with the same speed, and ignore air resistance, but not gravity. A long time later, but before any of the shells hit the ground, what shape do the shells make?



In the absence of gravity, the shells would always form a circle. Adding gravity simply shifts all of their locations downward by $gt^2/2$, so the shape is still always a circle.

5. A swimmer swims at speed v relative to still water. A river flows from a pier to a lake, with speed u. If the swimmer swims downstream from the pier to the lake, then back upstream, what was their average speed during the trip?

(A) v (B) $\sqrt{v^2 - u^2}$ (C) $\frac{(v-u)^2}{v}$ (D) $\frac{(v+u)^2}{v}$ **E** $\frac{v^2 - u^2}{v}$

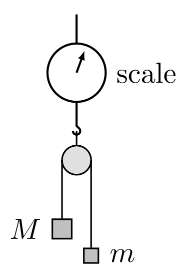
Assuming the distance from pier to lake is d, it takes time $d/(v+u)$ to get to the lake and $d/(v-u)$ to get back. This works out to distance $2d$ traveled in time $2dv/(v^2 - u^2)$, or an average speed of $(v^2 - u^2)/v$.

6. A block of mass m is at rest on a frictionless table. It is pushed horizontally by a constant force F, and has total momentum p when it reaches the end of the table. If a block of mass $2m$ is pushed across the table in the same way, also starting from rest, what is its momentum when it reaches the end of the table?

(A) $\frac{p}{2}$ (B) $\frac{p}{\sqrt{2}}$ (C) p **D** $\sqrt{2}\,p$ (E) $2p$

Since the forces and distances are the same, the total work done is the same. The kinetic energy is related to momentum by $K = p^2/2m$. Thus, $p \propto \sqrt{m}$, so the momentum of the second block is $\sqrt{2}p$.

7. Two blocks of unequal mass M and m are hung from the ends of a massless, frictionless pulley, as shown. The blocks are held in place, and the entire pulley is mounted on a sensitive scale.

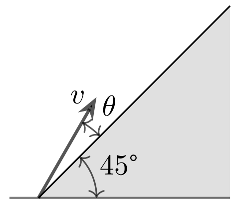


After the blocks are released from rest, but before either has fallen off the pulley, what is the scale reading?

A It is less than $(M + m)g$.
(B) It is equal to $(M + m)g$.
(C) It is more than $(M + m)g$.
(D) It is initially less than $(M + m)g$, then approaches $(M + m)g$.
(E) It is initially more than $(M + m)g$, then approaches $(M + m)g$.

When the blocks on a pulley are released, both blocks accelerate, with the heavier block accelerating downward. This means the center of mass is uniformly accelerating downward, which means the force acting on the pulley is less than the weight $(M + m)g$.

8. A ball is launched with speed v at an angle θ to a fixed ramp, which itself makes a 45° angle with the horizontal.



Assume that when the ball hits the ramp, it bounces elastically and frictionlessly. For what value of θ will the ball bounce off the ramp two times, and then return to its original launch point?

(A) 9.5° (B) 11.3° (C) 14.0° (D) 14.5° **(E)** 18.4°

Work with axes aligned with the ramp. In these coordinates, gravity has a component $g/\sqrt{2}$ towards the ramp, and $g/\sqrt{2}$ along the ramp. Meanwhile, the initial velocity has a component $v \sin\theta$ away from the ramp, and $v\cos\theta$ along the ramp.

A collision happens once the component of velocity away from the ramp goes from $v\sin\theta$ to $-v\sin\theta$, at which point the collision resets the velocity to $v\sin\theta$. Since three collisions happen (with the final one being at the original launch point itself), the total time must be

$$t = \frac{6v\sin\theta}{g/\sqrt{2}}.$$

In order to have arrived back the launch point at this point, the velocity along the ramp must have gone from $v\cos\theta$ to $-v\cos\theta$, which means

$$gt/\sqrt{2} = 2v\cos\theta.$$

Combining these results gives $\tan\theta = 1/3$.

9. Billy is leaning on a box of mass 30.0 kg, exerting a force 35.0° below the horizontal. If the coefficient of static friction of the box on the ground is $\mu_s = 0.400$, what is the minimum force needed for the box to slide?

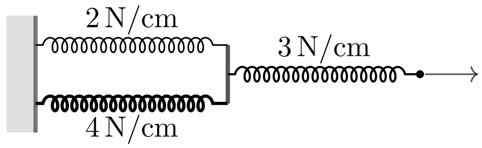
(A) 120 N (B) 147 N **(C)** 203 N (D) 224 N (E) 342 N

Suppose Billy exerts force F. Then the maximum friction force is

$$f = \mu_s N = (0.400)(300\,\text{N} + F\sin(35^\circ)).$$

The horizontal applied force is $F_x = F\cos(35^\circ)$. For this to move the box, we need $F_x > f$, and solving yields $F \geq 203\,\text{N}$.

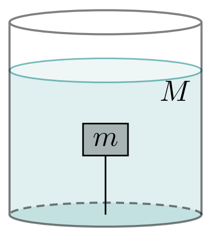
10. Two springs with spring constants 2 N/cm and 4 N/cm are connected in parallel. They are both connected in series to a spring of constant 3 N/cm, as shown.



What force must be exerted at the end to extend the system by 1 cm from its relaxed length?

(A) 0.5 N **B** 2 N (C) 3 N (D) 6 N (E) 9 N

11. Water with total mass M is poured into a cup of cross-sectional area A. A block of mass m, whose density is half that of water, is tied to a thin string. The string is attached to the bottom of the cup, and the block floats in the water as shown. The atmospheric pressure is P_{atm}. What is the total pressure force that the water exerts on the bottom of the cup?

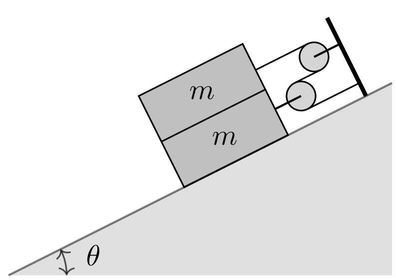


(A) Mg (B) $(M+m)g$ (C) $P_{\text{atm}}A + Mg$
(D) $P_{\text{atm}}A + (M+m)g$ **E** $P_{\text{atm}}A + (M+2m)g$

Consider force balance on the system of the block and water. The atmospheric pressure applies a downward force $P_{\text{atm}}A$, and gravity applies a downward force $(M+m)g$. Now, the buoyant force on the block is $2mg$ because water is twice as dense, which means the string applies a downward tension force mg to keep the block in place. To balance all these forces, the base of the cup must exert an upward force $P_{\text{atm}}A + (M+2m)g$.

There is also a shorter but trickier solution. The pressure force depends only on the height of the water, so it is unchanged if the block is replaced with water. Then the total mass of water is $M+2m$, so the upward force must be $P_{\text{atm}}A + (M+2m)g$.

12. Two identical bricks of mass m are attached to two frictionless pulleys by a massless string, on a fixed slope of angle θ to the horizontal, as shown.



Suppose that the coefficient of static friction between the lower block and the slope is μ, and all other surfaces are frictionless. What is the minimum value of μ so that the blocks can stay static?

(A) $\frac{1}{2}\tan\theta$ (B) $\frac{2}{3}\tan\theta$ (C) $\tan\theta$ (D) $\frac{3}{2}\tan\theta$ (E) $2\tan\theta$

Let the tension in the string that wraps around both pulleys be T. Applying force balance to the upper brick, we must have $T = mg\sin\theta$. The pulley applies a force $2T$ to the lower brick, so applying force balance gives

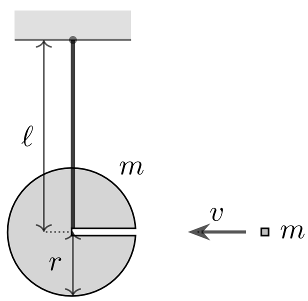
$$2T = mg\sin\theta + f_{\text{fric}}.$$

When the friction is just barely sufficient, we have

$$f_{\text{fric}} = \mu N = 2\mu mg\cos\theta$$

and combining these two equations gives $\mu = (\tan\theta)/2$.

13. A ballistic pendulum is designed as shown below.



A uniform ball of mass m and radius r is attached to a massless rigid rod, so that its center is a distance ℓ from the ceiling. The ball has a small tunnel hollowed out. A small block of mass m and speed v goes into the tunnel and collides at the center of the ball, perfectly inelastically. Subsequently, to what maximum height does the block rise?

(A) $\dfrac{v^2}{8g}$ (B) $\dfrac{v^2}{8g(1+r^2/5\ell^2)^2}$ (C) $\dfrac{v^2}{8g(1+2r^2/5\ell^2)^2}$

(D) $\dfrac{v^2}{8g(1+r^2/5\ell^2)}$ (E) $\dfrac{v^2}{8g(1+2r^2/5\ell^2)}$

We treat the impact by conserving angular momentum about the pivot, which is equal to $mv\ell$. The final moment of inertia about the pivot is

$$I = \frac{2}{5}mr^2 + 2m\ell^2.$$

To find the height of the highest point, we conserve energy,

$$2mgh = \frac{L^2}{2I} = \frac{m^2v^2\ell^2}{4m\ell^2 + \frac{4}{5}mr^2}.$$

Solving this for h gives

$$h = \frac{v^2}{8g}\frac{1}{1 + r^2/5\ell^2}.$$

Note that conserving linear momentum about the pivot would not give the correct answer, because the rigid rod provides a horizontal impulse during the collision.

14. A cylinder of water is suspended in a space station. Under the influence of surface tension, the cylinder splits into droplets. After a short time, viscosity causes the droplets to settle into static shapes. Neglect the effect of evaporation. Compared to the original cylinder, the final set of droplets will have

(A) Almost exactly the same volume and surface area

(B) More volume, but almost exactly the same surface area

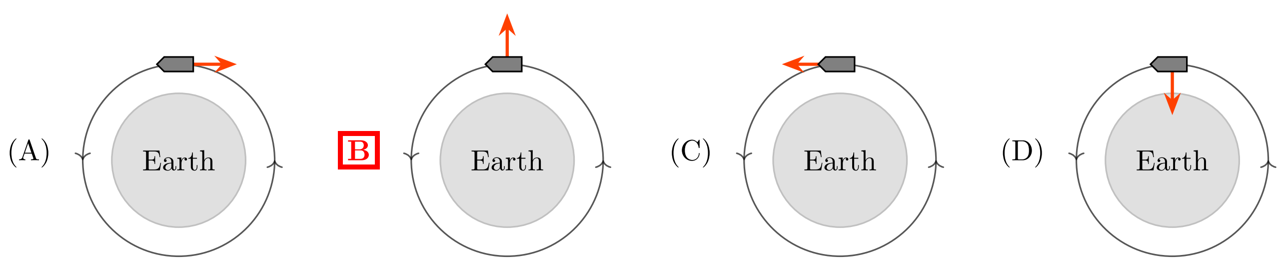
(C) Less volume, but almost exactly the same surface area

(D) More surface area, but almost exactly the same volume

E Less surface area, but almost exactly the same volume

Because water is almost incompressible, the volume stays almost exactly the same. During the process, energy is dissipated due to viscosity, and the energy due to surface tension is proportional to the total surface area. Thus, the surface area must decrease.

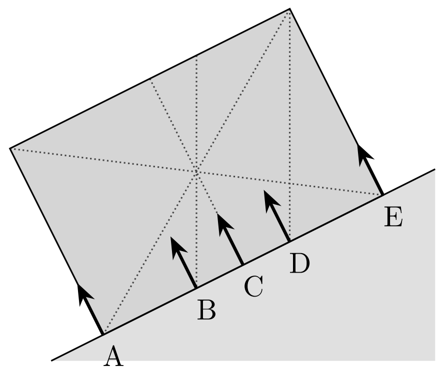
15. Five astronauts are orbiting Earth in a low circular orbit. They decided to go around the planet following the same orbit, but much faster, to set a new record for orbiting Earth. Four of them thought that they could quickly increase the velocity, then use a jet engine to maintain an orbit of the same shape; in the first four choices below, the red arrow denotes the direction that fuel will flow out of the jet nozzle. The fifth astronaut was skeptical about such a possibility. Who is right?



(E) It is impossible to follow the same orbit faster.

The centripetal acceleration required to follow a circular orbit is v^2/r. If the speed is increased, the inward acceleration must be increased, so the engine should be pointed outward to provide an inward force.

16. A uniform rectangular block is in static equilibrium on an inclined plane, and experiences gravity, static friction, and the normal force from the plane. Though gravity acts on the entire volume of the block, for the purposes of torque balance, it is equivalent to a single resultant force acting at the block's center of mass. Similarly, while the normal force acts on the entire bottom surface of the block, its torque is equivalent to a resultant force acting at a single point. Which of the following best shows this point?



(A) Point A, at the lowest point of the bottom side

B Point B, directly below the center of mass

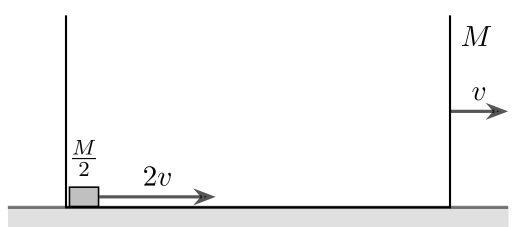
(C) Point C, at the midpoint of the bottom side

(D) Point D, directly below the top corner

(E) Point E, at the highest point of the bottom side

Consider the point of contact between the block and plane, directly below the center of mass. Gravity effectively acts at the center of mass, and thus exerts no torque about this point. The friction force points along the plane, and thus also exerts no torque about this point. Therefore, since the block is in static equilibrium, the net normal force must effectively be applied at this point, yielding choice (B).

17. A box of mass M is sliding to the right with velocity v on a frictionless table. A small puck of mass $M/2$ slides frictionlessly inside the box. Initially, the puck is at the left wall of the box, with a rightward velocity of $2v$ with respect to the table. After a time T, the puck collides elastically with the right wall of the box. How much longer will it take until the puck hits the left wall of the box again?

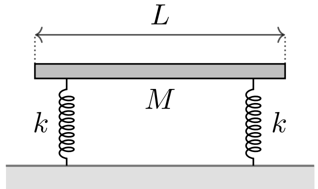


(A) $T/2$ **B** T (C) $2T$ (D) $3T$ (E) $4T$

As can be shown by working in the center of mass frame, during an elastic collision, the relative velocity of the two objects only changes in sign. Therefore, the relative speed remains the same, so the time until the next collision is also T.

18. **The following information applies to problems 18 and 19.**

A uniform rod of length L and mass M is placed with its ends resting on two identical springs of spring constant k, as shown.

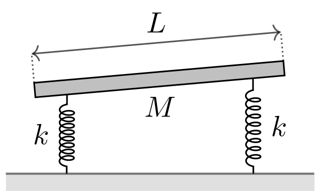


The rod is initially in equilibrium. If the rod is uniformly displaced downward and released from rest, what is the frequency f of its oscillations?

(A) $\frac{1}{2\pi}\sqrt{\frac{k}{4M}}$ (B) $\frac{1}{2\pi}\sqrt{\frac{k}{2M}}$ (C) $\frac{1}{2\pi}\sqrt{\frac{k}{M}}$ **D** $\frac{1}{2\pi}\sqrt{\frac{2k}{M}}$ (E) $\frac{1}{2\pi}\sqrt{\frac{4k}{M}}$

If the rod is displaced down by $\Delta\ell$, then the vertical net force it experiences is $2k\Delta\ell$. This is just like a spring-mass system where the spring constant is effectively $2k$, so the angular frequency is $\sqrt{2k/M}$.

19. Next, the rod is brought back to equilibrium. It is slightly rotated about its center of mass, then released from rest.



What is the frequency f of its oscillations?

(A) $\frac{1}{2\pi}\sqrt{\frac{k}{M}}$ (B) $\frac{1}{2\pi}\sqrt{\frac{2k}{M}}$ (C) $\frac{1}{2\pi}\sqrt{\frac{3k}{M}}$ (D) $\frac{1}{2\pi}\sqrt{\frac{4k}{M}}$ **E** $\frac{1}{2\pi}\sqrt{\frac{6k}{M}}$

If the rod is rotated by a small angle θ, the net torque on the rod is $\tau = 2(L/2)(kL\theta/2) = kL^2\theta/2$. The rotational form of Newton's second law, $\tau = I\alpha$, gives

$$\frac{1}{12}ML^2\alpha = -\frac{kL^2}{2}\theta$$

or equivalently $\alpha = -(6k/M)\theta$, which has the form of a simple harmonic motion equation with $\omega = \sqrt{6k/M}$.

20. A uniform, hollow sphere is initially at rest on a horizontal plane. The plane begins to accelerate horizontally to the right, with acceleration a. If friction is high enough to prevent the sphere from slipping, what is its translational acceleration?

(A) $2a/3$ to the left (B) $a/3$ to the left (C) zero
D $2a/5$ to the right (E) $2a/3$ to the right

It's easiest to begin by working in the noninertial frame of the plane. In this frame, there is a fictitious force Ma on the sphere, directed to the left, effectively acting at its center. Taking torques about the contact point with the ground, the angular acceleration of the sphere is $\alpha = 3a/5R$, so the acceleration of the center of mass is $\alpha R = 3a/5$ to the left. Transforming back to the original frame, the acceleration of the sphere is $2a/5$ to the right.

21. Amora and Bronko are given a long, thin rectangle of sheet metal. (It has been machined very precisely, so they can assume it is perfectly rectangular.) Using calipers, Amora measures the width of the rectangle as 1 cm with 1% uncertainty. Using a tape measure, Bronko independently measures its length as 100 cm with 0.1% uncertainty. Which of the following are closest to the relative uncertainties they should report for the area and the perimeter of the rectangle, respectively?

(A) 0.1%; 0.2% **B** 1%; 0.1% (C) 1%; 0.2% (D) 1%; 1% (E) 1%; 2%

Calculating the area involves multiplying the two measurements, and so the uncertainty in the area depends on the relative uncertainties of the inputs. Amora's 1% relative uncertainty dominates Bronko's 0.1%, so the result carries her 1% uncertainty. (It remains 1% because the area depends on each input to the first power.)

Calculating the perimeter involves adding the two measurements, and so the uncertainty in the perimeter depends on the absolute uncertainties of the inputs. Bronko's 0.1 cm absolute uncertainty dominates Amora's 0.01 cm, so the result carries his 0.1% uncertainty. (Note that while the perimeter is approximately twice Bronko's measurement, the *relative* uncertainty is unaffected by this observation, so the answer is not 0.2%.)

22. A cannon just above the surface of a spherical planet with mass M and radius R launches a particle with speed v, where $\sqrt{GM/R} < v < \sqrt{2GM/R}$. The initial velocity of the particle makes an angle θ with the vertical direction. Neglect drag. For what θ will the particle never collide with the planet?

(A) All possible launch angles, $0^\circ \leq \theta \leq 90^\circ$

(B) $\cos^{-1}\left(\sqrt{\frac{v^2R}{GM} - 1}\right) \leq \theta \leq 90^\circ$

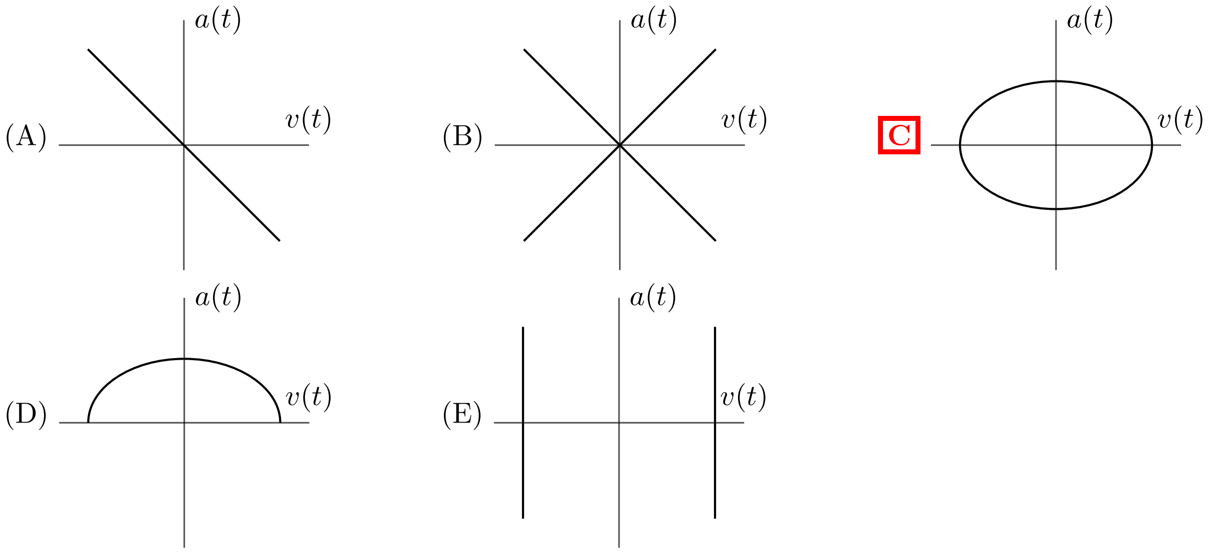
(C) $\sin^{-1}\left(\sqrt{\frac{v^2R}{GM} - 1}\right) \leq \theta \leq 90^\circ$

D Only $\theta = 90^\circ$

(E) A collision will occur for any value of θ

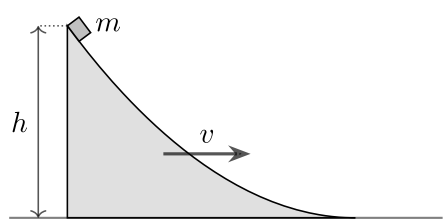
The launch speed is above the circular orbit speed $\sqrt{GM/R}$, so a launch angle of exactly 90° will result in an orbit that doesn't collide with the planet; the orbit is an ellipse tangent to the planet's surface at the launch point. On the other hand, the launch speed is below the escape speed $\sqrt{2GM/R}$, which implies, by Kepler's first law, that the orbit is a closed ellipse. Therefore, if the launch angle was anything besides 90°, the orbit would have to intersect the planet. Therefore, the answer is (D).

23. A mass attached to a spring is performing simple harmonic motion, with velocity $v(t)$ and acceleration $a(t)$. Which of the following could be a graph of the curve $(v(t), a(t))$ over a complete oscillation?



By conservation of energy, $E = kx^2/2 + mv^2/2$. In addition, the acceleration is $a = -kx/m$, which implies that $E = k(ma/k)^2/2 + mv^2/2$. Therefore, when a is plotted against v, the result is an ellipse. All combinations of signs of a and v are possible, so the ellipse occupies all four quadrants, giving choice (B).

24. A ramp with height h is moving with fixed, uniform speed v to the right. A small block of mass m is placed at the top of the ramp, and is released at rest with respect to the ramp.



The block slides smoothly to the bottom of the ramp and onto the floor. How much kinetic energy does it gain in this process? Neglect friction.

(A) mgh (B) $mgh + mv^2/2$ **[C]** $mgh + mv\sqrt{2gh}$
(D) $mgh + mv\sqrt{gh} + mv^2/2$ (E) $mgh + mv\sqrt{2gh} + mv^2$

In the frame of reference of the ramp, the block starts at rest, and ends up with horizontal velocity $\sqrt{2gh}$ by energy conservation. Thus, in the original frame, it has velocity $v + \sqrt{2gh}$, which means

$$\Delta K = \frac{1}{2}m((v + \sqrt{2gh})^2 - v^2) = mgh + mv\sqrt{2gh}.$$

In particular, mgh is not the correct answer because the horizontal motion of the ramp also does work on the block as it slides.

25. Two masses are initially at rest, separated by a distance r, and attract each other gravitationally. If their masses are m and $2m$, then they will collide after a time T. How long would they take to collide if they both had mass $2m$?

(A) $\left(\frac{2}{3}\right)^{3/2} T$ (B) $\frac{3}{4}T$ (C) $\sqrt{\frac{2}{3}}T$ **[D]** $\sqrt{\frac{3}{4}}T$ (E) $\sqrt{\frac{8}{9}}T$

Compare the relative speed of the masses when they are a distance $r' < r$ apart. In the first case, if the mass $2m$ has speed v_1, then the mass m has speed $2v_1$ by momentum conservation. By energy conservation,

$$\frac{1}{2}m(2v_1)^2 + \frac{1}{2}(2m)v_1^2 = 2Gm^2\left(\frac{1}{r} - \frac{1}{r'}\right).$$

Therefore, the relative speed is

$$3v_1 = \sqrt{6Gm(1/r - 1/r')}.$$

In the second case, both the masses have the same speed v_2, where

$$\frac{1}{2}(2m)v_2^2 + \frac{1}{2}(2m)v_2^2 = 4Gm^2\left(\frac{1}{r} - \frac{1}{r'}\right).$$

Therefore, the relative speed is

$$2v_2 = \sqrt{8Gm(1/r - 1/r')}.$$

In the second case, the relative speed is always $\sqrt{4/3}$ times higher than in the first case, when the masses have the same separation r'. Therefore, the time to collision is $T\sqrt{3/4}$.